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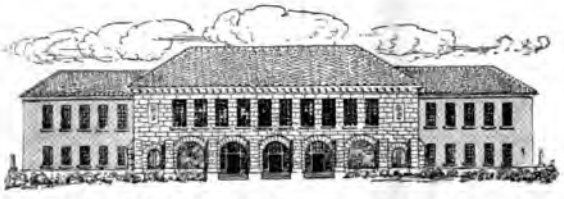

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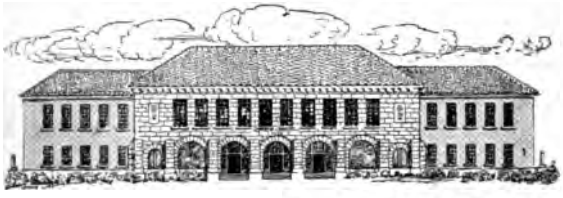

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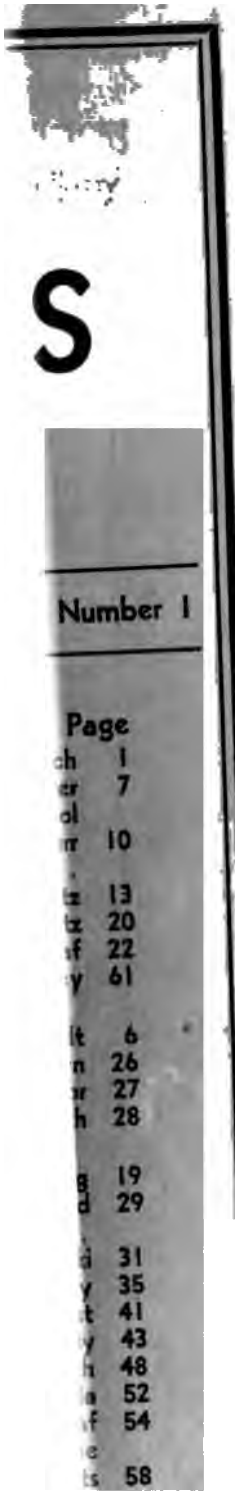

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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XLIV Number 1

Importance of Mathematics in General Education

By E. R. Breslich
The University of Chicago, Chicago, Illinois

Mathematics, like any other subject, is important to the extent to which it supports and contributes to the purposes of general education. For children are sent to school in order that they become useful members of the community and contribute to the welfare of society. The mathematics referred to in this paper designates that subject matter and training which is important to the entire school population, as distinguished from the special mathematics needed for professional and technical education. It is the mathematics which aids in the preparation of pupils for efficient participation in the activities in which they engage later as adults, and for the assumption of their share of social responsibility.

From the beginning of the American high school to the present time, the teachers of mathematics have insisted that the learner may be benefited greatly by the study of their subject. However, the pupil population of the early high school differed from the present day one in that its members belonged to a rather small and select group. The boys came to school to prepare themselves for leadership in the community. They planned to be ministers, doctors or engineers, which naturally made mathematics an important part of their education. The girls, not to be outdone, preferred the same kind of education. Pupils took school work seriously. Neither pupils nor parents were critical about the courses prepared and recommended by the teachers.

With the growth of the schools, the high school population kept on increasing and toward the end of the nineteenth century approximately half a million pupils were enrolled. Now they formed no longer a select group, and differed widely in interest, industry, and ability. Many "took" mathematics who were not interested and considered it a useless study. Many others were not able to do the work prescribed in the mathematical courses. This part of the school population kept on increasing and created serious problems for the teachers and administrators. The question was raised whether schools should insist that these pupils take the mathematics presented in the courses in algebra and demonstrative geometry. If not, the teachers faced the problem of formulating a more suitable curriculum, one more attractive and more profitable.

The mathematics to be required of all pupils. During the past thirty years, a great deal of work has been done to solve that problem, both by individuals and by

large committees. The aim has been to develop a new course intended to satisfy the needs of those not taking the traditional sequence of algebra and geometry. As far back as in 1923 the National Committee on Mathematical Requirements in recommending such a course says: "The work of the ninth grade should be so planned as to give the best mathematical training which the pupils are capable of receiving in this year and which will be most useful to them with little reference to future courses which they may or may not take." It should be noted that this is not a course for a special group, like "vocational" mathematics, but one containing the mathematics important in the education of all pupils.

The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics of 1940 made a study of the problem and reached the conclusion that mathematics is important in general education because the average citizen of today needs considerable mathematical knowledge in the activities and experiences of every day life, and because mathematics supplies a means of understanding important aspects of the world. A similar view on the importance of mathematics in general education is expressed by the Commission on Post-War Plans in 1944 and 1945. It recommends a course that develops "mathematical competence" for the ordinary affairs of life as a part of general education, appropriate to the major fraction of the high school population.

As a matter of fact, the citizen of today is dependent on mathematics. He must balance his budget, adjust his expenses to his income, and compute his taxes. As a member of a community or of a state he must participate in community life and should interest himself in the conduct of its business and in its public enterprises. His ability to understand them depends on his mathematical education.

One step in the problem of developing a course containing the mathematics every person should know is to determine the mathematical needs of people in their lives at home and in their every day activities. Studies of this type have taken much time and labor of individuals and committees. They have disclosed considerable agreement. A list of the proposals common to the reports of the National Committee of 1923, the Joint Commission of 1940, the Commission on Post-War Plans of 1944 and 1945, the Committee of Pre-induction Courses of 1943, and the Committee on Essential Mathematics for Minimum Army and Navy Needs of 1943 provides a good outline of the minimum knowledge of mathematics so important to general education that it should be required of all pupils. Such a list contains the following five mathematical requirements:

1. Arithmetical deficiencies of individuals and groups should be removed before the pupil graduates from the high school. This may be accomplished by relating arithmetic closely to algebra and geometry. Such topics as the following should be given special attention: proficiency in the fundamental operations, ratio and proportion, measurement, percentage, tables and verbal problems.

2. An educated person should be familiar with the important properties of the common geometric forms and with indirect measurement.

3. A limited amount of algebra should be taught, including the basic concepts, fundamental operations and simple equations and formulas. Further training in solving problems is essential.

4. There should be instruction in the graphical techniques.

5. Some work in statistics and in numerical trigonometry should be offered.

This is a very conservative program. It comprises the work commonly outlined for junior high school pupils. However, the average citizen of today needs a wider knowledge of mathematics and any school offering such a program should make it possible for those who succeed in the minimum course and who enjoy mathe-

matics to continue further study in the traditional sequence of courses.

The values to be derived from the study of mathematics. In selecting the content of the minimum requirements the committees have taken the "practical" point of view: mathematics is important in general education because people can use it advantageously in modern life; because the newspapers and magazines of today are filled with discussions of a quantitative nature which an intelligent reader is expected to comprehend, but which he fails to appreciate unless he has had the necessary mathematical training; and because people need to interpret complicated numerical data, to understand quantitative studies of social phenomena and to recognize fallacies in conclusions. Hence, high school education should provide pupils with a knowledge of the mathematical concepts, principles and processes which they will need as adults to understand and often to solve problems in which quantitative data and relationships are involved. Many writers and school administrators have rated the utilitarian values as the most important in general education. Indeed, some regard them as the only worth-while values. They would limit mathematical instruction entirely to the practical functions, making it no more than a "tool" subject.

While it is true that the utilitarian values make mathematics indispensable in modern life, the fact should not be overlooked that the study of that subject offers much that contributes to the enjoyment of the learner. Abstract mathematics alone does not hold the interest of many pupils very long, but they will also tire of practical mathematics and applications. This was recognized by the Educational Policy Commission of 1938 when it speaks of the "need of developing an appreciation of the 'cultural values' of mathematics." The National Committee of 1923 goes one step farther and submits a list of the cultural aims including: appreciation of beauty in geometric forms of nature, art and industry; ideals of perfection as to logical structure; precision of statement and thought and logical reasoning; appreciation of the power of mathematics; the role mathematics has played in the development of civilization, in particular in the sciences, industry and philosophy. The parent and the school advisor who tell Mary that there is no sense in her taking high school mathematics for the simple reason that she will probably never use it should give serious thought to her loss of the cultural values. They are depriving her of an enjoyment to which she is entitled, the loss of which she will feel deeply in later years, and for which she will blame them most heartily. Persons with no mathematical training will never reach full appreciation and enjoyment of the beauties of the forms in nature, art and architecture, for they have no knowledge of geometric laws and relationships and cannot recognize even the basic forms. They will always remain curious and mystified by such phenomena as the eclipse, the phases of the moon, and the movements of the planets and stars. They fail to understand our own solar system, to say nothing about the universe. To most people the history of the race is fascinating. Mathematics is closely related to the development of civilization and can never be separated from it. Often civilization stands still waiting for mathematics to advance until it can show the way to proceed. Unlike other innovations, mathematics is not the product of one or a few geniuses. It is the outcome of the labor of many men and women and comes from all parts of the world, but always its contributions have aided the progress of the race.

Teachers have been so busy developing skills and understandings for practical uses that the cultural values have not received the attention they deserve. Fortunately, there is available considerable reading material which pupils may read and enjoy. Such pamphlets as *The Story of Numbers* (American Council of Education) are helpful, and books like the following should be on the shelves of every

high school library: Dantzig's *Number, The Language of Science,* Hogben's *Mathematics for the Million* and Bell's *Men of Mathematics.*

In the early high schools much importance was attached to the "disciplinary" values of all subjects. Thus, it was assumed that the mental training derived from mathematics carried over automatically to other situations, even non-mathematical situations. It was believed that the individual who acquires skill in reasoning in mathematics would be able to reason in any other subject as well; the training in memory obtained in mathematics would be helpful in any situation where memory is used. The development of educational psychology in the early decades of the present century led to many experiments to determine the amount of transfer of training in one subject to other situations. The results were disturbing. It was found that transfer was not nearly as great as had been supposed. The Committee of Ten of 1893 did not yet question the disciplinary values of mathematics and the College Entrance Board a few years later still continued to accept this point of view, but educators began to interpret the small amount of transfer to mean that the idea of transfer may be disregarded as if it did not exist, and that it could be written off as a value of the study of mathematics. Accordingly, the leaders in the reform movements of mathematics which began at the turn of the century kept quiet about the disciplinary values and turned their attention to the utilitarian and cultural aspects of the subject. The reforms advocated by Perry, Klein, Borel and Moore were concerned largely with the selection, organization and presentation of instructional materials.

Later experiments with transfer established the fact that the amount of training depended on certain conditions such as the methods of teaching that were being used, and that under proper conditions mental training in one field could transfer to another. With this came a change in the attitude of educators and psychologists. It is significant that the National Committee of 1923 again classifies the aims of mathematical instruction as utilitarian, cultural and disciplinary, and that it lists as the primary purposes of the teaching of mathematics: powers of understanding and of analyzing quantitative and spatial relationships; habits of thought and of action; insight into and control over environment, all of which are disciplinary aims.

If transfer exists the study of mathematics deserves indeed a very important place in education. Since it is a major function of the school to train pupils in exact thinking the entire school program should contribute to the improvement of the quality of the pupils' thinking. All subjects should provide such opportunities but no subject equals mathematics in developing the power of correct thinking. "Correctness of thinking," says Hutchins, "may be more directly taught through mathematics than in any other way." Mathematics is a way of thinking and can be taught as such. It continually exhibits the processes of thinking in correct simple form, and frowns upon hit or miss methods. It stresses thinking in terms of relationships that exist between facts. The technique of thinking which it uses is the same as that employed in the experiences and relations involved in the social and economic problems encountered by adults in everyday life.

Mathematics lends itself readily to training in reasoning. In daily life, situations constantly arise in which people must proceed with caution and must make careful use of the principles and processes of correct reasoning. They must be able to separate facts from assumptions and to draw correct inferences from them. The reasoning so characteristic of mathematics is essentially the type required in non-mathematical situations where conclusions are to be drawn from given assumptions. Mathematics trains in the processes of drawing correct conclusions. This gives it the right for an im-

portant place in the curriculum of the schools.

However, the teachers should not forget that there is an important "if" attached to the foregoing statements, because transfer does not take place "automatically." Forty years ago Professor Snedden of Columbia University said: "It is useless for teachers to go around talking about the values of mathematics. They exist if and only if they are attained by the pupils." Transfer depends on the way mathematics is taught. "The teacher," says Hedrick, "must teach the pupils how to transfer his training to other and different fields." "A subject must not merely teach particular items," says Judd, "but must stimulate generalization." Thus, there is little training value in having pupils reduce fractions in arithmetic to a simpler form unless they are led to understand the principles underlying this process. Most of the errors and confusion in this particular operation are due to the fact that pupils do not understand the difference between factor and term; have never understood that when a factor of a given number is divided by a second number the given number is divided by the second number; and that when numerator and denominator of a fraction are divided by the same number the value of the fraction is not changed. When the mature mathematician "cancels," he is aware of these principles, but to the pupil not properly taught, cancellation means little more than the crossing out of a number.

In algebra, ability to solve problems does not come automatically from the teaching of specific problems, however many there may be, but from teaching the methods of solving problems through the use of a large variety and number of problems. There is a vast difference between teaching problems and teaching problem solving.

In many schools demonstrative geometry is still taught by the following procedure: first, the pupils study the proofs; then the proofs are memorized; third, the pupils recite the proofs in class; lastly, the pupils are tested by test items calling for the reproduction of proofs. This cannot develop reasoning power. To be sure, memory must always play a part in studying, but emphasis in geometry must be on training in the processes of correct reasoning. Moreover, this must be done in all mathematical situations, and in numerous non-mathematical situations. Since transfer according to Judd is a form of generalization the training must be such as to encourage generalization. That places a grave responsibility on the teacher. He will have to learn how this may be done. Helpful discussions are found in the yearbooks and the journals of mathematics. An excellent piece of work is that of Fawcett in his investigation on "The Nature of Proof" published in the *Thirteenth Yearbook* in 1938. The report of the Progressive Education Committee on the *Functions of Mathematics in General Education* published in 1940 will be most helpful to those who wish to learn more about developing problem-solving ability. The techniques of solving problems, quantitative and mathematical or non-mathematical, encountered by people in every day life are much the same. They call for analyses and careful plans of procedure. Training in attacking and solving problems will be helpful to pupils in and out of school and later in adult life.

For many pupils the power of clear thinking, logical reasoning and problem solving will be of far greater value than a meager knowledge of mathematical facts. The development of these powers makes mathematics more important in general education then the subject itself. "Divested of much of its conventional content and formal organization," says the Commission on the Function of Mathematics in General Education, "mathematics as a mode of thought and an instrument of analysis has an indispensable function in education."

Professor Judd once told a story of a meeting of a commission on teacher train-

ing. The question came up as to the characteristics of a good teacher. One member of the committee suggested that each person present should recall to his mind the best high school teacher he had as a pupil and then determine why he considered him the best. The interesting fact came out that most members named the teacher of geometry, and the reasons were: he taught us how to think; he taught us how to attack problems; he taught us how to reason; etc., all disciplinary values. None mentioned content or even practical applications.

There are other disciplinary values which make mathematics important in education, e.g., effective habits of thought and action; the habit of attacking problems with confidence and assurance; the habit of overcoming obstacles rather than of avoiding them; the habit of carrying tasks to completion; habits of neatness, accuracy, orderliness, attention to detail, observation, reflection, concentration, and constructive imagination, and not the least important, the habit of correct study. All these are important in education. They were always supposed to be by-products of mathematical training and the idea that teachers should and could train pupils is still foreign to many. It is all very well to select better subject matter, to organize it better and to improve presentation, but this is not sufficient. A large per cent of the difficulty with mathematics arises from poor attitudes toward school work. When the teacher reports that John has the ability to do the work but fails because of lack of effort the solution lies in doing something that changes his attitude by arousing genuine interest.

It has been attempted to show that mathematics is worthy of an important place in education. The reason lies in its utilitarian, cultural and disciplinary values. The last two can be attained through improved methods of teaching and do not come automatically from the study of the subject. They must be planned for in the daily work of the teacher. After he has made his plan for the topic to be taught, he must still find a definite answer to the question: where in this plan are the opportunities for deriving the cultural and disciplinary values, and by what method may they be attained by the pupil? Systematic attention to these questions brings surprising results and will hasten the time when the importance of mathematics in general education will not be questioned.

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The Rate of Progress of Pupils in Arithmetic in the Elementary School

By C. C. GROVER
Assistant Superintendent of Schools, Oakland, California

THE doctrine of individual differences is now generally accepted by educators and its implications are considered in the development of instructional programs in most schools. This doctrine assumes that there is wide variation in abilities, needs, and interests among pupils; that these differences become greater as pupils progress through the grades; and that such differences in ability and interests necessarily result in wide variations in achievement in the various school subjects.

Undoubtedly the most difficult and perplexing problem a teacher faces in a modern elementary school is the adjusting of instruction to pupils differing in the characteristics mentioned above. This problem is present in the teaching of reading, spelling, art, music; in learning to play baseball, in dancing, and, in fact, in every task undertaken in the public schools. The necessary adjustment is particularly difficult in the teaching of arithmetic. It is believed by some psychologists that facility with numerical relationships is a special ability and that satisfactory achievement in mathematics is not only dependent upon general intelligence and previous preparation, but upon this special ability as well. Be that as it may, certainly individual pupils in an arithmetic class differ in intelligence, in the mastery and understanding of number operations, in interest in the subject, in rate of learning, in background of experience, and in many other ways.

Many educators believe that individualizing instruction provides the best solution to this problem. Perhaps, ideally, it does. Such a method of organization and instruction, however, presents many difficulties for most schools. There are many reasons for this. Most of these difficulties depend ultimately upon the cost factor. Individual instruction, to be effective, requires small classes, efficient clerical service, and ample supplies. Since these conditions cannot be met in the usual public school system, other methods of providing for individual differences must be developed. The plan most frequently used is the organization of instructional groups of pupils within the classroom. Under this plan, each instructional group is taught as the abilities and needs of its members seem to require. This is not an unsatisfactory method of teaching. Even with small classes, many teachers prefer it to the individual method of instruction.

Assuming this type of organization, an important question regarding the pupil's rate of progress must be answered—If he is not able to attain the goals for his grade shall he have instruction adjusted to his needs and abilities? On the other hand, if he is above average in his ability to do the work assigned to his grade, shall he be asked to "mark time"? A pupil in the fifth grade, for example, whose ability and previous preparation indicates fourth grade work, is given material at that level; a pupil in the same class, who by the same reasoning should be doing work assigned to the sixth grade, continues to do the work of his class, i.e., the fifth grade. This latter answer, which seems to be the more commonly accepted one, does not seem to be quite fair to the superior student in mathematics.

The reason for this situation is not difficult to find. Teachers and administrators are fearful that pupils leaving a class and teacher where they have been permitted to advance in the textbook or

course of study at varying rates and going to a new class will have such a wide range in achievement as to make instruction for the next teacher very difficult. These difficulties, it is said, will be aggravated when the pupils for the new class come from two or more teachers or when they leave one division of the school system and go into another, e.g., from the elementary school into the junior high school.

There are, of course, elements of truth in this position. It is always easier to assume that all pupils in a given grade or class are equal in ability and achievement than it is to recognize that pupils differ and to provide for these differences. There are some reasonable adjustments, however, which will minimize the difficulties of this wide spread in achievement.

One such adjustment is in the efficient use of the educational materials provided by the school for use in the grades in question. The school can develop an organization so that these materials of instruction will be sent where they are needed and not arbitrarily assigned to a grade or a room. For example, the fourth grade teacher could reasonably expect to have assigned to her room: textbooks used in the third, fourth and fifth grades; flash cards, practice sheets, and drill materials used in these grades; concrete materials, measuring instruments, reference books, and illustrative materials usually assigned to third, fourth, and fifth grade classrooms.

Also the instructional groups within the room can be reduced in number. It is not contemplated that there be eight or ten small groups of pupils in one class working at various places in the curriculum; but rather that there be two or at most three instructional groups within a given room. Under such an arrangement, a group of superior pupils in the fourth grade might have assigned to them the work in the arithmetic curriculum usually given to the fifth grade; if there were only one or two such pupils in the class they would not be given this assignment but their needs would be met in some other manner.

Enrichment? Certainly, but who is to furnish it? To provide enrichment in arithmetic is a much more difficult and time-consuming task than to enrich the language arts curriculum. Materials are available, it is true, but they are much more difficult to locate; they have not been organized for use in a classroom; and the manner in which such materials may be used profitably is not clearly understood. The preparation of elementary school teachers is much less adequate in mathematics than it is in English or social studies. Fewer teachers are intensely interested in mathematics than are interested in the language arts. For these and for other reasons not enumerated, a fair appraisal of the situation leads to the conclusion that in the average elementary school classroom at the present time the teacher is not able to provide an enriched program for superior pupils in arithmetic.

Let it be said immediately that there is hope for the future. Our better teacher-training institutions are aware of the inadequacy of their programs for the training of their students for the teaching of arithmetic and are making plans to improve this condition. As the culture in which we live makes greater demands upon citizens in the understanding of numerical relationships, those responsible for planning education of youth are recognizing the need for a different kind of teaching in the field of mathematics in both the elementary and the secondary schools and much better results have been secured in recent years. It is not unreasonable to expect that the next decade will see even greater improvement in the training and preparation of young teachers than has the last one.

Are we then to cease to attempt to enrich the program in arithmetic? Certainly not. We should put more emphasis upon it. There are many kinds and many levels of enrichment. Superior teaching in one classroom may be enrichment in

other room; enrichment today becomes rely good teaching tomorrow. The posi- n being taken here is not concerned h the amount or desirability of enrich- nt. That enrichment is desirable is ten for granted. That it constitutes a isfactory and adequate provision for ted pupils is another matter and the inion expressed here is that it does not so. The problem of enriching the teach- ; of mathematics in the elementary ool deserves more extended treatment n can be given here, but a brief dis- sion of it will be given. At the outset night be well to consider some common ctices which are not enrichment. The lowing, in the opinion of the author, finitely may not be classed as in that tegory:

1. Assigning to superior pupils a larger number of the same kind of examples or problems given to the entire class as a regular lesson.
2. Assigning to superior pupils more examples or problems of the same general type but of somewhat greater difficulty than those given to the entire class in the regular lesson.
3. The practice of assigning superior pupils to assist the less able pupils in regular class work.
4. The preparation of original examples or problems similar to those used in daily assignments and based upon the class text or course of study.

It is believed that true enrichment of athematics may be found in the follow- g general areas:

1. Greater attention to the understanding of the number system—its origin, development, internal consistency, advantages, etc.
2. Increased emphasis upon the applications of mathematics to problems of society, e.g., relation of assessed valuation to tax rate.
3. Greater facility with mental manipulation of number by means of "short cuts" and other such devices, e.g., use of aliquot parts of the dollar, multiplication of one two-digit number by another two-digit number, checking multiplication and division by "casting out nines," etc.
4. Group projects involving numerical relationships, such as, scale drawing of floor plan of a building in which group is interested and consideration of cost of its construction.
5. Group or individual study and report on such topics as the calendar, origin of various weights and measures, the international date line, etc.
6. Judicious use of games, puzzles, odd and unusual facts, magic squares, and the like, simply for fun. The use of this material in its place and at the proper time will rejuvenate the most moribund class.

In summary, it may be said that differences in ability and achievement are always found among pupils in any class. A skillful teacher will take pupils where he finds them, divide his class into instructional groups as needed, and teach them as seems best in the particular situation. Allowing the better pupils to progress more rapidly through the organized curriculum in arithmetic is a reasonable procedure. There are many sound reasons for such a plan and there seems to be no valid objection to it. Without laboring the point, it seems fair to say that in the average elementary school classroom there is no other adequate method of dealing with the problem of individual differences. With a three to four year range of achievement commonly found in an upper elementary grade classroom, why have instructional groups within the classroom, if the upper group proceeds at the same rate as the middle group? Certainly pupils with superior ability in mathematics profit greatly by the opportunity to progress more rapidly.

What Principles and Applications of Statistics Should Be Taught in High School and Junior College?

By IRVING W. BURR
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1. INTRODUCTION

A WORD or two on the nature and importance of statistics might not be amiss. A series of numerical results which vary from measurement to measurement, person to person, place to place, time to time, or object to object are almost certain to be affected and to vary, at least partly, because of random causes or "chance." Such data, whether we like it or not, are statistical data. In analyzing such statistical data and making decisions from them, it is only natural that one should use methods primarily designed for such data, namely, statistical methods. Such methods can minimize the chances of wrong decisions and even estimate the probability of error, and in addition can enable one to learn things from statistical data which would otherwise be missed altogether. Furthermore they can facilitate the design of experiments from which a maximum of information may accrue. Limitations of space preclude the giving of illustrations to show how ubiquitous and important chance variation is. Suffice it to say that statistical methods and thinking are essential to maximum success in fields such as the following: education and psychology, biology, economics, government, actuarial science, manufacturing, and all experimental physical science.

Although statistical background is important to the student, both for his vocation and in his function as a citizen in a democracy, statistics is unfortunately either absent or forms only a minute part of many curricula.

2. STATISTICS IN OUR TEACHING OF OTHER SUBJECTS

A student once said he liked mathematics because the answer was always perfectly definite. If such is at least partly true in conventional elementary mathematics it cannot be said of statistics. There you get an answer and some measure of its unreliability or "fuzziness." A graduate student, when asked in an oral examination why one might call statistics "fuzzy mathematics," in effect replied, "Because the thinking is unclear." He missed the whole point that so much of science is made up of not what is absolutely true, but only what is probably true with some kind of random error, and that fitted scientific laws are not pure geometrical curves but are bands of probability. Such a concept can help discourage some of the all too common tendency of people to think in absolutes and stereotypes.

The writer believes that it is possible to include some of the statistical viewpoints and even some statistical subject matter in the algebra and trigonometry courses. For one thing it would seem well to use more practical numbers in problems instead of such whole numbers and simplified data that some students immediately begin to look for the error if the answer shows any signs of being complicated. It is natural to have a diameter .321″ and not one quarter of an inch. The writer's plea is that sometimes the problems contain the kind of approximate and rounded off numbers one meets in practical life. Such numbers are statistical in that they involve a random error. There are many opportunities in trigonometry for bringing in the notion of approximate numbers and variation, which opportunities are apparently being increasingly seized upon. Probability chapters have long been in algebra books, and chapters on statistics

are being included sometimes. The main thing in such work is to use data that have meaning to the student such as those to be mentioned later on in this paper. Frequency tabulations; calculations with approximate numbers finding the maximum, minimum and assumed answer; scatter diagrams to study relationships and sampling experiments are examples of items which can be included in an algebra course with reasonable grace.

If statistical thinking and concepts can be introduced into ordinary mathematics courses it is even more true that statistics should be in other courses. For example, we may consider the strength of evidence in history or civics; national statistics and forecasting in economics; variation in laboratory results in chemistry, physics, and biology; probability in heredity; variation in shop performance; variation in test grades, placement scores, etc., and misuses of statistics may be discussed in any course, especially current events or English courses. But for teachers of other subjects to use statistics in the approach to their subject presupposes that they have some statistical background and use it in their work. This poses a gigantic educational problem which will take years to solve. The increasing amount of statistics being taken in colleges and graduate schools will gradually help. Incidentally a knowledge of statistics will help mathematics teachers by giving them a further example of the importance and practicality of mathematics to bring before their students.

3. Introductory Courses in Statistics, High School

It is possible to teach a statistics course to high school students, but it is difficult. If one spends much time on sampling statistics and little time on purely descriptive statistics, the course is likely to be much more lively. But this is difficult because of the early emphasis on descriptive statistics in most present-day texts. The writer does not know of any sufficiently elementary statistics text in which the emphasis is as just suggested.

Another point worth mentioning is that a good understanding of statistical reasoning makes considerable demands upon the maturity of the student. This need not be a crucial stumbling block if it is kept in mind. Every teacher wants to teach the maturer students, but who is going to mature them? It may be a bit like the story of the little red hen.

It is well also to keep in mind that the subject of statistics is most emphatically one of those subjects which can be as dry as dust or fascinating, depending upon how it is presented.

The writer here presents a list of examples and problem material of the kind which will help motivation.

a. Frequency distribution of arrival times of trains or numbers of letters in words for a page.
b. Examination grades, frequency distribution, was one test harder than another, correlation of test grades.
c. Analysis of placement scores, etc.
d. Proportion of children of each sex in families of class members, vs. 50-50.
e. Coin tossing experiments (can easily be overdone).
f. Probabilities in card games expected vs. observed results.
g. Distribution of birth dates through the year and the probability of a coincidence of birthdays.
h. Actual height vs. guessed height for all or some of class.
i. Frequency distribution of coin dates and distribution of sample means from such a ragged population.
j. Prediction of scores for next game of school and distribution vs. actual.
k. Why gambling does not pay, but without so much emphasis that it has the actual effect of popularizing gambling.
l. Ways of estimating attendance at games or population, area sampling.
m. Height vs. weight for class (anonymous data from sheets returned in class).
n. Analysis of laboratory data from a course in the school.
o. Each person make two points on a piece of paper at 3″ as closely as possible without a ruler. Pass in, repeat. Analyze results: correlation, average error, any bias or improvement, any "lone-wolf"?
p. Non-sense correlation, height vs. change in purse. Is there any relationship?
q. Graphs of cost of living, prices, production,

real value of money vs. history. Correlate promising cases.

r. Choose by lot five people in class, note heights, find mean. Repeat building a distribution of means. Compare with the true known mean and standard error of the mean.

s. Accuracy of measurement. Each count number of e's on a page in a reasonable amount of time. Tabulate results and compare with correct result. Curve shape? Repeat on another page. Significant improvement? Continue till learning curve tapers off.

t. Heights of basketball squad vs. school as a whole.

u. Weather data locally. Distribution of cloudiness. At end of a month have each member of class guess at the number of days having precipitation. Any bias? Variability of estimates.

v. Multiplication and division of approximate numbers: the assumed answer, maximum and minimum, number of places to retain, relative error, etc.

w. National polls and sampling.

x. Each member of a class measure three objects with a micrometer and analyze results. Any mistakes? What is the true dimension?

y. Criticism of questionnaires in school, advertisements.

z. Class keep on look-out for misuses of statistics everywhere.

aa. The importance of not thinking in stereotypes. Saying "He is an athlete," says practically nothing about him. The variation may be so enormous that an average or stereotype is meaningless.

bb. Statistical method and scientific method.

cc. Guess answers such as averages, standard deviations, correlation coefficients, significance, etc. in advance of calculation.

4. Basic Courses in Statistics, Junior College

The problem of statistics courses for the junior college is more pressing, because the demand is being more keenly felt there than in high schools. Fortunately the currently available text books are more suitable for such students, because they are more mature, have probably had more mathematics and have taken or are taking concurrently courses such as biology, economics, business and engineering wherein statistical methods can be used. Thus the illustrations of the instructor can be more meaningful.

It is just as important to provide motivation and practical applications as in high school. The subject and its importance must be continually sold to the student. The interest level can reach the heights or the depths, depending on the instructor.

It is my belief that some sort of statistical subject matter now is or will relatively soon be included in the curricula of junior colleges, whether directly in statistical courses or indirectly in other courses. Such an important tool for thought as statistical methods provide cannot be ignored.

5. A Suggested Outline of Topics

The succeeding outline of topics for the statistics course for high school or junior college is to be thought of as one person's idea of what would make a good course:

a. What is statistics? Importance, place in thought

b. Condensing data: frequency tabulation, and graphs. (Cumulative frequency graph later on and only if the need becomes apparent to the class)

c. Mathematical summarization
   i. Averages: mean, median, mode
   ii. Variability: range, standard deviation (mean deviation)

d. Sample vs. population
   i. Introduction by experiments
   ii. Distribution of sample statistics: mean, median, range, standard deviation, proportion
   iii. Tests for compatibility

e. A generalized frequency polygon, the normal curve

f. Testing a hypothesis

g. Significance of differences

h. Correlation: scatter diagram, fitted straight line, correlation coefficient

i. Time series by graphs

j. Review of misuses, statistical thinking and its relation with scientific method

k. Statistics as a field and as a tool

l. Any other topics which one can include in a one or two semester course.

Perhaps the fundamental thing is to bring the students to the point where they will never again look at a number with the same naive, trusting confidence they had as grade school children.

Mathematical Understandings and Judgments Retained by College Freshmen*

By BEN A. SUELTZ
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BACKGROUND OF THIS REPORT

SEVERAL years ago the writer undertook a comprehensive study of the measurement of mathematical understandings and judgments at the elementary and junior high school levels. Phases of this study have been described and results have previously been reported.[1] During the past two years the investigation has been carried into the college field. No attempt has been made to evaluate in terms of typical college-level materials. Rather, the materials developed for use in the junior high school grades have been applied to college students in order to determine what types of understandings and judgments have been retained or augmented since their initial period of study. A number of incidental studies such as the relationship of results of tests of understanding and judgment to success in college mathematics and to general college success have been made but the results of these will not be presented at this time. In this report, the implication for shifts in the curricula on the secondary level will be emphasized.

AIMS OF INSTRUCTION AND EVALUATION

Measurement and evaluation should be held subservient to the valid aims of mathematical instruction. The writer believes that, for the large majority of public school pupils, the major aim of instruction should be one of functional competence in terms of the social, economic, and cultural life of the individual. This in turn implies the abilities: (a) to sense a mathematical situation which may be encountered through various media, (b) to understand the essential mathematics of the situation, (c) to be able to do the necessary thinking and computing to reach a conclusion, (d) and finally the will or feeling of compulsion to act in terms of the mathematics of the situation. This may appear to be a somewhat narrower view of functional competence than that stated by Betz[2] but such is not necessarily the case. It is consistent with the theses of the Commission on Post-War Plans of the National Council.[3]

Measurement in mathematics has tended to be confined to skills and computations and the applications of these skills in written problems. But even the casual observer knows that the mathematics of the real world is very much broader and that it is encountered in many different ways. The exercises used as a basis of this report admittedly have the limitations of removal from the real world because they necessarily have been presented by the "paper and pencil" technique. However, they attempt to place mathematical ideas in functional settings and thereby conjure a real situation in the student's mind.

COLLEGE STUDENTS

The students used in this report are enrolled in a state teachers college. They are average college students; their mean intelligence as measured by the American

* Presented at the Twenty-Seventh Annual Meeting of the National Council of Teachers of Mathematics, Baltimore, April 2, 1949.

[1] See: *The Forty-fifth Yearbook*, Part I, of the National Society for the Study of Education, 1946; *The Elementary School Journal*, February, 1947; and THE MATHEMATICS TEACHER, October, 1947.

[2] Betz, William, Functional Competence in Mathematics—Its Meaning and Its Attainment. THE MATHEMATICS TEACHER, May, 1948.

[3] The Second Report of the Commission on Post-War Plans, THE MATHEMATICS TEACHER, May, 1945.

Council Psychological Examination is slightly above the national average for college freshmen and the standard deviation of their scores is about three-fourths of the national group. Practically all of them have studied at least one year of mathematics beyond grade eight and the large majority have studied algebra and geometry in the ninth and tenth grades. A small minority have had at least three years of mathematics in high school. More than ninety per cent of them are from New York State. The test of Mathematical Understanding was taken by 377 college freshmen and the test of Mathematical Judgment was taken by 323 freshmen and sophomores who were beginning mathematics in college. The percentages shown with the sample exercises are the per cents of correct responses. The exercises selected for discussion were chosen for the purpose of illustrating certain generalizations that are apparent from the tests as a whole.

Mathematical Understanding

Mathematical understanding, as used in this investigation, includes the understanding of concepts, principles, and relationships inherent in a program of meaningful mathematics in the elementary and secondary schools. Not only have the results on tests been considered but many pupils have been interviewed immediately after taking the tests in order to determine if the items were valid for a particular age or grade. The test of understanding contained 53 items. Representative sample items are reproduced on page 15 so that the nature and scope of the test may be judged.

The response to exercise No. 1 should be compared to that on a similar item where students were asked to identify which one of four Roman numeral numbers was incorrectly written and in which only 19% were able to pick the correct answer. We ask whether or not intelligent adults should be able to comprehend these numbers and if so, what would constitute *a reasonable per cent*? Exercises 2 and 3 deal with fractions. Interviews with students revealed that many of those who chose the wrong answer for No. 2 were timid about the fraction $2\frac{1}{2}/6\frac{1}{2}$. They had never considered that such a fraction might exist and those who accepted it as correct could not see that it equaled 5/13. One might question whether $\frac{1}{8}$ quart (Ex. No. 3), as a bottle, is a real situation. I am told by some students that this is now called $\frac{1}{8}$ gallon. However, this involves a type of mathematical understanding which is not easily comprehended even by high school graduates.

Note the per cents correct for items 4 and 5. They show very low understanding. These exercises are beyond the usual limits of instruction in the elementary school. They are not taught in the typical high school. But they are real in the world of functional uses of percentage. Why doesn't the high school assume responsibility for such learnings?

Item No. 6 is fairly well understood. This is something that most junior high schools teach. Another similar item dealing with the size of arc for a constant angle was not well mastered. Only one student in the more than 90 interviewed had ever seen a cubic foot such as that illustrated in item No. 7! Yet the concept of cubic foot is a very useful functional item. How do schools develop an understanding of cubic foot?

Items Nos. 8, 9, and 10 appear on a part of the test in which each item may have one or more correct answers. These items were scored "number right minus number wrong." That is, if a person chose one wrong answer and one right answer his score was zero. For tabulation purposes and for computing per cent correct, items on which a student received a positive score were counted correct. Students knew there was a penalty for wrong answers.

The concept of "dimension" (No. 8) is very confused. This appears to be true at all levels from grade seven into college. In general "dimension" is confused with "side" in the sense that two lines forming

an angle are believed to illustrate something having two dimensions. Perhaps this is an unimportant item of understanding but it is a common item of the vocabulary of mathematics teachers. Exercise No. 10 seems to illustrate poor thinking and understanding in terms of a comparatively simple algebraic property.

Mathematical Judgment

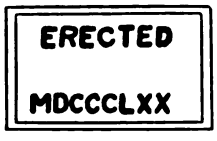
These exercises are designed to require an element of thinking or judgment which goes beyond the simple recognition of some basic mathematical concept or principle. The samples given here were selected to show a variety and range of type and ability.

Note items 1 and 6. Results are about the same. It was noted that many students who correctly estimated the length of the pencil did so by using some measure such as the length of a finger joint. Those who did well on exercise 6 sketched a line to indicate a right angle and then drew their judgment. Two questions arise: First, "Are judgments of this type educationally and functionally significant?" and second, "If some students have learned methods of judgment or approximation, why not teach these to all students?" It is interesting to note that on both of these exercises results for college students are about the same as for pupils in grade 8.

Item No. 2 is interesting for two reasons. College results are very high while correct answers on these same items for students in grades 7, 8, and 9 were less than 50% in each case. In general, junior high school pupils resorted to a great deal of "pencil and paper" work while college students tended to draw their conclusions mentally.

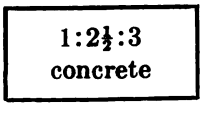
Exercise No. 3 involves a type of mathematical relationship that some elementary school pupils discover independently. Results for grades 7, 8, and 9 showed 11%, 15% and 40% correct respectively. College results are only slightly better. Most wrong answers were "(D) You can't tell." What should the public schools do about the development of such types of mathematical meaning and understanding?

Sample Items from Test on Mathematical Understanding Given to 377 Students and Showing Per Cent of Correct Answers



1. The date on the cornerstone of the old courthouse is given at the left. What year is shown by the Roman Numerals?

   (A) 1820 (B) 1870 (C) 1537 (D) 1920 1. *62%*



2. The numbers in the box at the left give the proportions for mixing concrete. This means 1 part cement, 2½ parts sand, and 3 parts of stone. What fraction of the mixture is *sand?*

   (A) $\frac{1}{2\frac{1}{2}}$ or $\frac{2}{5}$ (B) $\frac{2\frac{1}{2}}{6\frac{1}{2}}$ or $\frac{5}{13}$ (C) $\frac{1}{3}$

   (D) you can't tell exactly 2. *55%*

3. How many bottles like the one at the left can be filled from one gallon of water?

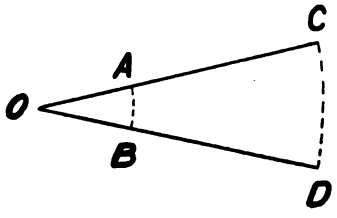
   (A) 5 (B) 4 (C) 4⅕ (D) 1¼ 3. *66%*

4. A mixed food for poultry contains "one fourth of one per cent" salt. How is this per cent written as a decimal?

   (A) .0¼ (B) .025 (C) .0025 (D) .25 4. *27%*

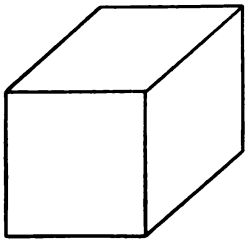
5. A home economics worker noted that large oranges contain 150% *more* juice than small oranges. *How many times as much* juice do the large oranges contain?

(A) 2 times (B) $1\frac{1}{2}$ times (C) $2\frac{1}{2}$ times (D) $1\frac{1}{3}$ times 5. *16%*



6. In the construction at the left, OC is made 3 times as long as OA. The angle BOA has 25 degrees. How large is the angle DOC?

(A) 25° (B) 28° (C) 50° (D) 75° 6. *80%*

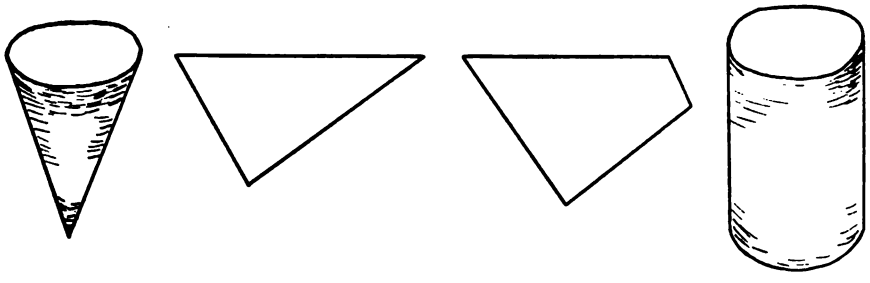


7. The cube at the left represents a cubic foot. If one layer of cubic inches is placed in the bottom of the cubic foot, how many cubic inches would be in the layer?

(A) 12 cu.in. (B) 48 cu.in. (C) 144 cu.in. (D) 1728 cu.in. 7. *54%*

*8. Which of the following objects have two dimensions?

(A) (B) (C) (D) 8. *37%*



*9. In which of the following formulas would the value of the left member be doubled when the value of r in the right member is doubled and other values remain the same?

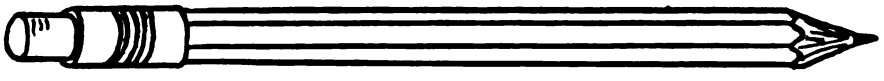
(A) $d=2r$ (B) $c=2\pi r$ (C) $V=\frac{1}{3}\pi r^2 h$ (D) $A=\pi r^2$ 9. *73%*

*10. The root of an equation is: $x=-4$. Which of the following equations based upon this root are correct?

(A) $\frac{1}{4}x=-1$ (B) $4x=16$ (C) $x^2=-16$ (D) $x^3=-64$ 10. *36%*

SAMPLE ITEMS FROM TEST ON MATHEMATICAL JUDGMENT GIVEN TO 323 COLLEGE STUDENTS AND SHOWING PER CENT OF CORRECT ANSWERS

1. How long is the pencil that is drawn below?



(A) $3\frac{1}{2}$ in. (B) $4\frac{1}{2}$ in. (C) $5\frac{1}{2}$ in. (D) $6\frac{1}{2}$ in. 1. *46%*

* *From a group,* where one or more answers are correct.

2. Elsie plans to make sandwiches like the four shown at the left. How many sandwiches will she get from a loaf that has 20 slices of bread?

(A) 10 (B) 20 (C) 40 (D) 80 2. *72%*

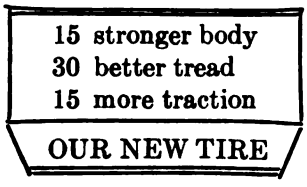
3. Mary was dividing by 28, the first figure in the quotient was 5 and the remainder was 26. What would be the next figure in the quotient after she has "brought down" a figure?

(A) 1 or 2 (B) 4 or 5 (C) 8 or 9 (D) you can't tell 3. *49%*

	seeds	plants
Corn A	5	4
Corn B	10	8
Corn C	50	40

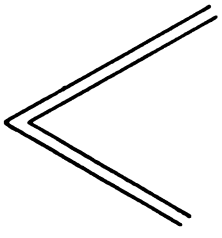
4. The chart at the left shows how many plants sprouted from three different samples of seed corn. Which is most likely to produce 80 plants for each 100 seeds in a bushel of corn?

(A) corn A (B) corn B (C) corn C
(D) all three samples are equally likely 4. *19%*



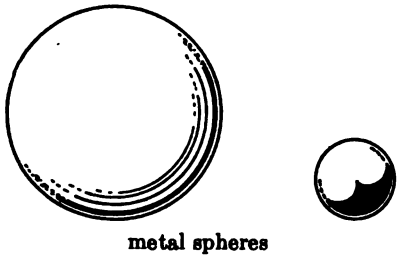
5. A new tire is advertised using per cents as shown at the left. By what per cent is the new tire better than the old tire?

(A) 60 (B) 20 (C) $22\frac{1}{2}$
(D) you can't tell the per cent exactly 5. *20%*



6. Two roads intersect as shown. Which of the following is the best estimate of their angle of intersection?

(A) 30° (B) 40° (C) 50° (D) 60° 6. *44%*



metal spheres

7. The large sphere has a diameter approximately three times as large as the diameter of the smaller sphere. If the smaller sphere weighs 20 pounds, how much would you expect the large sphere to weigh?

(A) 60 lb. (B) 125 lb. (C) 250 lb.
(D) 500 lb. 7. *17%*

*8. Which of the following have values that can be expressed with whole numbers or decimals so they can be set exactly on an adding or computing machine?

(A) π (B) 7.2 (C) $\sqrt{2}$ (D) $(\frac{1}{2})^2$ 8. *60%*

*9. Mr. Sharpe bought a farm. He said the value of the house was "less than worthless." Which of the following might be the value expressed by Mr. Sharpe?

(A) 1¢ (B) 0 (C) −100 dollars (D) \$0.00½ 9. *37%*

* From a group where one or more answers are correct.

Exercise No. 4 is not valid for junior high school pupils because they failed to sense the element of the relationship between the size of the sample and the reliability of the sample. This was also true for college students. However, if that is an important functional item of learning, it is not reasonable to expect that college students should recognize the situation?

Item No. 5 is interesting and disturbing. Results are only slightly better on the college level than on the junior high school level. This is the type of situation that may be called "sucker bait." Students tend to want to compute when they see numbers and that results in erroneous conclusions. Of course that is what they learned to do in schools.

Several items such as No. 7 have been used with various levels of pupils and the results are uniformly low. These exercises include comparisons of similar objects of both two dimensions and three dimensions. Even college students fail to recognize that this is an application of a proposition which they glibly recite. In the real world these judgments usually must be made in terms of an optical impression; as for example in the buying of fruits. What shall we do about this situation? Demonstrative geometry of the tenth grade seems to have done little or nothing. Of course there are other more meaningful methods of teaching comparisons; why not use them?

Exercises Nos. 8 and 9 may have one or more correct answers. The results for No. 8 are surprisingly good. Many people chose "(B) O" as the answer for No. 9. Some students justified this by the statement that nothing is less than worthless.

Summary

Approximately two thousand junior high school pupils and one thousand college students have taken tests designed to measure the Mathematical Understanding, Mathematical Judgment, and Mathematical Computations in functional situations. These tests have the limitations of the "paper and pencil" technique. Testing in real functional situations would require taking pupils to the grocery store, to street intersections, and to other places where mathematical situations exist. The author has done some of this but has found it rather time consuming. He has also experimented with testing based upon bringing real materials before students. The conclusions here stated are derived not only from the sample items that have been presented but also upon the broader experience in evaluation.

1. Pupils and students at all levels tend to learn those things that the teacher expects of them and to learn in a pattern set by the teacher. For example, a school that glorifies abstract computation of arithmetic and algebra tends to surpass in that type of work and may or may not achieve good results in understanding and judgment. The converse is also true but to a lesser extent because understanding and judgment usually involve some computational elements. The implications for curriculum and for teaching are obvious. What do you want pupils to achieve in mathematics?

2. Weaknesses tend to persist from one level to another. This is particularly true of arithmetic learnings. Pupils who did not comprehend fractions and percentage in the elementary school continue in low comprehension into college. This continuing weakness should probably be blamed upon the school as well as upon the pupil. Mathematics tends to be a segmented group of materials rather than a program of continuous growth in the basic areas of arithmetic, geometry, and algebra.

3. Certain types of exercises in which the mathematical element is not complicated and is found in situations in which older students have had experience prove a good deal easier for the older group. This may be an argument for basing learning upon real experience. It is surprising to find that many college students have had little opportunity to learn from

seeing and handling real things or even the standard models.

4. The extension of learnings in arithmetic is neglected in the typical secondary school program. This is most lamentable if, as most citizens believe, arithmetic is the most functionally useful part of mathematics for most people. Anyone who is acquainted with current arithmetic courses of study knows that these do not attempt to teach all of the arithmetic that the competent citizen requires. Consider percentage: where can a young person enlarge his understanding, extend his computations beyond the simplest cases, and learn to judge in terms of per cent in most of our junior and senior high schools? What is the prime aim of instruction in mathematics if not to provide a basis for more competent and intelligent participation in the civilization in which we live?

5. Schools tend to place too much dependence upon "paper and pencil" work. Many high school and college students set down figures and compute without doing any critical thinking. Furthermore they tend to use pencil and paper for simple things that ought to be done mentally. A considerable part of the mathematics of the real world is encountered orally and visually and requires only a little mathematical understanding upon which a valid judgment or decision should be made. The great dependence upon paper and pencil is not restricted solely to those of lower mental ability.

6. Confusion concerning the purpose of study and the values to be derived from the study of demonstrative geometry persists in the minds of students. In the recognition of geometric figures and concepts in the application of many geometric principles, those students, who studied the informal and experimental type of geometry found in many junior high schools, tend to do as well as many students who spent a whole year at demonstrative geometry.

7. Students tend to do comparatively well in abstract algebra such as computing with negative numbers and in solving comparatively simple equations but they tend also be to little concerned with the understanding and significance of what they are doing. In general "algebraic tricks" rather than algebraic thinking is most characteristic of students who have studied only one year of algebra.

LETTERS

In the February 1949 issue of THE MATHEMATICS TEACHER, Lurline Stewart did a splendid job in regard to the slang expressions used in mathematics classes. She covered all of the pet peeves in the field with one exception. I would like to add that exception to the list.

The slang expression that I find used in all mathematics courses from Intermediate Algebra to Analytic Geometry is "bring over." (Perhaps the reason I do not hear it in any course more advanced than Analytic Geometry is that by the time my students have reached Calculus most of them have been exposed to my teaching and know that "bring over" is taboo.)

The word "transpose" is bad enough, but "bring over" is the most objectionable expression that I have heard used in mathematics. In solving an equation such as $x+4=9$, the students "bring over" the four and find $x=5$. This is all right as long as you do not ask them what they mean, and in equations like the above the slang expression often goes unheeded. But, in equations such as $3x=12$, the students also "bring over" and get the result $x=9$. When asked just what they mean by "bring over," half of the students have no idea and the other half say "transpose."

Now I admit that it is much quicker and easier to say "transpose" than it is to say "subtract 4 from both sides of the equation," but wouldn't it be just as easy and quick and far more meaningful to say "subtraction axiom"?

Is it against the principles of education to teach new words? Why should not students learn new terms in mathematics? Students today are ever increasing their vocabularies with more formidable words than axiom. Most students in high school and college are familiar with such words as atomic bomb and nuclear fission. These terms are much more complex than axiom.

May I add my plea to the plea of Miss Stewart that in high school and college mathematics we omit the slang from our courses and teach the meaningful expressions in their place.

MARGARET F. WILLERDING, Harris Teachers College, St. Louis, Mo.

Mathematics for the Million, or for the Few?*

By WILLIAM BETZ
Rochester, New York

IT IS now thirteen years since the appearance of a much-debated volume entitled "Mathematics for the Million." That title was decidedly misleading. In reality, the book presupposed a mathematical maturity ordinarily achieved only by "the few." However, it served to focus attention on the real question at issue, namely, what kind of mathematical education should the schools provide not merely for a gifted minority, but also for the average citizen?

Ever since the organization of the International Commission on the Teaching of Mathematics, at the Fourth International Congress of Mathematicians, in Rome, Italy, in 1908, published reports on the mathematical curricula of the participating nations have gradually become available. In the United States, summaries of these documents have appeared in such publications as the National Report of 1923 and the Fourth Yearbook of the National Council of Teachers of Mathematics. They made it quite clear that in the majority of the reporting nations the schools provided one type of education for the masses and a radically different kind of education for the "classes." That sort of program is possible only in a dual system of education consisting of elementary schools for the many and secondary schools for the few.

However, in the United States that arrangement has long been considered undemocratic. Instead, the prevailing policy is that of admitting "all the children of all the people" to every kind of public school. Accordingly, our schools have become genuine educational laboratories on a huge scale. And it may well be that both our successes and our failures in the field of mass education hold valuable lessons for the other nations. In the light of this background, let us briefly examine what has been done here, or is being proposed, by way of answering the question embodied in the title of this paper.

Three principal approaches have been tried or suggested in American secondary schools in their attempts to provide functioning types of mathematical training for *all* American youth.

The *first* is anchored on the popular doctrine that "life situations" should be the primary basis of all curricula. But since mathematics is a *system* of ideas and processes, whereas life situations are incurably *unsystematic*, this approach has failed completely wherever it has been tried. It has always resulted merely in a sort of chaotic "mathematics without mathematics," and it has ignored fundamental aspects of the problem we are considering.

The *second* approach hopes to find dependable answers in the recommendations of authoritative committees and in the techniques of curriculum workshops and laboratories. However, the thousands of mathematical syllabi now crowding the shelves of our curriculum morgues have merely dramatized a hopeless confusion of objectives. And even the reports of *national* committees have regularly been attacked by leading educators as mere reflections of unsupported private opinion.

There remains a *third* approach, often explored partially, but never with anything like scientific completeness or thoroughness. It is that of making a really dependable, full-length study of the role of mathematics in the modern world, from both a practical and a cultural standpoint.

To accomplish that, we should have to look carefully at many aspects of the

* A report submitted to Section VII of the International Congress of Mathematicians, at Cambridge, Massachusetts, September 1, 1950.

question. How and where do we need mathematics in our homes, our vocations and professions, as wage earners, as citizens, as readers, as students, and so on? We should have to find out how mathematics is related to the industrial, commercial, and engineering enterprises of the nation. And in what way do mathematical modes of thinking, attitudes, and types of appreciation permeate the very warp and woof of modern civilization? To obtain the information we need, it would be necessary to analyze the work of many groups of people, such as businessmen, builders, contractors, mechanics, machinists, designers, draftsmen, economists, scientists, statisticians, accountants, navigators, and pilots. When such studies become available, we are sure to discover the utterly untenable character of the idea that only the "few" need genuine mathematical training. On the contrary, it will be found that mathematics is a subject of global significance, and that a citizen of the modern world simply cannot afford to be ignorant of a basic knowledge of mathematics, which must surely involve a little more than grocery store arithmetic and a few rules of mensuration.

It is believed that only on the basis of such objective findings will it be possible to obtain a convincing and enduring foundation for the mathematical curricula of the modern school. It will then be our major task to build a two-track program, or even a three-track program, for the divergent needs of those seeking an immediate life preparation and of those who aim at foundational studies in our colleges and universities.

This brief report is submitted in the ardent hope that it may enlist the active concern of another International Congress of Mathematicians pertaining to this "unfinished business," namely, our common problem of providing more adequate curricula both for the many and the few. Is it too much to expect that a work which was begun so auspiciously in 1908, at the instance of our own David Eugene Smith, be resumed and eventually brought to more complete consummation? It is therefore urged that Section VII of this Congress transmit to the International Mathematical Union an expression of its profound interest in this problem, together with the definite suggestion that proper steps be taken forthwith to initiate its solution.

(At the second session of Section VII, on September 2, Mr. Betz presented the Resolutions which follow. They were adopted unanimously, and will be transmitted to the newly organized International Mathematical Union.)

Whereas the work done by the International Commission on the Teaching of Mathematics, dating back to the Fourth International Congress of Mathematicians in Rome, Italy, in 1908, resulted for the first time in a comprehensive picture of the actual mathematical curricula of the leading nations, and

Whereas a still larger task remained entirely unfinished, namely, that of undertaking a dependable, scientific study of the role of mathematics in the modern world, from a standpoint of both "the many" and "the few," and

Whereas the National Council of Teachers of Mathematics, at its recent meeting (August 21–23, 1950) at the University of Wisconsin, strongly urged that this unfinished business be brought to the attention of Section VII of the present Congress and also of the International Mathematical Union, therefore be it resolved

First, that Section VII of the present Congress gives its unqualified approval to the plan of having such a study made, and

Second, that it wishes to call upon the International Mathematical Union to initiate this program through the continuation and revitalization of the International Commission on the Teaching of Mathematics, helping this Commission to investigate the significance of mathematics in all walks of life, and to make available its findings as a basis for more authoritative and more convincing mathematical curricula throughout the world.

The Use of Simple Algebra in Business Arithmetic

By WILLIAM L. SCHAAF
Brooklyn College, Brooklyn, N. Y.

SOME years ago the writer overheard a conversation between two college students, both majors in mathematics, one of whom was preparing to take an examination for a license to teach business arithmetic. Upon being asked by his companion what he thought of the text he was studying, he remarked that business arithmetic seemed to be a collection of ingenious devices to achieve what might be done much more simply by the use of simple algebraic symbols and equations. While this observation could scarcely serve as an adequate characterization of business arithmetic, it nevertheless suggests that the use of a minimum amount of basic algebra might very well simplify many of the procedures of business arithmetic, and at the same time make them more meaningful.

Before suggesting specific instances where algebraic symbolism could be employed to advantage, let us see how little algebra is actually needed. In the first place, nowadays many elementary school and junior high school pupils, by the end of the eighth grade, have become familiar with simple formulas such as $A=l\times w$, $P=2l+2w$, $C=2\pi R$, $C=\frac{5}{9}(F-32)$, $D=R\times T$, $A=\frac{1}{2}bh$, $A=\frac{1}{2}h(b_1+b_2)$, and possibly a few others. This, of course, is all to the good. Once the learner has begun to understand the meaning of generalized numbers and literal expressions, such as d dollars, t hours, W pounds, h feet and the like, then to learn what simple equations mean and how to solve them is no longer such a formidable task. For most of the purposes to be presented shortly, it is sufficient to be able to solve the following types of equations:

(1) $ax=b$, or $x=\dfrac{b}{a}$.

(If 3 apples cost 12¢, one apple costs $\frac{12}{3}$¢, or 4¢.)

(2) $\dfrac{x}{a}=b$, or $x=ab$.

(If $\frac{1}{4}$ lb. costs 12¢, one pound costs 4×12, or 48¢.)

(3) $x-a=b$, or $x=b+a$.

(If 3 less than a number is 12, the number is $12+3$, or 15.)

(4) $x+a=b$, or $x=b-a$.

(If 3 more than a number is 12, the number is $12-3$, or 9.)

The transition and extension to these simple equations from the familiar processes of arithmetic is not particularly difficult for most ninth-grade pupils.

Another bit of algebraic technique which is often helpful and not especially difficult is the use of the identity $k(a+b)=ka+kb$, or, if you will, $Km+Kn=K(m+n)$. This may be presented inductively in terms of familiar number relations; e.g., $5\times13=(5)(10+3)=(5\times10)+(5\times3)=50+15$, or 65. In fact, learning this sort of relationship may give added meaning to the multiplication algorism in arithmetic when reteaching the ideas

$$\begin{array}{r} 13 \\ \times\ 5 \\ \hline 65 \end{array} \qquad \begin{array}{r} 10+\ 3 \\ \times\ 5 \\ \hline 50+15=65 \end{array} \qquad \begin{array}{r} 3+10 \\ \times\ 5 \\ \hline 15+50=65 \end{array}$$

of place value and carrying. It is even possible that some pupils may arrive at complete understanding of the process for the first time in their lives.

To illustrate the simplicity, convenience and effectiveness of the use of symbols and equations to indicate mathematical relationships between quantities used in

business, consider first the concepts of *margin* and *profit*. Basically, the important ideas are expressed by

$$S-C=M, \quad \text{and} \quad M=E+P,$$

where $S=$sales, $C=$cost, $M=$margin, $E=$expenses, and $P=$net profit. From these equations it is easy to discover other relationships; for example,

$$S=C+M,$$
$$P=M-E,$$
$$S=C+E+P,$$
$$P=S-C-E.$$

As another illustration, take the troublesome percentage problem, the so-called "Case III," to find the base when the rate and the percentage are known. Pupils should eventually appreciate the fact that the three "cases" of percentage are merely different aspects of but one basic relationship, viz.,

$$P=B\times R;$$

from this, if we are at home with simple algebraic transformations, it is very easy to see that

$$R=\frac{P}{B} \quad \text{and} \quad B=\frac{P}{R},$$

which are "Case II" and "Case III," respectively. If at the same time that we become familiar with such formulas we also get the habit of thinking and writing .06, .35, .8, for 6%, 35%, or 80%, the usual difficulties with Case III will tend to disappear. These relationships frequently arise in connection with the determination of list price, or selling price, or the amount sold on commission, or the additional discount to be offered, etc.

In problems relating to chain discounts, the following formulas should prove useful.

(1) Net cost when two discount rates d_1 and d_2 are offered:

$$C=P\times(1-d_1)(1-d_2),$$

where P is the list price.

(2) Equivalent single discount rate, D:

$$D=1-(1-d_1)(1-d_2),$$

which, like formula (1), can be generalized for any number of successive discounts.

In this connection, a short cut suggested for finding the equivalent single discount rate is to "subtract the product of the two given discount rates from their sum." Thus, discounts of 25% and 20% are equivalent to a single rate of (25%+20%) −(25%×20%), or 45%−5%=40%. The proof of this rule of thumb method, using formula (2) above, is as follows:

$$\begin{aligned} D&=1-(1-d_1)(1-d_2)\\ &=1-(1-d_1-d_2+d_1d_2)\\ &=1-1+d_1+d_2-d_1d_2\\ &=(d_1+d_2)-d_1d_2. \end{aligned}$$

Admittedly, for a pupil to follow the above derivation, he would have to understand and be able to use negative numbers as well as literal numbers. The need for such skills is frankly not particularly great, and is therefore not urged. However, it is very questionable if a short cut should ever be taught merely as a mechanical or rote procedure without rationalization or proof.

When teaching the topic of simple interest, it is doubtful whether many pupils ever really understand the nature of interest, mathematically considered. By and large it is learned more or less mechanically; thus, $I=P\times R\times T$, and nobody ever questions the significance of multiplying these three factors. To be sure, I is a mathematical function of P, i, and n, to use standard notation. But i is related to I and P, as a parameter: thus, when $n=1$,

$$\frac{I}{P}=i, \quad \text{or} \quad I=Pi,$$

which means that i is simply the fractional part which I is of P. This is an important concept which should be understood; interest at the rate of 4% per

annum means that each year, 1/25 of the principal is due as interest. It is important once more to get into the habit of thinking of i as a decimal fraction, representing so and so many units of money, as well as thinking of it as a per cent. Then relationships and the reasons for the relationships can take on additional meaning. Thus

$$\begin{aligned} S &= P+I \\ &= P+iP \\ &= P+.04P \\ &= P(1+.04) = 1.04P, \text{ or } 104\% \text{ of } P. \end{aligned}$$

When $n=2$, we have:

$$\begin{aligned} S_2 &= P+P(2i) \\ &= P(1+2i), \end{aligned}$$

and, in general,

$$S_n = P(1+ni).$$

In fact, the various relationships involved may be conveniently summarized, symbolically, as follows:

$$I=Pni \qquad S=P+I \qquad P=\frac{S}{1+ni}$$

$$P=\frac{I}{ni} \qquad P=S-I \qquad S=\frac{I}{ni}+I$$

$$n=\frac{I}{Pi} \qquad I=S-P \qquad I=S-\frac{S}{1+ni}$$

$$i=\frac{I}{Pn} \qquad S=P(1+ni)$$

It is to be understood, of course, that there is no thought of having pupils memorize a group of formulas such as this. The desired outcome is an *understanding* of the basic relationships, together with sufficient skill in the use of simple algebraic manipulations, so as to be able to derive and use the appropriate relationship in a given problem situation. Such learning should not and need not be mechanical; approached by teacher and learner in the right way, it can be of invaluable help in all sorts and varieties of problems.

The relationship of interest and interest rate to discount and discount rate is seldom clearly understood by pupils. Even adults frequently labor under the misapprehension that *discount* is simply "interest collected in advance," but a discount rate of 5% is mathematically *not* equivalent to an interest rate of 5% on the same loan for the same period. As a matter of fact, the true discount rate is the reciprocal of the interest rate; true discount is interest on the present value. True discount differs from bank discount in that it is equivalent to interest on a principal, while bank discount is interest on an amount (i.e., principal plus interest).

In order to understand mathematically the concepts of present value and true discount, it is again submitted that algebraic representation should be extremely helpful. Without laboring the point, we shall simply set down the important relationships in such a fashion as to exhibit the analogy between future value, present value, interest and discount. The conventional symbols are:

P = principal, or present value (face value)
S = principal + interest, or future value (maturity value)
I = interest
D = discount
i = interest rate (as a decimal)
d = discount rate (as a decimal)
n = time in years

We then have the following:

(1) $I=Pni$	(1a) $D=Snd$
(2) $S=P+I$	(2a) $S=P+D$
(3) $I=S-P$	(3a) $D=S-P$
(4) $S=P(1+ni)$	(4a) $S=\frac{P}{1-nd}$
(5) $P=\frac{S}{1+ni}$	(5a) $P=S(1-nd)$

Now by equating the right-hand members of equations (5) and (5a), it follows that

$$i=\frac{d}{1-nd}, \quad \text{and} \quad d=\frac{i}{1+ni}.$$

In other words, the interest is the future value of the discount, and the discount is the present value of the interest. It is to be noted also that the expressions $(1-nd)$ and $(1+ni)$ are reciprocals.

Similar observations could also be made with regard to genuine understanding of the nature of compound interest. To be sure, to express these relations mathematically involves the use of exponential notation, which was not included in our original premise of a "few simple algebraic notions." However, without even hinting at the binomial expansion of $(1+i)^n$, or even the mention of logarithms, the *meaning* and *significance* of $(1+i)^n$ as being $(1+i)\cdot(1+i)\cdot(1+i)\cdots$ to n factors can be made clear by simple numerical illustrations, where n is held down to small values. Certainly it should help the pupil to appreciate the contrast between simple and compound interest if he can see the mathematical difference between the expressions

$$P(1+ni) \quad \text{and} \quad P(1+i)^n.$$

How far this approach can be carried must be left to the judgment of the individual teacher.

We suggest without further comment a few other relationships in which algebraic expression could help understanding.

(1) The *cost of goods sold* during a given period:

$$C=I_1+P-I_2,$$

where I_1 = initial inventory, I_2 = final inventory, and P = purchases during the period.

(2) *Average inventory*:

$$I=\frac{I_1+I_2}{2}.$$

(3) *Annual depreciation* (straight line method):

$$D=\frac{C-S}{n},$$

where C = initial cost, S = scrap value, and n = estimated life of asset in years.

(4) *Book value* (straight line depreciation) after k years:

$$B_k=C-\frac{k(C-S)}{n}.$$

(5) *Amount paid on a fire insurance claim* under an 80% coinsurance clause:

$$A=\frac{PL}{.8V},$$

where A = amount paid on claim, P = face value of policy, V = value of the property, and L = amount of loss as determined by the insurance adjuster.

Discussion of the above formula (5) reveals that it holds only for values of P equal to or less than $.8V$, since in no case will the company pay more than the face value of the policy.

In conclusion, we urge once more that a greater disposition to use algebraic formulation to express quantitative relationships has much to commend it, provided, however, that such learning is meaningful rather than mechanical or superficial.

Plan to Attend
The Twenty-Ninth Annual Convention of the National Council, Hotel William Penn, Pittsburgh, Pennsylvania, March 28-31, 1951

A very good program has been arranged, including visiting Pittsburgh schools on Wednesday and Thursday, sightseeing trip on Thursday, a big general meeting on Thursday night with a reception following, banquet on Friday night and the convention luncheon on Saturday at noon. A full schedule of meetings, discussions, exhibits, films, demonstrations, and panels has been arranged for Thursday, Friday, and Saturday which should appeal to elementary, secondary, and college teachers and to supervisors and administrators. The complete program with announcements and preregistration form will appear in the February issue of the Mathematics Teacher.

Know Your National Council Representative

By KENNETH E. BROWN
Chairman of State Representatives, Department of Mathematics, University of Tennessee, Knoxville, Tennessee

MISS ANNIE JOHN WILLIAMS

NORTH CAROLINA REPRESENTATIVE

MISS ANNIE JOHN WILLIAMS, a high school teacher at Julian S. Carr Junior High School, Durham, North Carolina, has been an active member of the National Council of Teachers of Mathematics for twelve years. She reports that she finds the articles in THE MATHEMATICS TEACHER very helpful to her in teaching students in the junior high school. Her interest in the improvement of the teaching of mathematics is indicated by her attendance for six years at the Mathematics Institute at Duke University. The sole purpose of this Institute is to provide a mathematics laboratory that will contribute to the improvement of mathematics education. Her special interest in the improvement of the teaching of mathematics is in harmony with her work as National Council Representative. Each year she writes several hundred letters to teachers of mathematics asking them to join the Council—the only national organization devoted solely to their cause. Miss Williams will welcome the assistance of the mathematics teachers in North Carolina in helping her with the National Council work.

ARIZONA REPRESENTATIVE

Mr. Louis Leithold, Instructor in Mathematics, Phoenix College, is a native of California. He received both his B.A. and M.A. from the University of California at Berkeley and joined the faculty of Phoenix College in 1946. The following year he became the National Council Representative in Arizona. Although the number of teachers of mathematics in Arizona is not large, this group has organized the Arizona Mathematics Association which meets twice each year. This increased interest in mathematics education is also evidenced by the steady increase in the National Council members in Arizona.

MR. LOUIS LEITHOLD

The Affiliated Groups and the Delegate Assembly

JOHN MAYOR, *Chairman*
The Committee on Affiliated Groups

THE Affiliated Groups of The National Council of Teachers of Mathematics will be interested in making preparations during January, February, and March for the coming National Council meeting in Pittsburgh. Two meetings of the Delegate Assembly have been scheduled. The first meeting is on March 29, 8:00–10:00 A.M. and the second on March 30, 1:15–2:15 P.M. The Delegate Assembly will be concerned with relationships between the Affiliated Groups and the National Council. There will also be a section on Affiliated Groups at the Pittsburgh meeting. In this section a panel of presidents of Affiliated Groups in the Pittsburgh area will lead a discussion on the programs and plans of the various Groups in their own localities.

The official delegate of each Group will want to become familiar with the issues which will be discussed at the Delegate Assembly and the point of view of the Group, which he represents, on these issues. Through the *Newsletter of Affiliated Groups*, the Groups have been invited to submit problems for discussion early enough to make sure that it would be possible for each Affiliated Group to have considered these problems before the Assembly meeting. The names of the official delegates should be in the hands of the chairman of Affiliated Groups by February 1, 1951.

Questions which have been suggested for discussion at Pittsburgh, and on which the Delegate Assembly may wish to make recommendations to the Board of Directors or to the business meeting, are as follows:

1. Should any of the decisions of the First Delegate Assembly, as reviewed in the November issue of THE MATHEMATICS TEACHER, be re-examined in the light of the past year's experiences, to determine if additional action is desirable at this time?
2. By what means can the National Council better serve the various regions and Affiliated Groups?
3. How can a closer relationship be established between the program of State Representatives and the program of Affiliated Groups?
4. What means of determining problems for discussion and what procedures in the Delegate Assembly will serve the interests of the National Council most effectively?
5. Would a plan of rotation for publication of the *Newsletter of Affiliated Groups*, such as is practiced in the publication of the periodicals of some of the Groups, be feasible and how can this be worked out, if found feasible?
6. Does your Group need the services of a Speakers Bureau, and if so, how can such a Bureau best serve your Group and the cause of the improvement of mathematics instruction at all levels?
7. How can Affiliated Groups best contribute to the total program of the National Council and especially to the programs of the meetings, to its publications, to its investigational activities, to the promotion of good public relations, and to growth in membership?
8. Should the name of the National Council be changed? The vote at the First Delegate Assembly favored a change in name, but further consideration, especially of possible new names, seems desirable. Some of the suggested names are Council of Teachers of Mathematics, International Council of Teachers of Mathematics, and American Council for Mathematics Education.
9. Would the Affiliated Groups be interested in a cooperative plan for contest sponsorship?
10. How can Affiliated Groups arrange for sharing and exchange of studies in progress, teaching materials prepared, and any other activities which have been found especially valuable?
11. How can the National Council more effectively carry out its purpose to improve mathematics teaching at all levels of instruction?

The President's Page

You will be interested in knowing that your National Council has taken the first step in a big venture, an endeavor that will ultimately call for the help of individual members and of the affiliated groups.

For a long time our leaders have been telling us that we are many years behind in our mathematics education in this country, and that we are due for a reform. We recognize that all is not well with our programs of mathematics and mathematics teaching, and we are anxious for improvements to be made. Many have been saying that the National Council should be doing something about it and are looking to the National Council for leadership. A complete over-all study of the situation is suggested, the purpose being to bring about this much needed and long overdue reform. The problem is a huge one, an endeavor that will require the cooperation of several national organizations, committees, and agencies. The necessary study and research will require great financial support and will call into service the most capable people that can be found. It will take careful planning, and the work will extend over several years.

Despite the vastness of the task, it should be done. Some beginning should be made immediately. Some group of national repute and influence should take the initiative in the matter and suggest a plan of action. With this in mind and completely desirous of having the National Council do its part, the Board of Directors, meeting last August in Madison, authorized the appointment of a committee of five, this committee to be known as the Research Committee. The committee was appointed early in October and charged with the responsibility of studying the problem and making recommendations to the Board. The committee is a permanent one, membership to be increased or changed as deemed advisable. The committee made a preliminary report to the Board at the Florida meeting in December. What can and will be done remains to be seen, but we are depending upon the Research Committee to point out the possibilities and to develop plans whereby the National Council can participate in a long-range program of study and research on the problem. From the information thus gained, we would strive for the needed changes and improvements.

Indications are that the work of this Research Committee will bring about cooperation with other groups, such as the Policy Committee which is composed of representatives of the American Mathematical Society, the Mathematical Association of America, the Institute of Mathematical Statistics, and the Association of Symbolic Logic. This Committee is interested in the welfare of mathematics and mathematics teaching and we hope to have a representative of the National Council on this committee soon. It may very probably mean that the Cooperative Committee on Science and Mathematics Teaching of the American Association for the Advancement of Science, a committee on which the National Council has been represented for several years, will lend its support. It may lead to our being represented on the National Science Foundation, an organization whose members are appointed by the President of the United States, and which is now in the organizational stage. The Office of Education of the Federal Security Agency is showing a definite interest in mathematics and may soon have on its staff a specialist in mathematics. From such relationships and those of the National Education Association, it is highly probable that contacts with foundational grants can be made to support a research project such as the Research Committee will advise. The Mathematical Association of America and the National Council of Teachers of Mathematics have cooperated before on major endeavors and have produced valuable

results. It is time for them to do it again. The outlook for cooperation with other interested groups is good and the prospects for securing financial backing is promising.

Eventually many will be called upon to help with this project. The Research Committee may be enlarged and sub-committees formed. Let the members of the committee know that you are interested and willing to help. Write to them and give them the benefit of your ideas and suggestions. It is a huge task and they need your help. The members of the committee are: Chairman, Henry Van Engen, State Teachers College, Cedar Falls, Iowa; Howard F. Fehr, Teachers College, Columbia University, New York 27, New York; John J. Kinsella, New York University, Washington Square, New York 3, New York; F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee; Lenore John, The Laboratory School, University of Chicago, Chicago, Illinois.

H. W. CHARLESWORTH, *President*

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD

State Teachers College, Oneonta, New York

THE ART OF RECKONING. III. WHAT YOU CAN DO IF YOU KNOW THE DOUBLES

THE USE of doubling as a method of multiplication appears in the Ahmes Papyrus (*c.* 1650 B.C.) in Egypt. Together with finding half of a number (mediation), doubling (duplication) was ranked with addition, subtraction, multiplication, and division as a fundamental process in the work of the Englishman Sacrobosco in the thirteenth century. It has been associated with mathematical puzzles and through its connection with the binary series 1, 2, 4, 8, 16, . . . , it is related to the electronic computing machines.

When multiplying by the doubling process, the computer tacitly recognizes that any whole number whatever may be broken into parts that are numbers taken from the binary series. Thus the number 49 is made up of 32, 16 and 1. To multiply 103 by 49, the computer would write

1—	103
2	206
4	412
8	824
16—	1648
32—	3296

Notice that the multiplicand 103 is in line with the number 1; the double of 103 is in line with 2; the double of the double (i.e., four times) is in line with 4 and so on. Then since 49 is 32 and 16 and 1, the product of 49 and 103 is 32×103 plus 16×103 plus 1×103. So the computer checks the numbers opposite 1, 16 and 32 and then adds.

1	103
16	1648
32	3296
	5047

49×103 is 5047

In the Ahmes Papyrus, the details of the multiplication process are given in some cases, omitted in others. Multiplication as it appears in this document is summarized as follows:

"Direct multiplication by integers was generally confined to the multipliers 2 and 10; that is, in cases where the process was explicit, and it may be presumed that in all cases, they multiplied by repeated doubling, or occasionally multiplying by 10, and adding the products formed from multipliers that would make up the given multiplier."*

* *The Rhind Mathematical Papyrus*, vol. I, Arnold Buffum Chace, Mathematical Association of America, 1927, p. 2.

The products of 19 by 6 and 23 by 12 are given as follows:

	1	19	
	2	38	
	4−	76	
Total	6	114	

1	23
10−	230
2−	46
12	276

The Ahmes Papyrus seems to consider division as a second type of multiplication in which the product and multiplicand are given and in which the multiplier is to be found. For clarity, we shall use the terms of division. The writer of the papyrus suggests that the computer is to use successive doubles of the divisor and its multiple of 10 and from these to build up the dividend as nearly as possible, and then find the remainder needed to complete the dividend. It is easier to do this than to describe it. For example, to divide 657 by 34, we would proceed as follows:

Table of doubles		subtracting:	
1	34		657
2	68	16×34	544
4	136		113
8	272	2×34	68
16	544		45
		1×34	34
			11

Thus $657=(16+2+1)34+11$. Accordingly the quotient is $(16+2+1)$ or 19 and the remainder is 11.

The fact that any whole number may be expressed as the sum of terms chosen from the series 1, 2, 4, 8, . . . has a number of applications. One is the number puzzle that asks how one hundred ducats could be placed in seven bags in such a way that any number of ducats up to 100 could be paid out without opening any of the bags. The solution was to place one ducat in the first bag, two in the second bag, four in the third, eight in the fourth, sixteen in the fifth, thirty-two in the sixth and all the rest in the seventh.

A more practical application of the use of the series is in the manufacture of the weights formerly used by goldsmiths and others. The weights were made in the *value of* 1, 2, 4, 8, . . . units and the fractional weights were $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. The appearance of these weights was interesting. The smallest one was a flat cylinder of metal which fitted closely into a cylindrical cup which was the next larger weight. This in turn fitted into another cylindrical cup until the largest of the series was the case for the entire set and often took the form of a sort of metal pail with a hinged cover.

The goldsmiths' weights were the basis of a number puzzle which asked the very least number of weights that could be used to weigh any whole number of ounces from 1 to 40. By the device of putting the weights on either side of the scales at will, the weights 1, 3, 9, 27 would be sufficient. To weigh two ounces, the one ounce weight would be on the side of the object to be weighed and the three ounce weight on the other side. To weigh five ounces, the nine ounce weight would be on one side, the one ounce and the three would be on the side of the thing to be weighed.

A modern puzzle use of the binary series is in a series of cards with the number 1, 2, 4, 8, 16, . . . one at the top of each card. Then all the odd numbers up to the desired limit are written on the card labelled 1. On the card marked 2 are listed all the numbers which require 2 in their composition. Thus for the numbers from 1 to 16 the cards would appear in this form:

1	2	4	8
3	3	5	9
5	6	6	10
7	7	7	11
9	10	12	12
11	11	13	13
13	14	14	14
15	15	15	15

Then to find the number of which a person is thinking, ask him to select the cards that have that number printed on them. Add the numbers at the top of the cards that are selected and this will certainly be the number the other person had in mind.

A later article in this series will discuss the uses of the binary series in computation.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
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THIS section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

A NOTE ON THE USE OF FELT FOR THE MANUFACTURE OF VISUAL AIDS

Two reasons for the infrequent use of visual aids in the teaching of mathematics are (a) the length of time required for their manufacture compared with the brief space of time for which they are used in the classroom, and (b) the lack of adequate storage space for such apparatus, which must obviously be large enough to be seen by the "back row" boys. Often the choice of a suitable material eliminates these difficulties.

It is a matter of common experience that a piece of wool will readily stick to a rough tweed jacket or any piece of woolen material. In the same way, quite large pieces of felt placed face to face will adhere. If a drawing board covered with black felt is used as a background and the board held vertically for display to the class, then other pieces of brightly colored felt can be placed thereon. Once placed, these will neither fall nor slip, and with their use the whole range of area properties can be demonstrated—triangle, parallelogram, trapezium, circle, Pythagoras, etc.

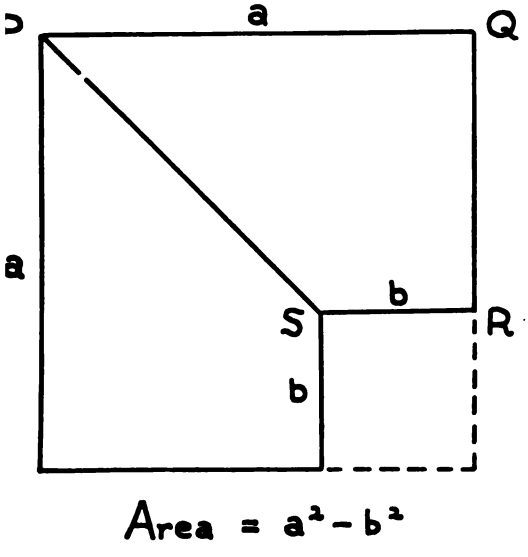

FIG. 1

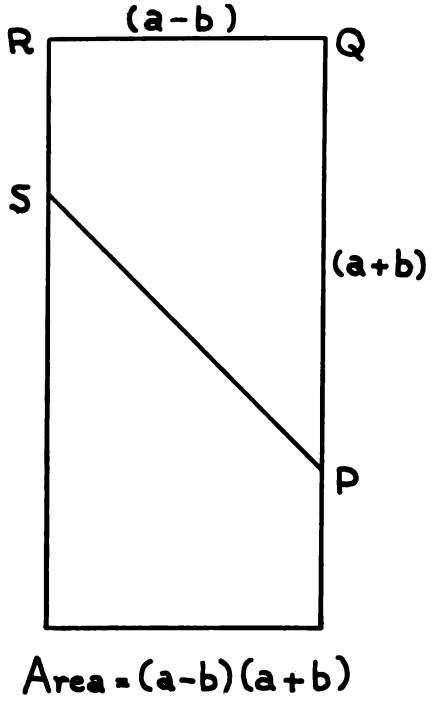

FIG. 2

* *Acknowledgement:* Drawings for publication in this section were produced by Patricia McGroder, Macalester College, St. Paul, Minnesota.

With an eight inch circle, half of red and half of yellow felt, cut into fourteen equal sectors, and two half-size sectors, the usual demonstration of the formula for area from the knowledge of the circumference is simple and expeditious. The tribulations of paper, paste and paint are avoided and a normally static demonstration becomes dynamic.

The bright colors in which these felts can be obtained make them admirably clear and easily seen. Preparation of the aids is speedy, and if each set is placed in a large labelled envelope storage is easy.

The following demonstration that $a^2 - b^2 = (a+b)(a-b)$, simpler than the usual one and more easily understood, does not seem to be widely known. The cutting of the two pieces of felt takes only a few minutes. (See Figure 1.) To complete the demonstration the trapezium $PQRS$ must be turned over and placed in the position shown in Figure 2.

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A Variable Parabola Demonstrator

A useful device for certain demonstrations concerning the parabola is illustrated in the accompanying figure. (See Figure 3.) A parabola is formed by a colored cord passed through eyelets at B_1, B_2, B_3, . . . B_n. A_1P is a rod from which the parabola is suspended. As A_1P is tilted at various angles with the horizontal the parabola changes dimensions; that is, the constants change, but the figure remains a parabola.

A wooden dowel one-half inch in diameter is ideal for the rod A_1P. Any kind of light weight string is suitable for the lines A_1B_1, A_2B_2, etc. Fish-line weights fastened to the ends of these strings will keep the parabola regular as the rod is tilted. For the eyelets at B_1, . . . B_n, one-half inch plastic rings, which can be obtained from any department store notions counter, will be found convenient. To fasten the strings at A_1, . . . A_n, holes should be drilled in the dowel, the strings pulled through, and wooden wedges (toothpicks or match stems) driven into the holes along side of the strings. The colored cord should be tied to the eyelet at B_n but simply threaded through the others so that it will slide freely when the rod is tilted. To keep the cord taut when the dowel is tilted a weight should be fastened to the cord at the left end.

The mathematics of this device is simple enough. Let A_1P and A_nB_n represent the x-axis and y-axis respectively in a rectangular coordinate system. The equation of the parabola in the position shown in Figure 3 will then be $x^2 = k(y+h)$, where k is negative. In accordance with this scheme A_1B_1, . . . A_nB_n are ordinates, and A_1A_n, A_2A_n, . . . are abscissas of the points B_1, . . . B_n.

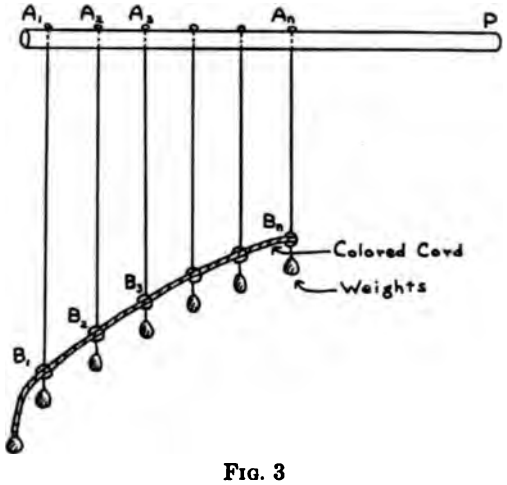

Fig. 3

In constructing the device the work can obviously be simplified by using the equation $x^2 = y$ to determine the shape of the curve, and then adding a constant length (A_nB_n) to each ordinate. It is an interesting problem for the student to show algebraically that the curve is always a parabola, no matter how the rod is tilted.

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IE PRACTICAL IDEAS ON MAKING CROSS NUMBER PUZZLES FOR JUNIOR HIGH SCHOOL STUDENTS

'oo often students become bored with ıding out answer after answer. The pose of this article is to suggest a ;ivating device which will enable the :her to re-create interest in solving :hematical problems. The device in- ves the construction of cross number zles which students can complete by ting in the correct answers to a set of blem exercises.

)irections for construction listed below l the accompanying illustration are gestive of what can be done. The puzzle ich is worked out is suitable for seventh l eighth graders. Teachers may con- uct puzzles themselves and use them regular classroom situations instead of ditional tests and written exercises, students and teachers can work them t together.

Steps to Follow

1. Select a basic square pattern to follow—6 by 6, 8 by 8, or 12 by 12.
2. Select from two to four definitions or problems per row horizontally; also follow this pattern vertically.
3. Each answer to a problem or definition should not be longer than can be written in four squares of any one row.
4. State each definition or problem very briefly. (See the sample puzzle.) When making problems for the horizontal rows very little difficulty will be found.
5. In selecting problems for the vertical rows a little clever thinking may be needed to find problems with answers that will fit the number combinations already given by the horizontal definitions and problems.
6. Select a title for each puzzle to add to its interest. The following are suggestive: "Areas and Perimeters," "Whole Number Processes," "Volumes," "Units of Measure," and "General Processes."
7. In the beginning, it is better to start with short puzzles than to plan long ones. In order to maintain student interest at a high level it is best not to include more than 25 items horizontally and the same number vertically.

The puzzles may be drawn and worked out on classroom blackboards or separate copies made for all students by duplicator processes. After presenting the idea to the class it is well to devote one full period to giving directions—i.e., showing the students how to outline the squares, and acquainting them with the numbering system. Students should be permitted to work at their own rate in class, and be encouraged to make similar puzzles at home.

Sample Puzzle: Areas and Perimeters

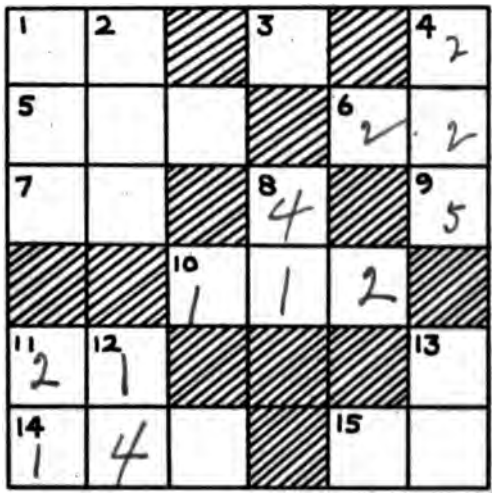

FIG. 4

Horizontal

1. Perimeter of a 9″ square.
3. Area of a 1″ square.
5. Area of a rug 9′×12′.
6. Circumference of a circle with a diameter of 7″.

7. Area of a rectangle $16'' \times 3\frac{1}{2}''$.
8. How many sides has a square?
10. Find the area of a triangle with a base of 16" and an altitude of 14".
11. Same definition as (11) vertical.
14. Perimeter of a wheel 49" in diameter.
15. How many 2" squares can be made to fit into a rectangle $4'' \times 10''$?

Vertical

1. Area of a room $30' \times 10\frac{1}{2}'$.
2. Area of a field 60 rd. $\times$ 10.1 rd.
3. Area of a unit square.
4. Area of a square 15 cm. on a side.
8. Perimeter of a fence 12 yd. $\times 8\frac{1}{2}$ yd.
11. Perimeter of a triangle which is 7" on each side.
12. Perimeter of a rectangle 4.8 cm. $\times$ 2.7 cm.
13. Perimeter of a lawn which measures $23\frac{3}{4}$ ft. in length and $21\frac{1}{4}$ ft. in width.

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A Trisection Device

A discussion of the problem of trisecting the general angle comes up perennially in plane geometry classes. That trisection is possible, even though it cannot be accomplished with straightedge and compasses, is common knowledge to most teachers of the subject, but a statement of this situation leaves curious students somewhat confused. The following device, which is an adaptation of the "tomahawk" described by Robert C. Yates in the *Eighteenth Yearbook*, can be produced with very few materials and should offer interested students some degree of satisfaction.[1] It is particularly useful for blackboard demonstrations.

Materials of construction include one coffee can top or other round flat disc approximately 5" in diameter, one strip of wood $\frac{1}{2}'' \times \frac{1}{2}''$ exactly three times the radius of the disc in length, one round $\frac{1}{4}''$ dowel approximately six times the radius of the disc in length, and two 3/16" stove bolts.

Figure 5 shows how the trisector is assembled. Let AB represent the $\frac{1}{2}'' \times \frac{1}{2}''$ wooden strip equal in length to three times the radius of the disc. Drill a $\frac{1}{4}''$ hole at the trisection point C. Note that AC is equal to the diameter of the disc, and CB is equal to its radius. Turn the wooden strip on its side and place it along a diameter of the disc. Drill two holes through the strip and the disc and secure the two pieces with stove bolts. Insert the dowel CD in AB at C. CD must be perpendicular to AB at C. Thus CD is tangent to the circular disc.

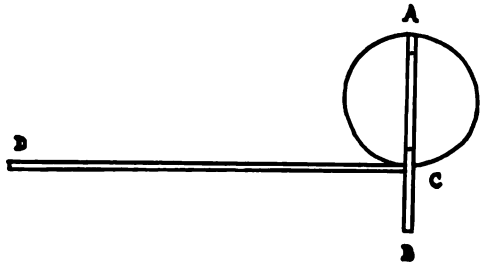

Fig. 5

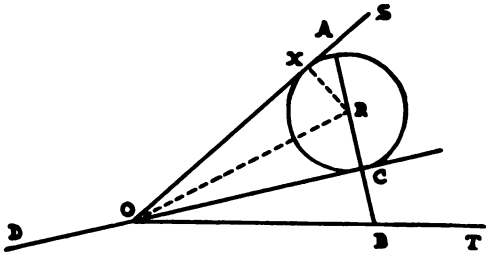

Fig. 6

Figure 6 illustrates how the device may be used to trisect an angle. To trisect angle SOT place the device so that B touches side OT. Adjust the device so that CD passes through O and the circular disc is tangent to OS. It can readily be demonstrated that right triangles XRO, ORC, and OBC are congruent. Hence angle COB is one-third of angle SOT.

[1] Yates, Robert C., "Trisection," *The National Council of Teachers of Mathematics Eighteenth Yearbook*, Bureau of Publications, Teachers College, Columbia University, 1945, New York, p. 148.

AIDS TO TEACHING

Edited by

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BOOKLETS

B. 42—How to Study—How to Solve

Addison-Wesley Press, Inc., Kendall Square Building, Cambridge 42, Mass.

Booklet by H. M. Dadourian; 5″×8″; 34 pages; $.25 each.

Description: This booklet is divided into two parts. The first gives hints on how to study. It speaks of the necessity for proper mental attitude, the helpful conditions while studying alone, benefits gained from studying in groups, and the correct approach for preparing for examinations. Such topics as the following appear: incentive, interest, self-confidence, common sense, regularity in study, concentration, orderliness, thoroughness, function of the classroom, function of the instructor, making summaries before examinations, and the will to learn. The second part gives fifteen specific hints for solving problems in mathematics. After the fifteen hints are given, they are discussed singly, using illustrations from college freshman mathematics.

Appraisal: Because the maturity of tone and the types of examples slant this booklet toward college topics it will be useful to college teachers and their pupils, and to secondary school teachers. It would be useful and interesting to have such a pamphlet prepared for secondary school pupils, or to have more of this material incorporated into regular textbooks at that level. There is no doubt that teachers could spend more time analyzing aloud, with some definiteness, the principles of method which apply to mathematics problems, or to studying in general. Many have not had the opportunity to formulate for themselves the principles they use, and so this book should help them do so. The greatest danger in using such a treatment is that teacher and pupils may both fall into the trap of thinking that analysis of problems, setting them up, classifying methods, and combining formulas by rule can ever replace the free-lance thinking which knows no rules or bounds. Some weak organizers can have their scope of problems broadened by mechanical rules, but the original thinker who does the problem not covered by the former rules will still get further in mathematics.

B. 43—Algebra Applied to Monroe Formula Analysis

Booklet; 8½″×11″; 24 pages; free.

B. 44—Machine Methods for the Extraction of Square Root

Booklet; 6″×9″; 11 pages; free.

B. 45—Machine Methods for the Extraction of Cube and Other Roots

Booklet; 6″×9″; 8 pages; free.

B. 46—Application to Simple Correlation

Booklet; 5½″×8½″; 8 pages; free

B. 47—Machine Shop Engineering

Booklet; 6″×9″; 27 pages; free.

B. 48—Machine Method for Use with "A Short Method of Evaluating Determinants"

Leaflet; 6½″×9½″; 4 pages; free.

B. 43 to B. 48 available from Monroe Calculating Machine Co., Inc., Orange, New Jersey.

Description of B. 43: This is actually a simple textbook in algebra summarizing the topics which an operator of calculating

machines would need to know. Although the introduction says it is for business figuring, it is material of general algebraic interest. Some of the topics covered will show this: symbolism, signs of operation and aggregation, addition, subtraction, multiplication and division of quantities, factors, equations, axioms, transposing (how did that word get in!), fractions, exponents, etc.

Description of B. 44: Three machine methods of finding square root are described: (1) subtraction method (successive odd numbers), (2) inspection method (algorithm method), and (3) Subtraction and division method (subtracting odd numbers followed by division to get final result).

Description of B. 45: A general formula for extracting any root of a given number by calculating machine methods is presented, and then examples are worked out for third and fifth roots.

Description of B. 46: A machine method is given for computing simple correlation from a series of paired, ungrouped data which results in ΣX, ΣY, ΣX^2, ΣY^2, and ΣXY at the end of the operation. These results are all found in the dials simultaneously.

Description of B. 47: The application of computing machines to certain machine shop engineering problems is explained for the following topics: jig boring machines, odd gear ratios, the design of long and short addendum gears, and square root (by the subtraction method).

Description of B. 48: Machine methods are described for obtaining an auxiliary matrix from any given matrix. This auxiliary matrix may be used to solve the linear equations of which the elements of the given matrix are coefficients, or it may be used to evaluate the determinant of which the elements of the given matrix are elements (if the matrix is square).

Appraisal of B. 43 through B. 48: B. 43 will be useful in any algebra classes whether they are interested in machine methods or not. It is a compact summary of a great deal of elementary algebra. B. 44 and B. 45 will be interesting only to groups which have machines for practice purposes. The former could properly be used in many secondary school classes, the latter probably not. Both may be useful in college business mathematics or statistics classes. On a secondary level the possibility of using these booklets as illustrative materials should not be overlooked: they could be worked through for understanding with no intention of teaching the skills through drill for complete mastery. B. 46 applies machine methods to correlation and so will be useful on a college level for the most part. B. 47 will serve as illustrative material in all types of mathematics classes of the kind of applications which machine shops use. In allied mathematics courses which are correlated with shop work they will be of very practical use. The final booklet B. 48 is of technical importance and may serve as illustrative material in college algebra classes or practical use in business and engineering mathematics courses, but not of any use on a secondary level. All the booklets reviewed seem to be carefully written except the last leaflet which seems to depend too much on a previous article referred to on the cover. The explanation is obscure without it.

B. 49—Your Money and the Federal Reserve System

The Federal Reserve Bank, Minneapolis, Minnesota.

Booklet; $10'' \times 12\frac{1}{2}''$; 32 pages; free.

Description: This booklet tells the story of money and the federal reserve system by discussing the following topics: What is money? How does money go around? How do you get your money? Where does new currency come from? Where does old currency go? Why will money buy things? How can so little cash finance so much business? Your money and the federal reserve system. How the Federal Reserve Banks serve industry, government, banks and agriculture. The federal reserve sys-

tem as a stabilizing force. Each topic is described concisely in words and illustrated completely with photographs.

Appraisal: This attractive booklet, written in very readable language and illustrated in a fashion similar to *Life* magazine, should be obtained by every mathematics and social studies teacher. Although it contains no mathematics other than simple data, it will give the social setting for a unit on banking. The photography is excellent and shows each step in the processes described in the text. The paper is heavy and durable. The titles are printed in green and the text is printed in heavy black type.

B. 50—Facts You Should Know about Life Insurance

National Better Business Bureau, Inc., Chrysler Building, New York 17, New York.

Booklet; 5½″×8¼″; 16 pages; $.05.

Description: An impartial treatment of the four fundamental types of insurance is presented in terms which are understandable for the layman. Information is given concerning the methods employed in calculating premiums and facts which must be considered if policies are to be purchased intelligently. The treatise is excellently handled.

The paper-covered booklet was not designed for prolonged use for cover and pages easily become detached and will not withstand much abuse. The print is clear and heavy type has been employed discriminately to emphasize important topics.

Appraisal: The material contained is pertinent to the new socialized junior high school mathematics curriculum and high school courses in general mathematics and consumer mathematics. This impartial treatment is superior to the sections on life insurance which are included in many mathematics texts. The booklet is excellently arranged and can do much to prepare students for making an expenditure which is a necessity rather than a luxury.

The booklet, as it stands, is appropriate for use on the high school level but may be adapted for use on the junior high school level by explaining such technical terms as "premium," "annuity," and "beneficiary."

Where school funds permit, it may be wise to purchase the booklets in quantities sufficient for each student. This expenditure is certainly justified by the results which may be easily attained. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

B. 51—Where Does the Money Go?

Union Dime Savings Bank, Avenue of the Americas, 40th St., New York 18, New York.

Booklet; 8″×10½″; 6 pages; free.

Description: The three-month expense record is designed as a guide for regular saving and wise spending. Each of the monthly records occupies two pages; seventeen columns are provided for recording expenditures made during the month. Among the expenses considered are savings, taxes, food, shelter, clothing, and advancement. Explanations appear at the heads of columns so that the budgeteer will be enabled to determine where his expenditures should be tabulated.

The booklet is printed in blue and red upon moderately thin white paper. It is not designed for durability but rather as a temporary expense account to be used only once.

Appraisal: The principal values of the expense record are in teaching the types of expenditures required and methods of wise budgeting. When used on the secondary level, the record will necessarily be somewhat artificial since most students have neither large incomes nor the expenses considered in the booklet. Nevertheless, a wise choice of money management problems by the teacher may result in real and lasting learning. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

B.52—Railroad 'Rithmetic Book I

Baltimore and Ohio Railroad Company; Baltimore, Maryland

Booklet; $8\frac{1}{2}'' \times 11''$; 60 pages; Free; Olive W. Dennis, Editor

Description: This is a booklet of railroad problems to supplement problems of the standard elementary textbooks. It contains ten units on topics such as: reading and writing numbers, using whole numbers, reading and using tables, using fractions, using decimals, finding per cent, solving two-step problems, using measurements, and using graphs. Answers are given for all problems in a separate section. The problems are real problems with data based on material supplied by many railroad companies. The problems are based on data such as miles of rail travel, cost of tickets, freight and maintenance, taxes, dimensions of cars, sharing freight charges, travel time, making change, areas, supplies, etc.

B.53—Railroad 'Rithmetic Book II

Baltimore and Ohio Railroad Company; Baltimore, Maryland

Booklet; $8\frac{1}{2}'' \times 11''$; 44 pages; 1950; Free; Olive W. Dennis, Editor

Description: This is a booklet of railroad problems to supplement problems of the standard textbook. It contains units on such topics as: percentage, passenger fares and freight fares, measurement, circle problems, cylinder problems, scale drawing, reading tables, and graphs. Answers are given for all problems in a separate section. The problems are all real problems with the data based on material supplied by many railroad companies. The problems are based on data such as the cost of supplies, amount of taxes, travel costs and data, car dimensions, volumes, sharing freight charges, distances and travel time, etc. The illustrations include time zones, scale drawings of cars, and a variety of graphs.

Appraisal of B.52 and B.53: These booklets will furnish the mathematics teacher with real life problems based on conditions encountered in the railroad industry since the statistical data and operating procedures have been obtained from railroad sources. As is so often true with real problems, the numbers involved are not always simple numbers, for example the investment return earned in 1947 was 3.41%. However, the booklet contains a great variety of problems suitable for supplementary work by the pupil with special ability or interest in railroads. The material is not copyrighted since the teacher is urged by the publisher to use it in courses of study or textbooks. It contains little direct advertising but, of course, it does contain problems that show the contribution of the railroads to society in such terms as economical transportation, payment of taxes, employment, etc. These are booklets that will furnish the elementary as well as secondary teacher of mathematics the kind of practical problems based on real data that are so often lacking in the textbook.

CHARTS

C.19—Graph Chart

The Mathaids Company; 336 Kirk Avenue; Syracuse, New York

Chart; $32'' \times 40''$; $8.50 (presswood) or $8.95 (Duron)

Description: A $32'' \times 40''$ tempered presswood board is coated with black "Liquislate"; horizontal and vertical lines are drawn at $1\frac{1}{4}''$ intervals to form a rectangular coordinate chart. The Duron model has "Liquislate" on the back and can be used as a portable blackboard. There is a 1-inch border around the top and right sides and borders of $3\frac{7}{8}''$ and $3\frac{3}{8}''$ on the left and bottom; these larger spaces are used for recording information concerning the graphs to be drawn.

Each corner of the board is reinforced with a brass jacket and metal eyelets protect the holes which facilitate hanging the

chart. Visibility of the squares is excellent due to the high degree of contrast between the "Liquislate" and bright yellow lines. The material is quite durable and the metal jackets will prolong the life of the chart. Chalk marks may be removed with an ordinary blackboard eraser but better results are obtained with the use of a clean rag.

Appraisal: This type of portable graph chart has been used extensively in classrooms which are not supplied with graphic wall blackboards. The problem of storage, however, makes this type of board less desirable than the flexible stencil-type or the blackboard chart which may be rolled up when not in use.

$8.50 or $8.95 is a high price to pay for this type of graph although it is sturdy and has several other desirable physical characteristics. (Reviewed by Bernard Singer, Hyannis, Mass.)

FILMSTRIPS

FS.74—Maps and Their Meanings

FS.75—We Live on a Huge Ball

FS.76—Flat Maps of a Round World

FS.77—Maps and Man

Popular Science Publishing Co.; 353 Fourth Avenue, New York 10, New York

35 mm filmstrips; Color and black and white; Approximately 50 frames each; Teaching Guide

Description of FS.74: This elementary filmstrip on maps begins by showing the relationship between pictures and maps. It then shows how symbols are used to indicate buildings, land and water forms, and other man-made or natural features. Examples of legends or keys for a map are illustrated. Direction and location on a map are related to east-west or north-south lines. The meaning and use of scale are illustrated in some detail. Maps using color to show relief or rainfall extend the information given by maps.

Description of FS.75: This filmstrip contains a series of pictures almost all of which are of globes. It shows the relationship between a map on a globe and a flat map. A series of drawings develops the idea of a North and South pole, equator and parallels of latitude to locate position on the earth. The relation of location and climate are presented briefly. The major emphasis of this filmstrip is the use of latitude to locate positions on the earth.

Description of FS.76: This filmstrip begins by showing the meaning of meridians of longitude on a globe. By numbering the meridians and using the latitude discussed in the previous strip of this series, the location of a city, Denver, is shown. The difficulty of making a two-dimensional map of the earth is illustrated by comparing the mercator, sinusoidal, and polar-equidistant projections. The kind of distortion involved is shown by comparing the map with the globe on the same frame. Several frames are used to give the pupil practice in using meridians and parallels to determine locations and distances with emphasis on longitude.

Description of FS.77: Maps such as weather maps, Polar Equidistant, projections, Mercator maps, maps of wind and ocean currents, maps of agricultural products, soil fertility, railroad networks, population, water resources, rainfall, and temperatures are used to illustrate the great variety and uses of maps. The convenience of a map to present a great deal of information concisely is emphasized.

Appraisal of FS.74 through FS.77: These filmstrips present an integrated elementary series on the reading of maps intended for use in the fifth, sixth, and seventh grade social studies classes. They will be useful for the mathematics teacher for background material when discussing projections and scale. However, most teachers will prefer to use real maps or a globe, to illustrate types of maps, symbols, or scale but the filmstrip is a convenient method of showing variety maps. The drawings are appropriate and clear with captions that require frequent class re-

sponses. A teaching guide which includes outcomes, generalizations, vocabulary, suggested activities and prints of every frame of the entire series will be most helpful for teachers.

INSTRUMENTS

I.28—World Hour Chart

Newsweek; 152 West 42nd Street, New York 18, New York

Chart; $3\frac{3}{4}'' \times 4\frac{1}{2}''$; Free

Description: The pocket-size chart consists of a piece of cardboard upon which are located the twenty-four time zones of the world. Two slots at the left top and bottom of the cardboard provide space for a non-ending tape upon which are printed twelve red digits and a like number in black to represent the twenty-four hours in each day. The tape may be turned until the correct Greenwich time is opposite the location of the observer. The chart is then in alignment for reading the times all over the world.

The chart is printed upon a sturdy piece of cardboard and will survive much use if care is taken to protect the fabric tape from damage.

Appraisal: The device may be used to motivate the study of time zones. With it the teacher may explain the concept of time variations due to revolution of the earth about its axis; meaning of the international date line becomes clearer when reference is made to the chart and students will enjoy checking answers to problems.

The chart is far too small to be used effectively as a demonstration device; each student should be supplied with one. (Reviewed by Bernard Singer, Hyannis, Mass.)

MODELS

M.15—Celestial Globe

Denoyer-Geppert Co.; 5235–5237 Ravenswood Ave., Chicago 40, Illinois

Globe; (G16 C 05) 16″; $40.00 (G12 C 05) 12″; $22.50

Description: The DG new 16″ celestial globe shows the important stars down to the sixth magnitude, distinctly on a light blue background. The latest constellation boundaries as fixed by the International Astronomical Union are used and the globe map is correct for Epoch 1930. It is mounted with either movable meridian with horizon (G16C5), Liberty stand with horizon (G16C05), plain stand (G16C1), or weighted base, movable meridian (G16C3).

Appraisal: If a student can pick up an object in his hands or twirl it around, he is more apt to learn the principles being put forth than is the case when all the work is done by the teacher. A globe such as this with all the essential markings would seem to be a most valuable instrument in the classroom. (Reviewed by Jacqueline LeMay)

PICTURES

P.9—Galileo

Denoyer-Geppert Company; 5235 Ravenswood Avenue, Chicago 40, Illinois

Picture; 26″×35″; Unmounted $2.00, five pictures $7.50, ten pictures $10.00; Lacquered panel, passe-partout edges $6.00; Lacquered panel, 3″ frame $10.50; Under glass, 3″ frame $12.50.

Description: Galileo is seen sitting at a desk, fingering a globe and writing. Another larger globe, a plumb-bob (possibly a sundial or groma) and an instrument which may be a telescope surround him. The composition and coloring in the picture are very attractive.

Appraisal: For static, artistic pictures of mathematicians or users of mathematics this is certainly much better than the usual head-and-shoulders presentation. The instruments, even though their use is not always clear, are suggestive of mathematical activity. With supplementary reading, such pictures would make mathematics seem to be the useful tool of adult workers, rather than a series of disconnected problems in a textbook.

A small leaflet showing this picture is available upon request.

SOURCES OF MATERIAL FOR LABORATORY WORK

SL.20—Thermium Film

Yoder Instruments; 140 North Market Street, East Palestine, Ohio

Protective Film; Several sheet sizes and roll sizes available; price varies depending on size.

Description: Thermium Film is a protective, transparent covering that can be applied to pictures, charts, drawings, records, etc. It protects them from dirt, smudging and tearing. It is applied by placing the sheet over the material to be covered and ironing over the sheet with a warm flatiron or specially designed Thermo-Welder. The film is available in a large assortment of cut sheet sizes as well as in roll form. The prices range from seventy-five cents for a dozen sheets $8\frac{1}{2}\times 11$ inches to $16.50 for a 100 foot roll of 25 inch width.

Appraisal: This protective film is completely transparent and colorless. It is easy to apply if one has the iron at the right temperature. It will be necessary for the teacher to experiment with different temperatures for the iron to determine the best temperature for effective use. (A small size Thermo-Welder for $4.25 will solve this problem.) It is an effective material to use for the protection of surfaces, even though it is expensive. Material covered by this film can be cleaned by wiping with a damp cloth. It is advertised that it will not deteriorate or discolor with age.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
School of Education, New York University, New York 3, N. Y.

MULTIPLICATION may be made comparatively easy if we follow the procedure illustrated by the Arab mathematician *Beha ed-din al'Amili, Mohamed ibn Hosein* (1547–1622). All that one needs to know is compressed into the multiplication facts limited to the first nine integers and into the proficiency in column addition of the first nine integers.

Let us illustrate this method by a few examples. We will first multiply 36 by 83.

Draw a rectangle containing four cells. Four cells are required because each of the numbers consists of two digits. If one of the numbers contains three digits and the other number contains four digits, then the required rectangle will contain twelve cells. Generally, if one number contains m digits and the other number contains n digits, then the required rectangle will contain mn cells.

The rectangle is so drawn that one of its vertices rests on a horizontal straight line as shown in Figure 1. Draw vertical diagonals of the cells of the rectangle until they meet the horizontal straight line AB.

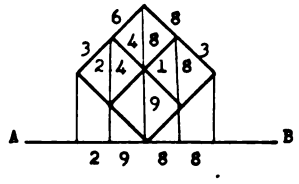

FIG. 1

Write the two numbers along the two upper sides of the rectangle is shown in Figure 1. Note that one digit is placed so that it occupies a position along the outer side of a single cell. Also note that every diagonal divides its cells into two *subcells*.

Multiply 6 by 8. The product is 48. Write this product in the cell which is ad-

jacent to 6 and 8. Furthermore, place one and only one digit in a subcell (as shown in the figure above).

Multiply 3 by 8 and write the product 24 in a manner described in the preceding paragraph. Note that the cell for this product is determined according to the principles which are employed in "tables" of numerical data. Then multiply 6 by 3 and write the product 18 in the manner described above. Finally, multiply 3 by 3. Note, however, that the product 9 is written in the subcell which is on the right of the diagonal of the cell, while the subcell on the left of the diagonal remains unfilled. The reader will surmise by this time that the subcell on the right of a diagonal is assigned to *units*, while the subcell on the left of a diagonal is assigned to *tens*.

After all the separate multiplications are completed perform column additions as indicated in the figure above. If the sum of a column is greater than 9, "carry" the units in the *tens* to the next column on the left. Thus the product of 36 and 83 is 2988.

Let us multiply 68924 and 3931. The procedure is exactly the same as described above. The required rectangle will contain 20 cells. This is shown in Figure 2.

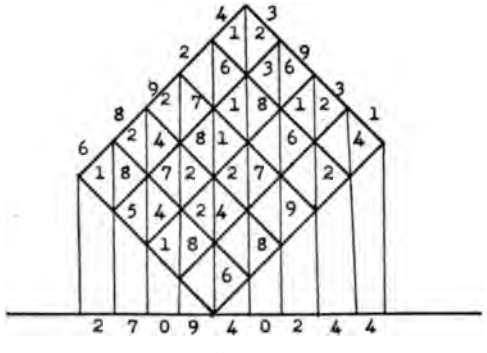

FIG. 2

The method of multiplication described and illustrated above can be theoretically developed on the level of eleventh year algebra. It may be noted that the vertical lines (the diagonals of the rectangular cells) represent the demarcation lines of the "places of significance" in the product of two numbers.

If we write the two numbers as

$$10^m a_m + 10_m^{m-1} a_{m-1} + \cdots + 10^2 a_2 + 10 a_1 + a_0$$

and

$$10^n b_n + 10^{n-1} b_{n-1} + \cdots + 10^2 b_2 + 10 b_1 + b_0,$$

where

$$a_i = 0, 1, 2, \cdots, 8, 9, \qquad i = 0, 1, 2, \cdots, m$$

and

$$b_j = 0, 1, 2, \cdots, 8, 9, \qquad j = 0, 1, 2, \cdots, n.$$

The product of the two numbers is obtained as the product of two polynomials. Since the diagonals of the cells provide for the "places of significance" and since at no time the product of a_i and b_j can exceed 81, two subcells are sufficient for the writing of the individual products of the coefficients a_i and b_j which are found in the polynomial representations of the two numbers respectively.

Here is an advance notice of the problem which will be discussed in the next issue of THE MATHEMATICS TEACHER. It is an interesting teaser for a Geometry classroom. Divide the area of a circle into n (n, any integer) equal parts, but the division of the circumference of the circle is not permitted, nor is the drawing of concentric circles or of parallel chords permitted. (These cases will be discussed in a subsequent issue of THE MATHEMATICS TEACHER.)

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

16. Algebraic Tick-Tack-Toe

This version of an old game was described to the editor of this department by Mrs. Lloyd Middleton Garris of the Lincoln Memorial High School of Palmetto, Florida.

Two students alternately fill in with signed numbers the cells of the ordinary tick-tack-toe diagram. Their object is to make each row, column, and diagonal sum to zero. The first student to make the sum of a row, column, or diagonal something other than zero loses the round.

Figure 1 shows a typical round played by "*A*" (italicized numbers) and "*B*". The sequence of play was as follows:

−9	−5	14
13	5	*−18*
−4	*0*	*4*

FIG. 1

A, 4; *B*, 5; *A*, −9; *B*, −5; *A*, 0; *B*, 14; *A*, −18; *B*, 13; *A*, −4. This game was won by *B*.

The possibilities of varying the game by such devices as choosing a sum other than zero, using literal terms also, or using other operations than addition are obvious as are also the uses of the game for motivating drill and reviews and for merely adding to the fun and interest to be found in a mathematics class or club.

The connection with magic squares is also obvious and might well motivate a report or project dealing with this latter topic.

17. Geometric Proof of the Addition Formulas for the Tangent

In this note the formulas for tan $(\alpha+\beta)$ and tan $(\alpha-\beta)$ are developed geometrically. Similar developments for sin $(\alpha\pm\beta)$ and cos $(\alpha\pm\beta)$, are given in textbooks on trigonometry, while tan $(\alpha+\beta)$ and tan $(\alpha-\beta)$ are commonly developed algebraically.

For the sake of simplicity the angles α and β are restricted to positive acute angles whose sum is an acute angle. An extension of the method of this note may easily be made so as to include all angles α and β for which tan α, tan β, tan $(\alpha+\beta)$ and tan $(\alpha-\beta)$ are defined.

Let (Fig. 1) α and β be two acute angles whose sum is an acute angle. Let P be any point on the positive end of the terminal side of $(\alpha+\beta)$. Drop the perpendiculars PA and PC to the initial and terminal sides respectively of α. From C drop the perpendiculars CD and CB to the lines PA and OA. Since the sides PC and PD of $\angle CPD$ are perpendicular to the sides of α, and since α and $\angle CPD$ are both acute, they are equal. Therefore, right triangles DPC and OBC are similar.

From Fig. 1,

$$\tan(\alpha+\beta)=\frac{AP}{OA}=\frac{AD+DP}{OB-AB}$$

$$=\frac{BC+DP}{OB-DC} \qquad (1)$$

since $AD=BC$ and $AB=DC$. When both numerator and denominator of the right side of (1) are divided by OB,

$$\tan(\alpha+\beta)=\frac{\frac{BC}{OB}+\frac{DP}{OB}}{1-\frac{DC}{OB}}. \qquad (2)$$

Now

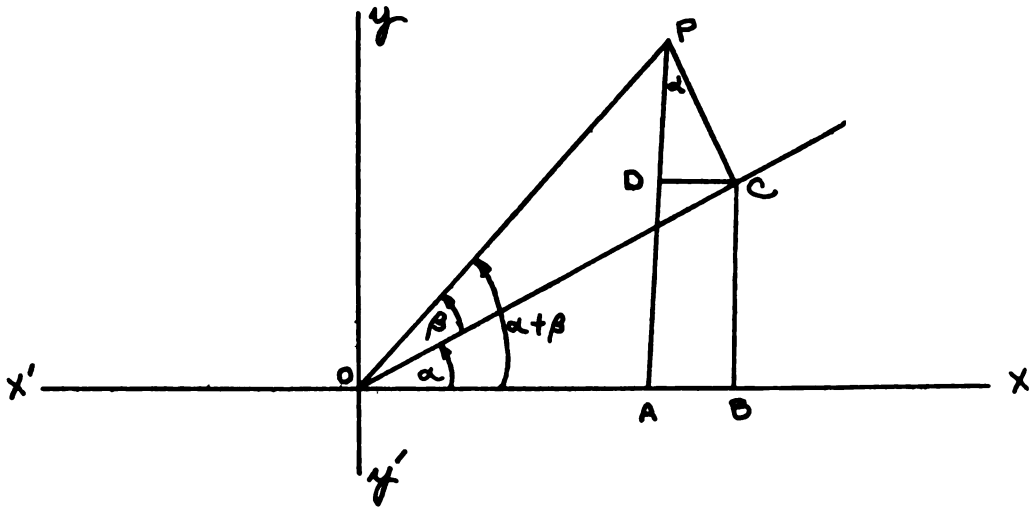

FIG. 1

$$\frac{DP}{OB}=\frac{DP}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB} \quad \text{and}$$

$$\frac{DC}{OB}=\frac{DC}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB}.$$

Consequently,

$$\tan(\alpha+\beta)=\frac{\frac{BC}{OB}+\frac{DP}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB}}{1-\frac{DC}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB}}. \quad (3)$$

From the similar triangles PDC and OBC we have

$$\frac{DP}{PC}=\frac{OB}{OC}$$

and hence

$$\frac{DP}{PC}\cdot\frac{OC}{OB}=1.$$

Also, from the same triangles

$$\frac{OC}{OB}=\frac{PC}{PD}$$

and consequently

$$\frac{DC}{PC}\cdot\frac{OC}{OB}=\frac{DC}{PC}\cdot\frac{PC}{PD}=\frac{DC}{PD}.$$

Then (3) becomes

$$\tan(\alpha+\beta)=\frac{\frac{BC}{OB}+\frac{PC}{OC}}{1-\frac{DC}{PD}\cdot\frac{PC}{OC}}. \quad (4)$$

Since

$$\frac{BC}{OB}=\tan\alpha, \quad \frac{DC}{PD}=\tan\alpha,$$

and

$$\frac{PC}{OC}=\tan\beta,$$

the substitution of these values in (4) yields

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}. \quad (5)$$

To prove the formula for $\tan(\alpha-\beta)$ take α and β acute, and $\alpha>\beta$ so that $\alpha-\beta$ is a positive acute angle. This restriction is made only for simplicity. Let (Fig. 2) P be a point on the terminal side of $\alpha-\beta$. From P drop perpendiculars PC and PA to the terminal and initial sides respectively of α. From C drop a perpendicular CB to OA and from P drop a perpendicular PD to CB. From Fig. 2,

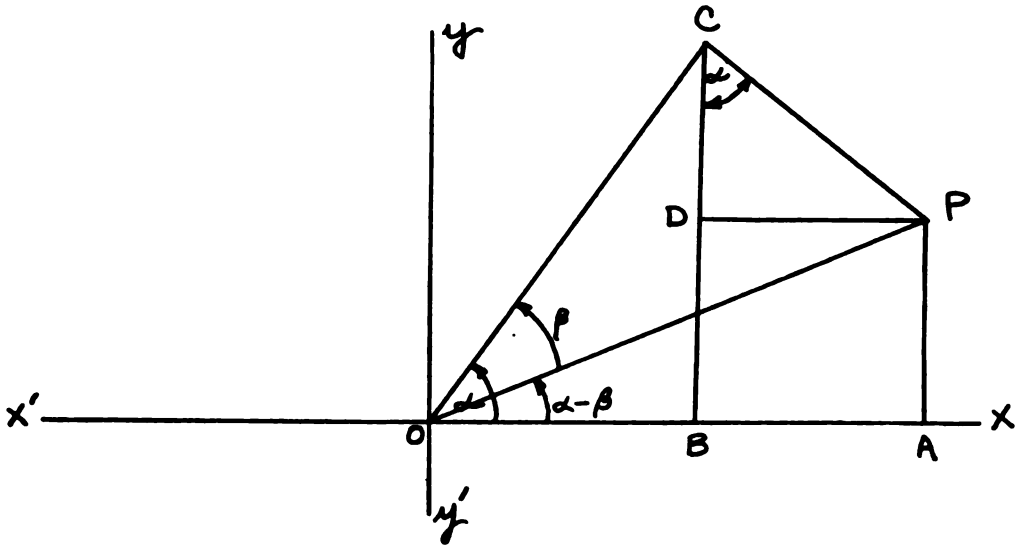

FIG. 2

$$\tan(\alpha-\beta)=\frac{AP}{OA}=\frac{BD}{OB+BA}=\frac{BC-DC}{OB+DP}, \tag{6}$$

since $AP=BD$ and $BA=DP$. If the numerator and the denominator of the right side of (6) are divided by OB,

$$\tan(\alpha-\beta)=\frac{\frac{BC}{OB}-\frac{DC}{OB}}{1+\frac{DP}{OB}}, \tag{7}$$

which may be written

$$\tan(\alpha-\beta)=\frac{\frac{BC}{OB}-\frac{DC}{CP}\cdot\frac{CP}{OC}\cdot\frac{OC}{OB}}{1+\frac{DP}{CP}\cdot\frac{CP}{OC}\cdot\frac{OC}{OB}}. \tag{8}$$

From the similar triangles OBC and CDP,

$$\frac{DC}{CP}=\frac{OB}{OC}$$

and hence

$$\frac{DC}{CP}\cdot\frac{OC}{OB}=1.$$

From the same triangles

$$\frac{OC}{OB}=\frac{CP}{CD}$$

and

$$\frac{DP}{CP}\cdot\frac{OC}{OB}=\frac{DP}{CP}\cdot\frac{CP}{CD}=\frac{DP}{CD}.$$

Equation (8) may now be written

$$\tan(\alpha-\beta)=\frac{\frac{BC}{OB}-\frac{CP}{OC}\cdot\frac{DC}{CP}\cdot\frac{OC}{OB}}{1+\frac{DP}{CP}\cdot\frac{OC}{OB}\cdot\frac{CP}{OC}}=\frac{\frac{BC}{OB}-\frac{CP}{OC}}{1+\frac{DP}{CD}\cdot\frac{CP}{OC}}. \tag{9}$$

Now

$$\frac{BC}{OB}=\tan\alpha,\ \frac{DP}{CD}=\tan\alpha,\ \text{and}\ \frac{CP}{OC}=\tan\beta.$$

Hence (9) becomes

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}, \tag{10}$$

which is the required formula.

By means of Figures 1 and 2 and the above methods, the formulas for cot $(\alpha+\beta)$ and cot $(\alpha-\beta)$ may be derived. Similarly formulas for sec $(\alpha\pm\beta)$ and csc $(\alpha\pm\beta)$, in terms of csc α, csc β, sec α, and sec β, which are not in general use, may be obtained. Thus it is possible to derive geometrically addition formulas for the six trigonometric functions.

Herbert L. Lee,
University of Tennessee,
Knoxville, Tennessee

18. The Nedians of a Plane Triangle

The *medians* of a triangle are the straight lines drawn from the angular points to the mid-points of the opposite sides. It is well known that the medians are concurrent, the point of intersection being the centroid of the triangular area. Fig. 2 illustrates this. Denoting the lengths of the sides of the triangle by a, b, c, and the lengths of the medians AD, BE, CF

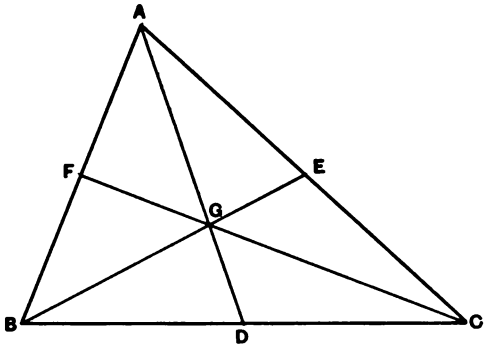

Fig. 2

by m_1, m_2, m_3, respectively it is easily proved that

(1) $m_1^2+m_2^2+m_3^2=(3/4)(a^2+b^2+c^2)$.

If the sides BC, CA, AB are divided at D', E', F', respectively (Fig. 3), such that

$$BD'=\frac{1}{N}BC,\quad CE'=\frac{1}{N}CA,\quad AF'=\frac{1}{N}AB$$

(where N is a positive integer) the lines AD', BE', CF', may be called the *nedians* (just as medians refer to the mid-points, nedians refer to the $1/N$ points).

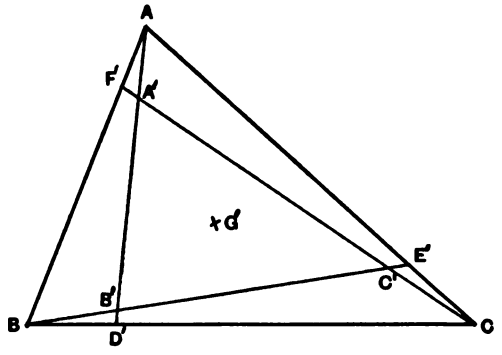

Fig. 3

Denoting the lengths of the nedians by n_1, n_2, n_3, respectively, it may be shown by analytical geometry that

$$(2)\qquad n_1^2+n_2^2+n_3^2=\frac{(N^2-N+1)}{N^2}(a^2+b^2+c^2)\cdot$$

Putting $N=2$, makes the nedians become the medians and the value of

$$\frac{(N^2-N+1)}{N^2}$$

becomes $\frac{3}{4}$, as in (1).

Putting $N=1$ places D', E', F' at C, A, B, respectively. Letting N increase without bound makes D', E', F' approach B, C, A respectively as limiting positions. In these two cases the nedians coincide with the sides of the triangle ABC and

$$\lim_{N\to\infty}\frac{(N^2-N+1)}{N}=1.$$

Calculus shows that

$$\frac{(N^2-N+1)}{N^2}$$

has a minimum value of $\frac{3}{4}$, which occurs when $N=2$, when the nedians become the medians.

The triangle $A'B'C'$ formed by the nedians may be called the Nedian Triangle. My colleague, Professor Petrie of the Ontario College of Education, proved by analytical geometry, that the area of the Nedian Triangle

(3) $= \dfrac{(N-2)^2}{(N^2+N-1)} \cdot$ (Area of the triangle ABC.)

The expression

$$\frac{(N-2)^2}{(N^2+N-1)}$$

equals unity when $N=1$, and also approaches 1 as a limit when N increases without bound. This expression has a minimum value of zero when $N=2$.

So far we have put $BD'=(1/N)BC$ and so on and gone around the triangle ABC anti-clockwise. We may also go around the triangle clockwise, making $CD''=(1/N)CB$, $BF''=(1/N)BA$ and $AE''=(1/N)AC$, (Fig. 4). We get a different set of nedians and for the sake of distinction we may call the first set the *forward nedians* and the second set the *backward nedians*, and the triangle $A'B'C'$ the *forward nedian triangle* and the

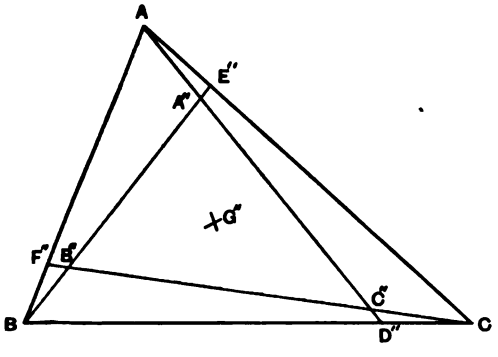

FIG. 4

triangle $A''B''C''$ the *backward nedian triangle*. Denoting the lengths of the backward nedians by n_1', n_2', n_3' respectively, we have naturally (by the same proof as for (2) and (3))

(4) $\quad n_1^2+n_2^2+n_3^2=n_1'^2+n_2'^2+n_3'^2$

and the

(5) Area of $A'B'C'$ = Area of $A''B''C''$.

The triangles $A'B'C'$, $A''B''C''$ although equal in area are not similarly-shaped. It is however, very interesting to find that their *centroids are common with the centroid of the main triangle*. The proof is easy.

If we divide each of the sides of the triangle ABC into N equal parts, and draw lines joining the vertices to the division points we get a figure like Fig. 5, which shows the case for $N=5$. The lines other than the forward and backward nedians defined as above need no special names, unless *intermediate nedians* be applied to them. It will be seen that the intersections of what may now be called corresponding pairs of forward and backward nedians lie upon the medians of the triangle ABC. That this should be so follows from *Ceva's Theorem* which states that "If three

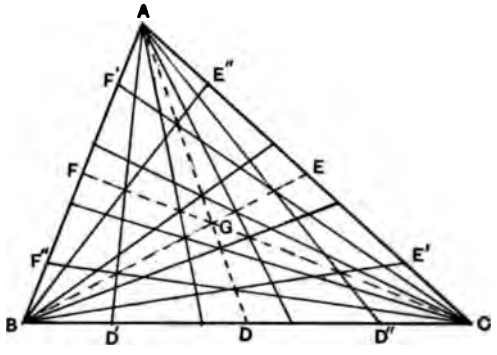

FIG. 5

concurrent straight lines are drawn from the angular points of a triangle to meet the opposite sides (Fig. 6) the product of these alternate segments taken in order is

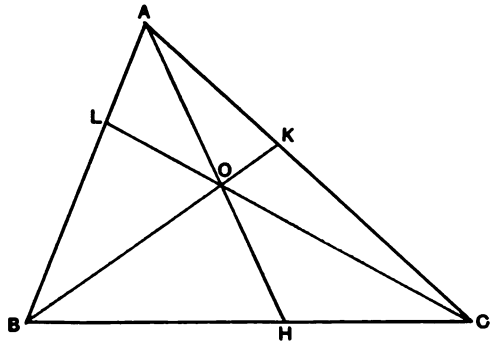

FIG. 6

equal to the product of the other three segments i.e. $BH \cdot CK \cdot AL = HC \cdot KA \cdot LB$. The converse to Ceva's Theorem is also true. These conditions are fulfilled in our array of nedians and medians, hence the prolific concurrency as illustrated in Fig. 5.

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APPLICATIONS

Edited by SHELDON S. MYERS
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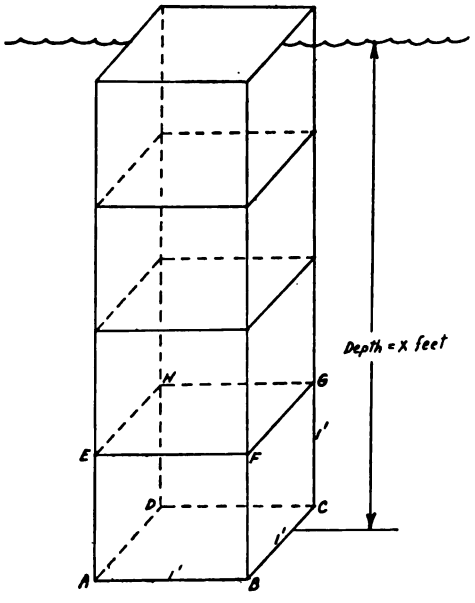
P.1 Gr. 10–12 USING THE METHOD OF FORMAL PROOF TO ESTABLISH TWO IMPORTANT PRINCIPLES OF PHYSICS

There are many who hold the erroneous belief that applications of high school mathematics to the sciences is impractical. This stems usually from the false assumption that too much time is needed to provide students with the scientific background necessary to understand the application. The applications of this department, and especially the one below, should prove convincingly that this need not be the case.

PRINCIPLE 1

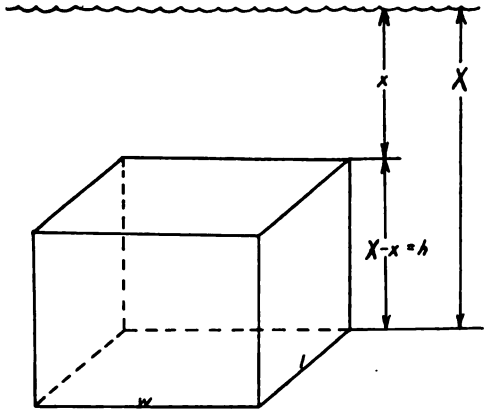


Given Data: Square foot $ABCD$ at depth of X feet in water
$EFGH-ABCD$ is a cubic foot
D is the density of water in pounds per cubic foot

Prove: Pressure, P, of water on $ABCD$ at x feet depth is xD pounds per square foot
or $P=xD$

Arguments	Authorities
1. Cube $EFGH-ABCD$ weighs D pounds.	1. Density (D) is the weight in pounds of a cubic foot. (Definition)
2. There are x cubic feet of water resting on $ABCD$.	2. Given that the depth of $ABCD$ is x feet and $ABCD$ is a square foot.
3. Total weight of the column of water on $ABCD$ is xD pounds.	3. Number of cubic feet times the weight of each cubic foot equals the total weight.
4. Pressure of $ABCD$ at x feet is xD pounds. $P=xD$	4. Pressure is the force in pounds on a square foot. (Definition)

PRINCIPLE 2



Given Data: Rectangular solid with dimensions l, w, h
Depth of top surface is x
Depth of bottom surface is X

Prove: The buoyancy of water on the rectangular solid in the figure is equal to the weight of the water displaced by the object.

Arguments	Authorities
1. Pressure, at x feet is xD lbs. per square foot. Pressure, at X feet is XD lbs. per square foot.	1. Principle 1 above: $P=xD$ Principle 1 above: $P=xD$
2. Area of top surface is lw Area of bottom surface is lw	2. Area of rectangle $=lw$
3. Force down on top surface $=lwxD$ Force up on bottom surface $=lwXD$	3. Total force = force on each square foot times the number of square feet. Also: pressure is exerted in all directions. (Postulate about liquids)
4. Buoyancy = Force up − Force down	4. Buoyancy is defined as the net upward push of the liquid on an immersed object.
5. Buoyancy $=lwXD-lwxD$	5. Substitution
6. Buoyancy $=lwD\ (X-x)$	6. Factoring
7. Buoyancy $=lwDh$	7. Substitution (see figure)
8. $lwh=V$ (Volume of solid)	8. The object is a rectangular solid.
9. Buoyancy $=DV$	9. Substitution
10. Volume of solid = volume of displaced water	10. An object displaces its own volume of liquid. (Postulate about liquids)
11. DV = weight of displaced water	11. Pounds per cubic foot (D) times number of cubic feet (V) equals total weight.
12. Buoyancy = weight of water displaced	12. Substitution

Principle 2 is known as *Archimedes rinciple* in a restricted sense. Archimedes rinciple applies to *any* shaped object mersed in *any* position in *any* liquid. gnificant questions can be raised with e students about the nature of proof in is situation: To what extent can the principle proven above be generalized without further proof to the broader principle of Archimedes? What additional points must be established before complete generalization is possible?

AR. 5 GR. 7–9 FAMOUS EXPLOSIONS ON THE EARTH

Here is a guide sheet developed by Dr. John Kinsella at the University School, Ohio State University, in 1943.

1. Mt. Etna once blew off 450,000,000 gallons of water as steam in two weeks. How many gallons were blown off in one hour? _____

2. The Valley of Ten Thousand Smokes in Alaska, in one year, sent up 300,000 tons of a gas, smelling like rotten eggs, called hydrogen sulphide. It also gave off about four times as much hydrochloric acid gas. (This acid is used to clean metals to be soldered together). This is _____ tons.

3. In June 1912, the Alaskan volcano Katmai exploded and hurled a cubic mile of ash and fumes into the air. What is a cubic mile? _____

_____

One hundred miles away, darkness occurred for about 3 days. Ashes may settle from 6 to 9 times that distance, or _____ to _____ miles.

4. In August 1883 near Sumatra, the volcanic island, Krakatao, exploded, blowing away two-thirds of the island. A cubic mile of rock dust rose 20 miles above sea level, traveling around the earth near the equator, making dusky sunsets for months. This explosion, heard 2,000 miles away, caused tidal waves that drowned 36,000 people.

a) From New York to San Francisco is about 3,500 miles. What fractional part of this distance was the explosion heard?_____. Reduce this fraction _____.

b) How many feet is 20 miles?_____.

5. In 1815 Tomboro, a volcano in the island of Sumbawa, near Java, blew away its top. This top was a cone about 4,224 feet high and 5 miles across at its base. What was its volume in cubic miles? _____ What fractional part of a mile is 4,224 feet? _____ Reduce this fraction _____.

6. Tomboro produced about 30 to 50 cubic miles of dust and coarser materials. This dust filled the upper air, greatly reducing the intensity of the sunshine and lowering the temperature of the earth. This caused the year 1816 to be abnormally cold which is why it is called the year without a summer.

a) If it had produced 8 cubic miles of dust, this dust could have been contained in a cube _____ miles by _____ miles by _____ miles in size.

b) A cube containing 27 cubic miles would be _____ miles by _____ miles by _____ miles.

c) A cube containing 64 cubic miles would be _____ miles long, _____ miles wide and _____ miles high.

d) Therefore 30 to 50 cubic miles of dust would be contained in a cube about _____ to _____ miles long, from _____ miles to _____ miles wide and from _____ to _____ miles high.

AL. 1.5 GR. 9–10 A SIMPLE FORMULA

$I = .000012\ L(T_2 - T_1)$

I = Increase in length of paving (cement) in feet due to heat expansion.

T_1 = normal temperature of the highway in degrees Fahrenheit

T_2 = higher temperature due to summer heat

L = length in feet of concrete highway section

Problems to work: Find I when $L = 30'$, $T_2 = 110°$; $T_1 = 60°$. How are concrete highways built to prevent cracking due to expansion? What does $(T_2 - T_1)$ mean?

The above example provides the opportunity to teach the increment notation, ΔT, to stand for $T_2 - T_1$.

C. Al. 2 Gr. 10–12 Second Degree Curves and Bridges

Here are two problems used in advanced algebra and senior mathematics by Eugene Smith, University School, Ohio State University.

A concrete overhead, parabolic in shape and 20 feet high, is to span a highway 16 feet wide. The problem is to determine how high above the road it should be for each of the 16 feet. Since the parabola is a symmetric curve, it will be necessary to compute these heights for only eight one-foot intervals measured from the middle of the road to the shoulders. The formula to be used is:

$$Y=-\frac{4H}{w^2}x^2+H$$

where H is 20 feet, w is 16 feet, and x is the distance in feet measured from the center of the road. Y is the corresponding height in feet. On graph paper draw to scale a plan of the overhead with values of Y measured to the nearest tenth of a foot.

Bridges over small rivers are occasionally elliptic in outline because of their beauty. Suppose you have to design and plan to scale an elliptic bridge with a span of 800 feet and a maximum above-water height of 40 feet. The formula to be used is:

$$Y=\frac{h}{S}\sqrt{S^2-4x^2}$$

where h is the maximum height above the water, S is the span, x is the distance from the middle of the river, and Y is the height in feet above the water for the corresponding value of x. Here again symmetry will make it necessary to compute only half of the values of Y. Choose your own interval so that your bridge design is possible without too much guesswork.

Here are the solutions to last month's problems:

C. Al. 1 Gr. 10–12 Arithmetic Series in the Home

(See last month's figure.) $a=99''$, $n=17''$, $d=5.8''$

$$L=a-(n-1)d$$
$$L=99-(17-1)5.8$$
$$L=6.2''$$
$$s=n\left(\frac{a+1}{2}\right)$$
$$s=17\left(\frac{99+6.2}{2}\right)$$

$s=894.2''$ or $74.5'$ A total of 74.5 feet of lath was needed to protect the cellar steps.

Al. 3 Gr. 9–10 Developing a Formula for Grading a Standardized Test of Fifty Items

Score = Rights − Wrongs $(S=R-W)$

$W=50-(R+0)$

Substituting: $S=R-[50-(R+0)]$

Simplifying: $S=2R+0-50$

Note on Ar. 4 Gr. 7–9 Snow on the Roof

A mathematics instructor in the public schools who is also a federal weather forecaster in Springfield, Ohio came up with these figures concerning the 23.9-inch snowfall last November. Based on a city area of 8,395 acres, an estimated 3,327,744,420 pounds, or 1,663,872 tons of snow fell on the city in four days. Figuring 480 snowflakes per ounce of snow, roughly 25½ trillion flakes fluttered down, but the count is not guaranteed. When melted down, this much snow amounts to 391,762,500 gallons, or about 27 days of normal pumpage by the city waterworks.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
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The Question: Does success with the formal steps of problem analysis in arithmetic tests insure success in solving whole problems?

The Study: Burch, Robert L. *An Evaluation of Analytic Testing in Arithmetic Problem Solving.* Duke University. 1949.

In our efforts to make students more successful in solving arithmetic problems, many of us, including authors of textbooks, have encouraged pupils to respond to such questions as, "What does the problem tell?", "What is to be found?", "What is to be done?" and "What is the best guess?". The study of Dr. Burch throws some doubt on the over-all effectiveness of this step-by-step approach.

Twenty one teachers in twenty classrooms and four schools were involved in the experiment over a period of two weeks in grades 4, 5 and 6. Two problem tests were devised, each consisting of thirteen problems whose solution depended on one arithmetic operation. Two problems involved addition, two subtraction, four multiplication and five division. The first test (S), of a multiple-choice type, required the student to select, for each problem, the correct response to each of the four questions listed in the preceding paragraph. The second test (O) consisted of the same 13 problems but required only that the problem be solved in the space provided.

Four experimental groups of classes were formed. Each group took one of the two tests during the first week of March, 1948, and the other two weeks later. The four groups were designated (O,O), (O,S), (S,S) and (S,O); the order of the letters indicates the order of taking the two tests. In addition to this testing interviews were held with 23 pupils of grade 4, 17 in grade 5 and 22 in grade 6. The purpose of these was to find out what means were used in solving the problems.

Findings

1. Success in selecting the correct process in solving the problems was usually "not dependent upon prior success with any certain step or combination of steps." (P. 121.)

2. Attempting to follow the step-by step procedure in problem solving "may actually lead to a lower level of performance on the total task." (P. 123.) In other words, the use of "steps" frequently causes confusion.

3. Furthermore, a large number of pupils who had little success in responding to the "step" questions (low S− Scores) were frequently successful in solving whole problems (high O− scores).

4. Students usually respond correctly to the question, "What is to be done?" after they react accurately to a combination of the other steps. "However, the correct response to the combination is probably not the factor which accounts for the correct determination of the algorism." This is supported by the results of the oral interviews which showed that the students did not use the formal step analysis in getting their solutions but, instead, used various cue words and phrases and capitalized on the elimination of the less plausible responses.

In concluding Dr. Burch observed that "when tests are constructed to measure separately the parts of a total task, the test builder must try to find whether performance in response to the parts is comparable to performance for the whole." (Pp. 125–6.)

The Question: What difference should philosophies of education make to the teacher of mathematics?

The Study: Ibrahim, Abdel-Aziz E. *Philosophies of Education, Their Implications for Mathematics Curricula and Classroom Procedures.* Ph.D. dissert. Ohio State University. Columbus, Ohio, 1948.

"ONE cannot teach successfully unless he studies the fundamental reasons for things. Unless practice stems from theory and returns to illuminate it, practice becomes in danger of being stagnant, and theory in danger of becoming futile" (p. 11). With this point of view Dr. Ibrahim proposed "to make explicit the implications for the teaching of mathematics in general education of each of the major educational philosophies and their associated theories of learning" (p. 7).

Idealism, realism and pragmatism were selected as the "major educational philosophies." Next, the essential ideas in each were explained and differences noted. Corresponding to each philosophy Dr. Ibrahim found that a particular psychology of learning was more appropriate than any other. For instance, by a deductive process he discovered that organismic psychology was more consistent with the definitions and assumptions in pragmatism than with those of idealism and realism. Finally, each of the three philosophies with their corresponding psychologies of learning were applied to mathematics teaching and curriculum problems.

Illustrations of the kinds of observations Dr. Ibrahim was led to make follow:

Teachers who believe that mathematics is a man-made creation will not guide learning in the same way as those who consider mathematical concepts, principles and propositions to be eternal truths.

Teachers who believe mathematics in its creation and development to be independent of the world of natural phenomena will not instruct in the same way as those who believe in the continuity of the abstract and concrete.

Teachers who believe that mathematics is made only by means of a hypothetico-deductive logic will not exhibit the same classroom procedures as those who see a place for intuition, induction, imagination and experimentation in the birth and development of the "queen of the sciences."

Mathematics educators who believe that a general mathematics program can be obtained by an analysis of "subjects" or of adult, life activities will not advocate the same learning experiences as those who take their cues from the present experiences and needs of youth.

In the terminal part of his study Dr. Ibrahim, aided by the assumption that the democratic ideal is the prime value in American education, emerged with a preference for pragmatism as a philosophy and organismic psychology as a theory of learning.

The study convinced this reviewer that mathematics teaching and curriculum making are honeycombed with contradictory ideas and practices despite the fact that mathematics itself is married to consistency.

ANNOTATED REFERENCES TO RESEARCH STUDIES

LOFLIN, ZACHARIAH L. *Mathematics in Chemical Engineering.* Ph.D. dissert. Teachers College, Columbia University. New York. 1949.

Only the simplest types of problems from algebra, analytic geometry, trigonometry and the calculus appeared. Algebra was the most widely used subject. The use of approximations and graphic methods, and the ability to read problems and frame equations were frequently needed.

This study should be of help in guidance and in the construction of mathematics curriculums.

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
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Mathematics and Engineering

ABOUT a year ago, writing in this magazine on the preparation of mathematics teachers, Professor Snader proposed an admirable nine-point program. One of his theses was "that the correlation of mathematics with related fields such as physical science, engineering, aviation, economics, manual arts, statistics, etc., requires teachers to be prepared through survey courses, directed professional reading and independent study." With this thesis, as with all his others, this Department is in thorough accord. If teachers are to become familiar with fields related to mathematics, fields in which they usually have had no special training, they must do it on their own or in conjunction with their professional training. In either case, a guide to the literature should prove helpful.

These bibliographic notes may also be of interest to those concerned with the mathematical training of engineers. As engineering advances, so do the demands upon the mathematical knowledge of engineers. The day has passed when a competently trained engineer could get by on a year of calculus and a half-year of differential equations. A considerable number of advanced mathematical concepts and procedures have already proved their usefulness in several engineering fields. Among these would be included: Fourier series, harmonic analysis, Gamma function, boundary-value problems, integral equations, contour integration, Fresnel integrals, Bessel functions, Legendre functions, Laplace's equations, vector analysis, matrices, the calculus of variations, the operational calculus. While not many of these subjects are now, generally speaking, part of the mathematical equipment of the average engineering graduate, more of them doubtless soon will be.

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Burington, R. S. and Torrance, C. C. *Higher Mathematics, with Applications to Science and Engineering*. New York, McGraw-Hill, 1939. 844 p.

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Fleming, Sir Ambrose. *Elementary Mathematics for Engineers*. New York, Chemical Publishing Co., 1941. 110 p.

Johnson, Walter C. *Mathematical and Physical Principles of Engineering Analysis*. New York, McGraw-Hill, 1944. 346 p.

Karman, T. V., and Biot, M. A. *Mathematical Methods in Engineering; An Introduction to the Mathematical Treatment of Engineering Problems*. New York, McGraw-Hill, 1940. 489 p. Fairly advanced.

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BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is available whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

ELEMENTARY SCHOOL

1. Texts

Arithmetic 7: The World of Numbers, by Dale Carpenter, Supervisor, Mathematics Education Section, Los Angeles City School Districts; and Elizabeth Cuthbertson, Teacher of Mathematics, Los Angeles. Cloth, 332 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $1.68.

Arithmetic 8: The World of Numbers, by Dale Carpenter, Supervisor, Mathematics Education Section, Los Angeles City School Districts; and George F. Drake, Jr., Teacher of Mathematics, Los Angeles. Cloth, 372 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $1.68.

Everyday Arithmetic, Junior Book 1, by Harl R. Douglass, College of Education, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Ridge Road School, Parma, Ohio. Cloth, ix+488 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $1.88.

Everyday Arithmetic, Junior Book 2, by Harl R. Douglass, College of Education, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Ridge Road School, Parma, Ohio. Cloth, ix+502 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $1.92.

HIGH SCHOOL

1. Algebra

Algebra: Its Big Ideas and Basic Skills, by Daymond J. Aiken, Lockport Township High School; and Kenneth B. Henderson, University of Illinois. Cloth, xv+409 pages, 1950. Harper and Brothers, 49 East 33rd Street, New York 16, N. Y. $2.48.

2. Special Courses in Mathematics

A Course in the Slide Rule and Logarithms, by E. Justin Hills, Los Angeles City Schools. Paper, iv+108 pages, 1950. Ginn and Company, Statler Building, Boston 17, Mass. $1.40.

A Manual for the Slide Rule, by Paul E. Machovina, Department of Engineering Drawing, Ohio State University. Paper, 78 pages, 1950. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $0.75.

COLLEGE

1. 1st Year Mathematics

Introduction to Algebraic Geometry, by W. Gordon Welchman, Massachusetts Institute of Technology, formerly Fellow of Sidney Sussex College, Cambridge. Cloth, x+349 pages, 1950. Cambridge University Press, 51 Madison Avenue, New York 10, N. Y. $4.50.

2. Trigonometry

Plane Trigonometry, With Tables, by H. M. Dadourian, Seabury Professor of Mathematics and Natural Philosophy (Emeritus), Trinity College. Cloth, x+109+84 pages, 1950. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $3.50.

3. Analytical Geometry

Elementary Analytical Conics, Second Edition, by J. H. Shackleton Bailey, School House, Lancaster, England. Cloth, 378 pages, 1950. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. $1.75.

4. Advanced Mathematics

Differential Equations, by J. E. Powell and C. P. Wells, Michigan State College. Cloth, vi+205 pages, 1950. Ginn and Company, Statler Building, Boston 17, Mass. $3.00.

Classical Mechanics, by Herbert Goldstein, Harvard University. Cloth, xii+399 pages, 1950. Addison-Wesley Press, Inc. Kendall Square, Cambridge 42, Mass. $7.00.

Lectures on Classical Differential Geometry, by Dirk J. Struik, Massachusetts Institute of Technology. Cloth, viii+221 pages, 1950. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $6.00.

Mathematics of Relativity, by G. Y. Rainich, University of Michigan. Cloth, vii+173 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.50.

MISCELLANEOUS

Theory of Mental Tests, by Harold Gulliksen, Princeton University, Research Adviser, Educational Testing Service, Princeton, N. J. Cloth, xix+486 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $6.00.

Anuario Estatistico do Brasil Ano X—1949, Instituto Brasileiro de Geografia e Estatistica. Paper, 775 pages, 1950. Servico Grafico do

Instituto Brasileiro de Geografia e Estatistica, Rio de Janeiro. (In Portuguese)

REVIEWS

Arithmetic 3: The World of Numbers, Dale Carpenter and Esther J. Swenson; *Arithmetic 4: The World of Numbers*, Dale Carpenter and Lester G. Anderson. New York. The Macmillan Co., 1950. Each iii+316 pages. $1.68.

The World of Numbers is a series of six books, grades three through eight.

The authors have been truly resourceful in capitalizing upon the daily contacts of children. They have used many and varied experiences as subject matter out of which a knowledge of arithmetic must grow.

Addition and subtraction facts are shown in attractive colored pictures of fruits, flowers, dolls, marbles, children, and other things familiar to the child in every day life. To introduce multiplication, Alice is pictured setting the table, using one knife, one fork, and two spoons at each of four places. Division is introduced in a candy store where candy canes are two cents each. Mary places her eight cents on the counter in twos and Bob places a cane near each two pennies.

After each new fact is presented there are story problems under headings that call for class discussion, and each group of problems stresses some particular activity in the child's daily living. There is much opportunity for class activity in building the various number facts.

Throughout the book the idea of interrelationships is stressed, emphasizing the fact that arithmetic is an organized system of related ideas. They are developed on the idea of making arithmetic meaningful to the learner.

Arithmetic Three and *Arithmetic Four* of this series are recommended to furnish pleasant study and practice material for meeting the specific aims and objectives of the "new arithmetic."—LORENA HOLDER, Dallas Public Schools, Dallas, Texas.

Practical Mathematics. Part I, Arithmetic With Applications (Fifth Edition), Claude Irwin Palmer and Samuel Fletcher Bibb. New York, McGraw-Hill Book Co., 1949. xii+179 pages. $1.80.

This book provides for adequate drill in the fundamental processes of arithmetic and includes an ample supply of practical problems selected from technical fields. A theoretical presentation of each topic combined with hints and illustrative examples makes the book adaptable for general use. Historical notes and information concerning simple mechanics add interest.—MARIE S. WILCOX, George Washington High School, Indianapolis, Indiana.

Mathematics in Daily Use (Revised edition), Walter Hart, Cottell Gregory, and Veryl Schult. Boston, D. C. Heath and Co., 1950. vii+376 pages. $2.04.

This text is specially designed by its authors to provide a course to meet the needs of those pupils for whom algebra will not be a profitable study or for those who will study algebra later. The authors point out that the book's "primary purposes are to produce appreciation of the presence of quantitative problems in daily life and to develop the understandings and skills that are needed to solve them."

One interesting chapter on leisure time problems includes puzzles, games of chance, and problems concerned with other leisure time activities such as knitting and crocheting. Other chapters on community activities and safeguarding family income provide pertinent problems regarding education and recreation; city, state, and federal expenses and income; and the distribution of individual incomes.

Various forms of motivation are employed in teaching the fundamental operations and their application to problems of home and school, and each of the ten chapters is followed by a review and test. The last twenty-four pages are devoted to "Tests Classified by Chapters," "Tests on Separate Processes," tables of measure, and a good index. In general, the many varied and interesting problems closely related to the experiences in the daily lives of the pupils, the attractive drawings and diagrams, and the excellent physical features of the book should assure the accomplishment of the purposes of its authors.—SADIE E. CLAPPER, Springfield High School, Springfield, Illinois.

Plane Geometry, Walter W. Hart. Boston, D. C. Heath and Co., 1950. x+382 pages. $2.00.

This textbook for tenth grade geometry classes is not just a revision of earlier textbooks by Mr. Hart; it is a new book with a new order of theorems. It would seem to be the culmination of much experience in teaching high school pupils and in writing textbooks. It is written with the normal class situation in mind—providing the minimum essentials for the slower learner, while giving sufficient material to interest and challenge the pupil of superior ability.

The introduction gives background and historical information and is followed by the first chapter which reviews geometry studied in previous courses. This makes a good "bridge" for the gap that is usually felt between previous mathematics courses and tenth grade geometry. Special attention is given to the development of scientific vocabulary.

Logical proof is developed and pupils are given many opportunities to prove "originals." Sets of supplementary exercises provide additional material to challenge the superior student. Two short chapters are given to *Analytical Geometry* and *Solid Geometry* in an attempt to interest the pupil in further study of mathematics.

The format is good, type is large and diagrams are clear and attractive.—K. EILEEN BECKETT, Lebanon, Indiana.

Analytic Geometry, L. M. Kells and H. C. Stotz. New York, Prentice-Hall, Inc., 1949. viii +280 pages. $2.85.

The authors state: "The aim of this text is to give such a presentation of the materials of analytic geometry that a student will be led to a deeper understanding of underlying principles while he is learning to apply them in solving the ordinary problems of the subject."

In keeping with this expressed aim, the authors introduce fundamental topics as early as possible and use these ideas repeatedly. Specific illustrations of this early treatment are vectors introduced on page two and translation of axis on page twenty-four during the study of graphs. This book gives an adequate treatment of polar equations and parametric equations as needed for the calculus.

The material is quite readable and the graphs are excellent for both two and three dimensional representation. There are ample problems.

Answers to all problems are included. Appendix I gives reference materials and Appendix II is a table of miscellaneous functions.

The author has written a text which is adaptable to courses of varied lengths.—F. R. Brown, Illinois State Normal University, Normal, Illinois.

Collected Mathematical Papers. George David Birkhoff. New York, American Mathematical Society, 1950. 3 volumes, lvii+754, 983, 897 pp. $18.00.

It is only with the publication of such a collection as this, of Birkhoff's papers, that one begins to get a really accurate idea of the scope of the accomplishment of a great mathematician. The broadness of the mathematical achievements of Birkhoff is already indicated by the several categories into which the collected papers have been divided. Volume one contains papers on boundary value problems and associated Sturmian problems, on differential equations, on difference equations, and four papers on dynamics. Volume two contains papers on dynamics and on physical theories. Volume three contains papers on the four color problems and miscellaneous papers. With relatively few exceptions the papers in all but the section on physical theories and the last section are of a research nature.

The final section contains two papers which appear in Yearbooks of the National Council of Teachers of Mathematics: (1) *A New Approach to Elementary Geometry* (with Ralph Beatley); (2) *Polygonal Forms*. The second paper refers to Birkhoff's theory of aesthetics, there being several other papers on this subject in the final section. The paper on elementary geometry is supplemented by a more detailed exposition in the paper *A set of postulates for plane geometry, based on scale and protractor*. The reviewer thought these papers extremely interesting and sides with Birkhoff in the view that there is no special virtue in sticking to the classical Euclidean approach to geometry in the standard high school course. The student at that level is in no position to really appreciate the postulational approach as typified by Euclid's geometry. It would seem that Birkhoff's method of basing geometry on concepts established in the prior training of the student is much more sound pedagogically. And there is absolutely no virtue in retaining Euclid for merely traditional reasons.

Among the expository papers of the final section the reviewer found *Mathematics: Quantity and Order* extremely interesting. A person with very little technical background can gain a little insight of some of the most profound problems of mathematics by perusal of this paper.

Included in Volume 1 are three obituaries of Birkhoff.—D. H. Potts. Northwestern University, Evanston, Illinois.

An Introduction to Probability Theory and Its Applications. Volume 1. William Feller. New York, John Wiley & Sons, Inc. 1950. xii+ 419 pp. $6.00.

It occasionally happens that when a first-rate mathematician "stoops" to writing a textbook the result is also first-rate. Such is the case in point. The author is one of the leading mathematicians of our day; his product is a highly interesting, skillfully written book which should prove to be greatly used in years to come. His treatment of the subject of probability is entirely mathematical, the speculative philosophical aspects having been properly divorced from the mathematical theory. In this volume the author has confined himself to consideration of "discrete sample spaces," thus requiring of the reader only an elementary mathematical background (a first course in calculus would suffice for much of the material). Problems involving more advanced mathematical techniques have been deferred to a second volume. However, such relatively advanced topics as infinite Markov chains and random walk are included.

From a pedagogical point of view the reviewer thought this book excellent. On the publisher's jacket one finds "a distinguishing feature of the book is the wealth of illustrative examples." This is not an overstatement. Each chapter ends with adequate problem sets. Answers are given in the back of the book.—D. H. Potts, Northwestern University, Evanston, Illinois.

Watch for the program for the 29th Annual Meeting of the NCTM in Pittsburgh, March 28-31, 1951 in the February issue. Make your hotel reservations now!

TEXAS' FIRST MATHEMATICS INSTITUTE
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e more evidence of living up to its philosophy of service, the University of Houston is g its first mathematics institute to be held on its campus from July 30 to August 2, 1951. ttee composed of two members of the College of Education staff, two members of the ıtics Department, and one from the College of Business Administration is already work- rery inclusive program which will attempt to meet the needs of classroom teachers at all l in as many areas of interest as possible.

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Third President

of

The National Council of Teachers of Mathematics

1924-25

and

Member of the Board of Directors

1932-34

RALEIGH SCHORLING
1887–1950

THE MATHEMATICS TEACHER

Volume XLIV Number 2

Raleigh Schorling 1887–1950

By E. R. BRESLICH
University of Chicago, Chicago, Illinois

BY NAMING the February issue of THE MATHEMATICS TEACHER the "Schorling Number" the National Council of Teachers of Mathematics is paying tribute to a man who devoted 40 years of his life to the cause of mathematics and was an outstanding leader in the field of education.

Raleigh Schorling was born in Batesville, Indiana on August 15, 1887. He received his elementary and secondary education in that state. He continued his studies in the Indiana State Normal School from which he graduated in 1909. During the next two years he studied at Indiana University and at the University of Michigan receiving the A.B. degree in 1911. He then enrolled at the University of Chicago and was granted the A.M. degree in 1914. Later he entered Columbia University and in 1924 the Ph.D. degree was conferred on him at that institution.

His teaching experience was varied and extensive. From 1904 to 1907 he taught in the public schools of Indiana. He was principal of an Indiana high school during 1907–1908. Next he taught mathematics at Short-ridge High School in Indianapolis. He then joined the University High School faculty at the University of Chicago, but left in 1917 to become principal of the Lincoln Experimental School of Teachers College, Columbia University. From there he went to Ann Arbor, Michigan as principal of the University High School, the University of Michigan. Lastly in 1926 he was appointed Professor of Education, Principal of the University High School, and Supervisor of Student Teaching at the University, a position he held until his sudden and unexpected death on April 22, 1950.

Professor Schorling was an outstanding teacher of mathematics. He possessed unbounded enthusiasm which he was able to pass on to his pupils and colleagues, and gained among his pupils the reputation of being the teacher in whose classes there was no napping. He was particularly interested in and sympathetic with the slow pupils and knew how to bring his teaching down to the level of their ability. At the same time he was keen in identifying pupils of superior ability. He aroused in them genuine interest in the subject and many took all the mathematics they could crowd into their programs and continued the study of the subject in college. Many men and women prominent today in the affairs of adult life gratefully remember the assistance and council he gave them when they were high schools pupils.

His activities as a high school teacher were not limited to his classes. He had a

keen interest in the various clubs and societies, school contests and athletics. His council and advice was greatly appreciated by the pupils and the administrative officers of the school.

Professor Schorling was a born leader and organizer. He was constantly helping pupils organize their entertainments, clubs and contests. In 1913 he was one of the group of Chicago teachers who founded and developed the Men's Mathematics Club which has been a definite influence for improving the teaching of mathematics and is as active today as it was in the early period. During the first three years he was the club's president. In February 1920 he took an active part in the group of teachers which founded the National Council of Teachers of Mathematics. He was the third president from 1924–1925 and served as a director in 1932–1934. *The First Yearbook* of that association was edited by him. Moreover, throughout his busy life he gave also generously his time and energy to the problems of other organizations, especially the Central Association of Science and Mathematics Teachers and the Mathematical Association of America.

During the last three decades a great deal of the work of improving the organization and teaching of mathematics has been done by large committees and commissions.

Professor Schorling was a member of the National Committee on Mathematical Requirements which made its famous report in 1923. He was a member of the Cooperative Committee on Science Teaching which reported in 1941, and of the Committee on Pre-Induction Courses in Mathematics which reported in 1943. He was chairman of the Committee on Post-War Plans, which made three reports, the first two being published in 1944–1945, and the last, known as the *Guidance Pamphlet in Mathematics for High School Students*, being published in 1947.

Perhaps his greatest influence on the teaching of mathematics was exerted by Professor Schorling's books and pamphlets. As early as in 1915 he began his career as an author when he published *Review of High School Mathematics* with W. D. Reeve as co-author. By 1950 his name had appeared on numerous books, pamphlets and texts on mathematics, statistics and student teaching. Most of these were published by the World Book Company, and John R. Clark, Roland R. Smith, Francis G. Lankford, Jr., Mary Potter, and Vera Sanford were co-authors.

With his death at the early age of sixty-two years the cause of mathematics has lost one of its most devoted and active supporters, but his work will be appreciated by the teachers of mathematics for many years to come.

"At the moment, mathematics in the high school is enormously popular. . . . Time and circumstances have played in our favor, but they may not do so a second time. We need to keep a discerning eye on the situation that may exist five years after the war ends. Unless we provide sensible programs (note the plural) in mathematics for all pupils of the secondary schools, each appropriate to the ability and need to be served, the devastating fire of criticism may again be directed at mathematics. At any rate, it should be.

. . . . . .

". . . No one should assume that all is well with the traditional courses. In most schools they are woefully out-of-date as regards both subject matter and method. . . . The unique values of the traditional courses cannot be achieved by a constant gearing down, nor by the inclusion of popular material in a futile effort to meet the mathematical needs of pupils who should not have elected the courses in the first place. The first step in the postwar period is to scrutinize the traditional courses in the light of recommendations that have long been widely approved."—SCHORLING, RALEIGH. "The Need for Cooperative Action in Mathematical Education," *The American Mathematical Monthly*, Vol. 52, April 1945, p. 195.

This issue of

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dedicated to the memory of

RALEIGH SCHORLING

Third President

of

The National Council of Teachers of Mathematics

1924-25

and

Member of the Board of Directors

1932-34

of the neighborhood boys left school early, but Raleigh graduated from the eighth grade in 1901 and his teacher urged the Kammeyers to send him on to high school.

Dad used to tell later how he worked all that summer on the farm (he began to help at home when he was just big enough to gather eggs), hoping that he would be allowed to go to high school but not daring to ask. Finally came the first morning of school. Dad got up at the usual hour of 4 o'clock, worked until seven, when Father Kammeyer said, "Hitch up the buggy, Raleigh. You can go to high school if you're so set on it." He finished the course in three years, starred on the basketball team, graduated with honors, and in the fall of 1904, barely seventeen years old, Raleigh Schorling took his first teaching position.

The same fine teacher who had insisted that Dad go to high school, now gave destiny another push by urging him to go to college, though no one from that county had gone up to that time. Since the Franklyn Township school, where Dad taught all eight grades and served as janitor for $18.00 a month and keep, did not start until after fall harvesting and stopped early in the spring, it was possible for the young teacher to attend summer terms at the Indiana State Normal School at Terre Haute. To earn his expenses, he ran a cooperative boarding house and operated a bookstore. He taught country school for another two winters and in 1907 became superintendent of schools and principal of the high school in Sunman, a town about three miles from his home.

During a part of the time at Terre Haute, Dad lived in the home of Professor James Baxter, a great and dedicated teacher of mathematics. It was he who urged Dad to go on to the University of Michigan, his own alma mater. So after completing the course at Terre Haute, my father spent two years at the University of Michigan, getting his A.B. in 1911, with election to Phi Beta Kappa.

The year after graduation from Michigan, Dad taught at Shortridge High School in Indianapolis. Then he accepted a position in the mathematics department of the University High School of the University of Chicago. Years later, I was shown the historic spot on the stairway where in September, 1912, he met Marie Louise Oury, who taught Latin and history there and who became Mrs. Schorling five years later. It was a family joke that Dad was always partial to brown eyes and married the most beautiful pair he could find, only to be rewarded by having three green-eyed children!

Those must have been good years. I have heard rumors of dinner dances at the Quadrangle Club (where the unmarried instructors lived), and of ice skating on the Midway. But as always, Dad was working hard. In all his life, he never took an hour off from a full time load to do graduate work, even on his doctorate. He received his M.A. at Chicago in 1914. In the high school, he coached the boys' soccer team, sponsored many student activities, and was voted the most popular teacher of the year. Summers he spent as a counselor in a Wisconsin boys' camp and still found time to co-author his first book, *A Review of High School Mathematics*, written with William D. Reeve, and published by the University of Chicago Press in 1915.

During the years at Chicago, Dad made many lifelong friends, among them Dr. Otis W. Caldwell (for whom my younger brother is named). When Dr. Caldwell was called to Teachers College, Columbia University, to organize its new experimental school, he asked my father to be the first principal. Raleigh Schorling and Marie Oury were married on July 26, 1917, and went to New York where Dad assumed his new duties at the Lincoln School.

However, the next spring, he interrupted his work to enlist in the army and was sent to a training school for Field Artillery

officers at Ft. Monroe, Va. 2nd Lt. Raleigh Schorling was demobilized in January of 1919 and returned home to find his family increased by a daughter, named Ruth Mary. Mother says that she was somewhat apprehensive about introducing us for I was a fretful baby. But Dad promptly put me on his shoulder and marched around the room to a tuneless chant of "Eefa safa safa safa eefa safa la." I don't remember that incident, but I know that with my brothers and with my own boys, Dad used this technique, with unfailing success, to secure delighted crows from a baby.

The little family returned to New York and shortly thereafter Dad's second book was published: *General Mathematics*, also written with Reeve. It was at this time that his thirty year association with John R. Clark began. Their first book, *Mathematics for the Seventh School Year*, appeared in 1922. His bibliography lists 41 books and uncountable tests and workbooks which they co-authored.

Although four colleges can claim Raleigh Schorling as an alumnus, he gave his heart to Michigan. Thus it was a dream come true when, in 1923, Dean A. S. Whitney asked him to come to the University of Michigan, with the rank of Associate Professor, to act as principal of the new University High School. Essentially a country man, he had disliked raising a family in New York City (the first boy, Donald Clark, named for John Clark, was born in May of 1922), and it was a joy to come back to peaceful, green Ann Arbor town. In the fall of 1923 the Schorlings moved to Ann Arbor and Otis was born there in November. The following year my father received his Ph.D. from Columbia University. He remained principal of University High School until 1926 and always was Supervisor of Instruction there. In 1927 he became a full professor.

Dad loved teaching youngsters and was loath to give it up. He continued to teach at least one high school class until 1930, and I was lucky enough to be a member of one of his last seventh grade classes. I know now, of course, that he was an inspired teacher. But then we only knew that math, previously despised by many of us, was FUN. We experienced the joy of learning. It was a demonstration class and we were used to visitors and observers but he taught for us. How we loved the "Contracts," when we chose the section of problems to solve, and were graded not only on the difficulty of the contract chosen, but also on the speed and accuracy with which we finished the contracted assignment. Incidentally, I can't remember that any caste distinction existed between those who chose the "A" or "B" contracts, and the less proficient students who did the "C" section. For the slower students were close to Dad's heart and he got pleasure out of making mathematics live for them.

One of the things I remember about that year is the awful day, funny in retrospect, when the back of the room was crowded with teachers in town for the MEA meetings, all waiting to see Dr. Schorling perform his famous demonstration of teaching the relationship between the volume of a cone and that of a cylinder of the same height and radius. It was a vivid demonstration, pioneering the use of visual aids in mathematics teaching, and although for many years now, my only acquaintance with cones has been with the variety containing ice cream, I could do it today. Anyhow, when the class had come to order and the visitors were agog on the edge of their chairs, Dad called the names of the four of us who had promised to bring the rice necessary for the experiment, and there was a ghastly pause. Not one of us scatter-brains had remembered the rice. Dad called me outside the classroom and handed me a quarter with a glare and instructions to "Go get that rice!" He must have stalled successfully until I arrived panting back from Kroger's with the rice, for it was an outstanding demonstration.

During those years the books came popping out like popcorn, and Dad performed the herculean task of revising each one every five years, as well as supervising the high school, teaching a full college load, serving on state and national committees, making speeches, attending conventions, and planning the work of thousands of student teachers who graduated from the University of Michigan during his 27 years there.

Our family life never suffered neglect because of Dad's professional life, for he was a devoted and imaginative parent.

Mr. Schorling with Rudy, His Daughter-in-Law and Ruth Schorling Rickard

He was a wonderful raconteur, and our bedtime stories, drawn from his wealth of experience, were eagerly awaited events. Dad's "way with children" consisted of treating them as individuals, and he was endlessly patient with us, particularly with the incessant questions which he was never too busy to answer. Every night at the dinner table he used to ask each child to relate any special achievement of the day, and how proud we were to tell of an "A" in spelling, and how solemnly he listened and gave praise where praise was due.

Dad was always a sportsman; he taught us tennis and golf, and took us to football and baseball games as soon as we were able to sit up in the stands. He was a rabid Michigan fan who missed very few home games from 1923 to 1949, and the Detroit Tigers ran the Wolverines a close second in his affections. One year he organized the neighborhood into two softball teams, the Mugs and the Wumps (whom he captained) and during the long summer evenings the fathers and youngsters played ball while the mothers formed a cheering section and supplied watermelon. Until he was well past fifty, Dad used to spend an occasional Saturday morning in the U High gym, shooting baskets with my brothers and their friends.

No story about my father would be complete without extensive reference to fish. He was an ardent fisherman, submitting eagerly to the utmost rigors of climate and early rising in pursuit of finny triumphs. His idea of bliss was to arise at 3 A.M. in the company of Marshall Byrn, his close friend since their college days at Terre Haute, and row countless miles, trolling for the elusive, small mouthed bass. Dad caught his biggest fish during his sabbatical leave in 1949, part of which he spent with us here in California. He was a modest man and never kept a press clipping of his professional accomplishments. But there is no modesty among fishermen, and when the sports page of the *Los Angeles Times* reported that the record halibut of the season had been caught at Malibu Beach by Raleigh Schorling of Ann Arbor, Michigan, he bought twenty copies to mail to the envious members of his fishing gang. That was a tremendous fish, 29 pounds of halibut, bigger than my wide-eyed son, who stayed a cautious distance from the monster until it appeared in small pieces on the table. To this day, on the rare occasions when small Robert Raleigh eats in a restaurant, he orders halibut and retells the story of grandpa's prodigious fish.

Dad liked to travel and did a lot of it. We used to tease him about being able to

find someone to pay his way on a trip he intended to take anyhow, and there was a lot of truth in our joking for he was in great demand as a speaker all over the country. I have before me the front page of the Alpena, Michigan, newspaper which Dad once sent me, for he relished a joke on himself. The biggest, blackest headline states that "FISHING SEASON OPENS TODAY," and the smaller headline announces that "Dr. Schorling Speaks to Teachers Association Here." That was no coincidence.

In 1929 my father went to Germany as one of a group of 30 American teachers invited there as guests of the Weimar republic. He found his childhood German very useful and was even able to speak to a group of teachers in Holland. When Dad had a sabbatical leave in 1937, the Schorlings with their three children gave Europe a foretaste of what was to come by invading en masse, complete with car. Dad spent a lot of time visiting German schools and was saddened by the changes wrought by the Nazis in a country which he remembered from his previous visit as a fine, young democracy. But he loved taking us up the Rhine, through the Black Forest, and into Denmark, Austria, and the low countries. The greatest hardship we suffered in Europe was having to wait until Monday or Tuesday to find out from the *Paris Herald* how Michigan's football team had fared on the previous Saturday.

In 1929 Dad had started the "Correlated Course" in Education, a type of course which has been widely adopted in other schools. It consisted of an intensive pre-teaching program at the University for six weeks, then six weeks of observation and practice teaching in the field and a return to the University to correlate these experiences and finish the academic work in Education. When I decided to become a teacher, I took the 1939 Correlated Course, which proved to be the most challenging and stimulating course I ever took, as I found myself for the second time one of my father's students.

A 29-pound Halibut which made the sports column of the *Los Angeles Times*. Biggest fish caught at Malibu Beach that season, March 1949.

My father genuinely loved his students and took a personal interest in them and their lives. He followed their athletic achievements and campus activities, called them by their first names, and knew each as an individual. In return they loved him for his wisdom, his friendly grin and quiet humor, told him their troubles, borrowed money from him, asked his advice about their careers, and expected him to find them jobs, which he did even during the depression years when jobs didn't exist.

Dad was a great believer in picnics and a notable camp cook. For many years the U High faculty had a picnic every spring and many of Dad's classes wound up the year with a June picnic. When we of the correlated course returned from our field

work all over the state of Michigan, we promptly planned a picnic. It followed the pattern of all the others which I remember. Dad purchased the meat and cooked it himself, with the aid of student assistants, who were permitted to gather firewood,

Mr. Schorling with Clyde Vroman, now Director of Admissions, University of Michigan

but not to tamper with his famous mixed grill. Before dinner we square-danced, played softball and even Farmer-in-the-Dell. Then, after eating steak-bacon-hamburg sandwiches, too full to move, we sat on the grass, swatting mosquitoes and singing college songs.

Sometimes people ask how Dad did so many things so well. I think the answer is that he never stopped working. Walking to and from school, he wrote his speeches. He used to sit at home in the evening, playing solitaire in front of the fire, while his mind busied itself with the chapter of a new book. Incidentally, he never kept lecture notes from year to year, but prepared his lesson plans daily for each course, which kept them fresh and topical.

During the summer of 1941 I was assigned the project of helping Dad with the revision of the seventh and eighth grade books, and had an opportunity to see him at work. (This was as a member of the Curriculum Workshop, one of Dad's innovations for graduate students in the School of Education summer session.) He had a hide-out in the basement, which had no telephone and was relatively free of interruptions. We called it the Black Hole of Calcutta, and Dad used to hibernate down there with six copies of the book being revised, a ream of paper, scissors, a pot of rubber cement, and some pencils. There he rewrote, clipping out and pasting into the new text those portions of the old one which merited inclusion in the new edition. Dad used to say that a good sentence was a day's work. Each sentence was cut and polished, reworded half a dozen different ways, and often discarded in the end for a better phrasing. The clarity and simplicity of his prose resulted from the hours he spent in choosing the one word that would exactly express his meaning. During the course of that summer I came to have a profound respect for the years of intensive labor represented by the shelf of books which bear his name.

When World War II came, Dad remembered that he was a farmer and took to victory gardening on a grand scale. He raised vegetables in such profusion that his friends almost cringed when they saw him coming, basket of broccoli in hand. He also did his bit on a larger scale by spending parts of 1943 and 1944 as a civilian in the Bureau of Navy Personnel in Washington, working on a mathematics text for the navy training program.

In the spring of 1945 came the blow from which my father never recovered. Sgt. Clark Schorling was killed in action in Germany and Dad aged overnight. He still had his warm, quiet friendliness, but the lop-sided grin was tired and not so wide as formerly and his step lost its bounce. He kept right on working, harder indeed than before, but with a little less joy and enthusiasm. Mother and Dad established through the Regents of the University of Michigan a "Clark Schorling Memorial Fund" to be administered by the School of Education: "To promote the peacetime equivalents for the challenges that war activities provide for some youth."

During the summer of 1945, despite his deep personal grief, Dad took a group of educators on a tour of army and navy training schools. From this trip came the monograph "Swords Into Ploughshares." In 1946 he authored the famous "Bill of Rights for Teachers," which was acclaimed the first statement of its kind and reprinted by more than 50 newspapers and magazines, including *Time* (August 5, 1946).

Nine more books were to appear in the next four years, the most important being the rewritten *Mathematics in Life, Mathematics for the Consumer* (credited with being the first comprehensive book on the subject), and *Student Teaching*, Second Edition, picked as one of the 37 outstanding educational books of 1949, and already adopted for use in over 100 colleges and universities. He continued to commute to Chicago while serving on four national commissions, and to cope with the increased University load caused by postwar enrollment. Honors and recognition in his chosen field came to my father. Harvard University hung in its library a picture of the Indiana boy who became one of the nation's foremost educators, and in April of 1950 he was notified of election to Phi Beta Kappa Associates.

However, that large, wonderful heart of his which he gave to all the world, now showed damage from the years of driving work. At the NEA meetings in Atlantic City he began to feel ill and was very sick from the first of April on, although he refused to let mother tell any of us about his illness.

The 21st of April, 1950, was a typical day in Dad's life. He met his classes, read proof on his last book (*Elementary School Student Teaching*, written with Max Wingo and published by McGraw-Hill in September, 1950) and drove over in the afternoon with mother to see how the painters were coming along on the new house they had been building all winter. It was also his last day for Raleigh Schorling died of heart failure early in the morning of April 22, at the age of 62.

My father will be missed by many people in the world, besides the members of his family. Perhaps the most sincere tribute paid him came in the unconscious pathos of a remark made by one of his students. Late this summer Mother met one of Dad's best students on the campus and asked him if he had found a teaching job yet. He said, "No, I haven't, Mrs. Schorling, but I'm sure Dr. Schorling would have found me one."

" . . . Competence in mathematics in many respects parallels literacy in communication. . . . It is nevertheless clear that modern technology has stepped up the minimum requirements of literacy in communication.

"In great-grandfather's day, when life was relatively simple, the ability to compute accurately when dealing with whole numbers, common fractions, decimals, and per cent was adequate for the common affairs of life. There are good reasons for believing that the minimum requirement in mathematics for effective citizenship is moving up and is already *higher than mere control of the four fundamentals of arithmetic.*

"The war has demonstrated in a dramatic way what is nonetheless true in peace time, but not so readily discernible; namely, that a boy lacking certain mathematical competencies is a pathetic victim of a ruthless system. . . .

"Thus, functional competence in the mathematics of common affairs seems as crucial as functional literacy in communication. Unfortunately, it is not generally recognized that the minimum mathematical training needed for citizenship is higher than the level commonly achieved by the product of our schools. . . . A program correlating and emphasizing industrial arts, science, and mathematics is probably a good approach to the *general education* of many pupils who are never going to be satisfied with a purely academic program."—SCHORLING, RALEIGH. "The Need for Cooperative Action in Mathematical Education," *The American Mathematical Monthly*, Vol. 52, April 1945, pp. 196–198.

Raleigh Schorling—Teacher

By JAMES B. EDMONSON
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WHILE the major contributions of Professor Raleigh Schorling in teaching, research, and writing were largely in the field of mathematics, he made an enviable reputation in other educational work. He had in influential part in the organization of the Lincoln School of Teachers College, Columbia University, and served as the first principal of the school. At the University of Michigan he organized the University High School and served as its first principal. He also planned and directed the first program in student teaching offered by the School of Education of the University of Michigan. In addition to his University duties he served on numerous state and national committees and was an exceedingly valuable member of such committees because of his unusual resourcefulness, marked initiative, and fine professional spirit.

Professor Schorling's interest in the general problems of teachers and teaching was unusually deep as evidenced by the fact that he contributed to seven books and six monographs relating to such problems. He was also the author of an extensive list of articles on educational issues. His best known books, outside of the field of mathematics, were in the field of student teaching; and through these books Professor Schorling exerted a nationwide influence on the quality of the experiences provided for prospective teachers. Among the monographs to which he contributed were such titles as: "Swords Into Ploughshares—What Civilian Education Can Learn from the Schools of the Armed Forces," "Social and Economic Trends and Their Educational Implications" (with H. Y. McClusky), and "The Techniques of Textbook Authors" (with J. B. Edmonson). The foregoing titles are indicative of the wide range of Professor Schorling's interests. His articles in educational magazines dealt with such issues as the youth problem, objectives of secondary education, camping problems, and curriculum issues. His most widely known article was doubtless the one entitled "An Evolving Bill of Rights for Teachers," in which 12 rights of the classroom teacher were defined. The article was designed to stimulate discussion of the conditions that might make teaching more attractive to larger numbers of able young people and to make it possible for competent teachers now in service to do more efficient work. This article has been widely quoted in books and magazines, and thousands of reprints have been made available to discussion groups.

Professor Schorling was very highly regarded on the campus of the University of Michigan, and the Board of Regents paid a high tribute to him in a memoir which was adopted on May 20, 1950. Part of this memoir is as follows: " . . . he was alive to the general problems and needs of our school systems and his judgments, expressed in his frequent contributions to educational journals, were heeded by the members of his profession and by laymen alike. . . . Above all, Professor Schorling's broad human sympathy and his instinctive friendliness and helpfulness made themselves felt in all his personal and official relations with his students and fellow faculty members."

Professor Schorling had a host of friends and admirers, and there were innumerable demands on his time for conferences, committee work, and addresses before organizations. He was a splendid colleague and enjoyed unusually high standing in University circles. His achievements brought high distinction to the School of Education at the University of Michigan, and he will long be remembered for his outstanding services as a scholar, a teacher, a writer, and an educational leader.

Raleigh Schorling in World War II

By JOSEPH J. WICKHAM
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ıRING World War II, Raleigh Schor- gave extensively of his time and 'ledge to the Training Program of the ɔd States Navy. The idea of a Navy ing program was not entirely new. In as far back as World War I, the Navy nstituted a system of Training Course uals for use by enlisted men who were ing for promotion in their respective . However, the available training ıes in all ratings were limited in ɔer and, at the time of Pearl Harbor, derably outdated. Naval Aviation ıwhile had become a large and ex- ely important part of the Navy. e were more than 22 regular aviation gs and 13 specialist designations, but four training manuals.

arked by Admiral A. C. Read of NC-4 and approved by Admiral King, f of Naval Operations, the Training ses Section was given the task of ıring a series of training courses for enlisted men in Naval Aviation. fully selected specialists, both mili- and civilian, were brought into this on to write original books for the ıotion of aviation ratings. Because ı ratings were largely technical, it rally followed that each trainee ed a basic understanding of mathe- cs. Many books on mathematics available, but none were aimed at ıviation personnel or, in fact, at any e Navy's ratings.

hat sort of person was needed to pre- a mathematics book for enlisted men ıe Navy? Naturally, someone well ıded in mathematics. But he must be an educator with a sympathetic rstanding of American youth and extensive experience in writing. After derable sorting and sifting of possible ıtors to do the work, the final selection was Raleigh Schorling. He came to the Navy shortly after Admiral Nimitz's letter concerning our high school graduates' deficiencies in mathematics had received wide publicity. He felt that the Admiral's criticism of mathematics education was a personal challenge to him to do an important work.

Raleigh Schorling was pleased to be able to give assistance to service men, but he realized at once that he was face to face with a new venture. With this mathematics book there would be no teacher to explain, correct, and reteach; there would be no teacher's manual and no key. "These new books," he was told, "must be books the men will want to read—and in reading them will absorb their substance. To be effective, we have to give them what they like."

Swinging quickly into the spirit of the project and showing that versatility which was characteristic of his thinking, Dr. Schorling soon came up with a workable program. He visited the sailors at the Chicago Pier, Jacksonville, Bainbridge, and other training centers. There he discussed with the men the types of problems that they daily encountered. He sorted and correlated the information he gathered and began rapidly to build a new mathematics textbook. He quickly brought to bear his vast fund of knowledge in the mathematics field and took great joy in creating with a free hand. A few chapter headings from his Navy Training Manual, *Mathematics*, will serve to show the spirit of his work: "Trouble shooting your weaknesses," "Vulgar fractions," "Very vulgar fractions," "New wine in old bottles," "We escape chaos," etc. The following excerpt illustrates the fresh approach he brought to the project.

"If you were sick, you might find it comfort-

ing to have the doctor tell you that your illness is NOT due to bad teeth, bad food, or bad heart. But you still wouldn't know the cause of your trouble. Even if the doctor tells you that your trouble lies in your respiratory system, you can't be sure whether it is tuberculosis, cat fever, pneumonia, a sinus infection, or what have you. Probably your doctor would start a more detailed diagnosis. That's EXACTLY what this study guide is going to do to spot your addition troubles."

For the enlisted men his style of presentation was most effective, and when salted down with real mathematics, it was unbeatable.

So successful was this initial attempt to give mathematics to the men of Naval Aviation, that Raleigh Schorling was commissioned by the Battleship Navy to do a similar text. The "Doctor," as he was called by officers and sailors alike, worked long hours to help meet the deadline on his text. This was not the five-year plan of the textbook publisher, but the yesterday-was-too-late plan of the speeded-up war program.

An anecdote may help to point up a particular Schorling characteristic. During the strenuous weeks of writing, he often spent Sundays at his Navy desk writing and rewriting, organizing and reorganizing material. Many of his younger associates were glad to take a one-day rest, but Dr. Schorling continued to drive himself. On one rainy Sunday he was carried past his bus stop at the Navy Department and found himself at the rear of the building, which was enclosed by a 12-foot-high cyclone fence. Only the front gate was open, but the entire building was well patrolled by Marine guards. Dr. Schorling, anxious to get to his work and a bit provoked because he was outside the fence and standing in a downpour, decided he would save time by crawling under one of the closed gates. Almost immediately he found himself staring into the business end of a Springfield rifle backed up by a businesslike and suspicious Marine. No, he wasn't shot, but the Navy officer in charge had some careful identifying and extensive explaining to do before Dr. Schorling was released by the Marines and allowed to report for work.

Dr. Schorling's contribution to the war effort was fully appreciated by the Navy. Out of more than one hundred training courses produced by the Training Courses Section during his tenure, his was the only one to carry a personal identification as follows: "Acknowledgment is accorded Dr. Raleigh Schorling, whose teaching methods and procedures are incorporated in this book."

RALEIGH SCHORLING stands the Regimental Review with (l. to r.) Captain John S. Baylis, Commanding Officer, Commander N. S. Fulford, Executive Officer, Commander A. E. French, Training Officer, Commander W. G. Hodgson, Captain of the Yard, and Lt. L. K. Alway, SPAR Training Officer.

As Others Saw Him

By PHILLIP S. JONES
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DR. HOWARD BATCHELDER of Indiana University writes of his former teacher:

"Dr. Raleigh Schorling's untimely death was felt deeply by thousands of his students who worked closely with him and who came to know him well, his passing marked the loss of a personal friend and a great teacher. Yet, characteristic of all great teachers, the influence he had in shaping the professional thinking and careers of his own students will continue to be felt, in turn, in the lives of their students through the years to come."

This widening circle of influence upon others is the result not only of a teacher's ideas but also of his personality. It is hoped that some appreciation of Raleigh Schorling's personality is at least partially presented here in brief notes from only a few of his many pupils and co-workers.

ALICE M. HACH of Slauson Junior High School in Ann Arbor completed her master's degree under Raleigh Schorling while also teaching for him in the University High School. She writes as follows:

"'God grant me the serenity to accept things I cannot change—Courage to change things that I can—Wisdom to know the difference.' This quotation printed on a large placard stood in a prominent place on a desk in an education office. Opposite the desk a collection of cartoons dealing chiefly with education was displayed on a bulletin board. A small snapshot of a fishing trip, an unusually 'large catch,' was among the display. Against one of the walls were many shelves of books while in the background near the windows were two large foliage plants. The name, Raleigh Schorling, was on the door.

"It was a busy office because here was a man who saw no task impossible, no difficulty insurmountable, and who had the vision and courage to pioneer when convinced of some great need. 'We must come to grips with this,' Dr. Schorling would say. Then with his unusually keen insight he would survey a situation, analyze it, and be ready for action if action seemed advisable.

"Yet Dr. Schorling was never too busy to listen to the personal problems, the requests, and the needs of those who came to him. 'Let me know how you are after the operation. I shall be anxious to hear,' Dr. Schorling said to me when I was in his class for the first time. Nor did Dr. Schorling forget the situation. He inquired a number of times regarding my progress, provided opportunity for me to complete the course by independent study, and arranged for the examination to be taken in my home.

"His enthusiasm for teaching was reflected in a remark made by him in the summer of 1949. 'My one wish is to live long enough to have a helicopter to go up north and fish and come back here to teach my classes.' Throughout his teaching Dr. Schorling never forgot the child. During one of his classes after a detailed study of record keeping and anecdotal records he warned, 'Don't forget when you have records that you also have Mary.' Never was the child forgotten or lost in the maze of theory and method, for anything Dr. Schorling offered was practical and useful.

"One afternoon Dr. Schorling stepped into his methods class and with a twinkle in his eyes set up a hypothetical classroom situation—a skillful teacher was developing a topic in mathematics with a junior high school class. The pupils were challenged and enthusiastic. Just at the critical point in the development the bell rang for passing. Should the teacher quickly give the answer or should the teacher postpone the work and ask the children to save their papers for the next day? After agree-

ment by the class that the work should be postponed, a second question was raised. 'What might you as a teacher anticipate for the beginning of the next period?' At this point many suggestions were given such as the following. 'Many pupils would discover the answer before class.' 'Pupils would get help from their parents.' 'The children would ask pupils in other classes.' Each time Dr. Schorling would shake his head and say, 'Well, maybe, but probably not.' Finally after, the class was at a loss for ideas, Dr. Schorling chuckled and said, 'You would probably find that they had lost their papers!'

"Dr. Schorling was fascinated by an amaryllis that was blooming in my classroom. He admired the flower a number of times and studied the growth curve made by a seventh grade class. It was evident that Dr. Schorling liked flowers. 'You should see our garden,' was a comment Dr. Schorling often made. In the fall, choice chrysanthemums were lying on the counter in the high school office brought by Dr. Schorling to be shared with his teachers.

"This is Dr. Schorling as I knew him—my teacher, my counselor, my supervisor, and more than anything else my friend. The office door no longer bears the name, Raleigh Schorling, but in the lives of those who knew him, and through his writings for those who did not know him personally the door is not closed, for Raleigh Schorling has taken his place among the immortals."

Hope H. Chipman and Katharine Hill, mathematics critic teachers in the University High School of the University of Michigan knew him both as the chairman of their department and as a co-worker in teacher training who often called upon them to share the burden of instruction in special methods. The fact that their notes were written without collaboration makes the similarity of their comments add emphasis and conviction to that which they say. HOPE CHIPMAN writes:

"Dr. Schorling's sympathetic interest in his associates quickly made the members of his department think of him as a close friend. I know that I started out feeling very much in awe of him but I was soon asking his opinion on my insurance program, on some small investments I wanted to make, and on such matters as where to buy the best seeds and plants in the spring and where to find a man to put on our screens. We knew about his family and he knew about ours. If there were illness or a death in one of our families we knew his prompt expression of sympathy was honestly felt. We heard about his garden and his fishing success and his latest trip. It was fun to have him come in to sit on a table with his feet swinging while he told about an exciting meeting he had been attending in New York or Chicago or Los Angeles.

"There probably has never been a busier man, but when I went to his office I could be sure of a welcome, and that he would give my question his attention and helpful answer.

"One of my first contacts with him illustrates a happy facet of his character. At the end of the very first afternoon of classes in my experience at University High School, a warm sunny September day, Miss Hill and I started to leave the building together. As we walked down the hall toward Dr. Schorling's office he motioned to us to come in. There was some pleasant discussion about how the day had gone and then he said, 'Get out into the sunshine now. And remember, in this school when you do not have a class your time is your own. You do not need to feel that you must be here every minute of the day.' That gave me a wonderful feeling and it illustrates one of the traits that made Dr. Schorling such a fine man with whom to work. He trusted us all to do our best and left us free to do our work in our own way."

KATHARINE HILL uses the following words to tell how he appeared to her.

(Continued on page 99)

The General Mathematics Movement and Raleigh Schorling's Significant Contribution to It

By WILLIAM D. REEVE *and* JOHN R. CLARK
Teachers College, Columbia University, New York, N. Y.

E reform movement in the teaching econdary mathematics, variously n as fused mathematics, correlated matics, integrated mathematics, mental mathematics and general matics, was initiated and promoted group of teachers in the Chicago during the first two decades of the ry. Among the group in order of identification with it, were George s of the School of Education of the rsity of Chicago, Ernst Breslich, . Matthews, William D. Reeve, and gh Schorling, of the University of go High School, Harold Rugg of the l of Education of the University of go, and John R. Clark of the Chi-Teachers College.

his teacher training courses in the l of Education at the University, /. Myers attacked the traditional ization of arithmetic, algebra, geome-nd trigonometry as being out of line he demands of effective learning. He d that the traditional organization s artificial barriers, blocks, and ng difficulties. Moreover, he said such an organization may have l for the scholar and the mathe-ian, but that it is not a sound organi-l for the immature, beginning pupil. December 9, 1902, Eliakim Hastings e, head of the mathematics depart-of the University of Chicago, made amous presidential address before American Mathematical Society[1] in . he argued that the water tight compartmental organization of mathematics should be abandoned and advocated a closer correlation between mathematical subjects. Moore's speech did a great deal to stimulate further thinking and activity not only in the Chicago area, but elsewhere as well.

Early in the century Myers enlisted the cooperation of the members of the department of mathematics[2] in the University of Chicago High School in thinking through the problem of reorganization of the content material for teaching purposes. In 1903–4 a tentative program in correlated mathematics was worked out, revised and then used in mimeographed form as a text in the first year classes of the University of Chicago High School in 1904–5. The material was revised, remimeographed, and tried out again in the first year classes. After it was revised once more the course was published as a text.[3] In September 1909 a revised and larger edition of the above text was published with two other teachers[4] cooperating. Early in 1910 the University of Chicago Press published a second text called "Second Year Mathematics for Secondary Schools" which was used in the second year classes of the University of Chicago High School.

The above texts were then used for four or five years in the first two years of the University of Chicago High School, where Breslich was now the head of the

[1] oore, Eliakim Hastings. On the Founda-f Mathematics. *Science*, N. S. Vol. XVII, 1–416, March 13, 1903. Also The First ok of The National Council of Teachers hematics, pp. 32–57, 1926.

[2] They were William R. Wickes, Ernst R. Breslich, Harris F. McNeish and Ernest R. Wreidt.

[3] *First Year Mathematics.* University of Chicago Press. 1906.

[4] They were Arnold Dresden and Ernest L. Caldwell.

mathematics department. Through the influence of Charles H. Judd, Eliakim H. Moore and Franklin W. Johnson and with the approval of Myers, Breslich began a revision of the texts mentioned above and entirely rewrote both volumes which later[5] appeared under his name as sole author.

Reeve went to the University of Chicago High School in 1910, Schorling in 1912 and Horace C. Wright shortly after. These men all read and criticized Breslich's work before it went to the printer. Through the efforts of the above group of teachers the movement for a more modern type of mathematics waxed strong and grew in prestige.[6]

Many mathematics teachers went to the University of Chicago High School to observe the classroom instruction of Breslich, Reeve, Schorling, and Wright and to exchange ideas with them concerning the new organization of the subject matter.

Reeve left Chicago in 1915 to go to Minneapolis as head of the department of mathematics in the University of Minnesota High School. Since he and Schorling were interested in developing a general mathematics course they proceeded to prepare an experimental edition which was tried out by Reeve with a number of cooperating schools in the twin cities, Minneapolis and St. Paul. After considerable discussion with these schools and after careful revision this material was published in 1919.[7] Schorling, wen Lincoln School of Teachers Col 1916 as head of the mathematics ment and shortly induced Clark him there. Both of these men b vigorous promotion of the con general mathematics and later rated on some junior high schoc along general mathematics lines.

In 1917 Harold Rugg and Cla lished a monograph called "S Method in the Reorganization of Grade Mathematics," which stin further interest and discussion merits of general mathematics. T followed in 1918 by their text book ninth grade called "Fundamen High School Mathematics."[8] I Reeve published a second book[9] grew out of his experimental w Minnesota.

From this time until his death Sc was an ardent supporter of general matics. Moreover he showed great into the problems of improving the ing of mathematics and contin through his lifetime to be a keen a stant contributor to the literature field. We know how happy he alw to see the old formalism and cc mentalization of subjects repla more unified, correlated, integra general mathematics. In his passi field has lost a great teacher and standing leader.

[5] Breslich, E. R. *First Year Mathematics*, 1915, and *Second Year Mathematics*, 1916. University of Chicago Press.

[6] In 1917 Breslich published a third book called *Third Year Mathematics*, which was also published by the University of Chicago Press.

[7] Schorling, Raleigh and Reeve, Wi *General Mathematics*, 1919. Ginn & Co.

[8] Rugg, Harold, and Clark, John R., *mentals of High School Mathematics*. Book Company, 1918.

[9] Reeve, Wm. D. *General Mathemati* II. Ginn & Co. 1922.

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The Twenty-Ninth Annual Meeting of the National Council
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Testing

By MARY A. POTTER
Supervisor of Mathematics, Racine Public Schools, Racine, Wisconsin

EARLY in Raleigh Schorling's career he was fascinated by the possibilities of testing as a scientific technique to use in increasing competence in mathematics in American education. Although he had an unusual flair for dramatizing the principles of mathematics, he considered teaching a science as well as an art and used tests as scientific tools to improve the outcomes of instruction.

Educational testing was in its infancy when Mr. Schorling began to teach. But few tests in mathematics had been standardized and—like pioneer work in many fields—these tests were important not because of their quality but because they showed the power of a new method of evaluation of pupils' attainment in mathematics.

To Mr. Schorling, devising a test was not an end to be achieved, it was merely a step toward the end of better teaching. From his President's address entitled "Improving the Quality of Scholarship" printed in the *First Year Book* of the National Council of Teachers of Mathematics to his recent pleas for "functional competence in mathematics," he strove for a higher level of skills, understanding, and thinking from the children educated in our schools, and helped focus attention of educators upon the general lack of accomplishment in arithmetic, backing his statements by per cents of failures made by large numbers of pupils upon certain exercises found in standardized tests.

Following Mr. Schorling's concern with the teaching of mathematics in the six upper grades, he—usually in collaboration with others—wrote tests in the various subjects commonly taught at those levels—arithmetic, algebra, and geometry. He did not confine himself to tests of achievement but developed a new type called instructional tests, a form widely used later in many other school subjects. In line with modern trends, he became interested in and collaborated in constructing a test in functional thinking. He encouraged the construction of new type examinations by students in his classes and his last work in testing was to sponsor a "Test in Functional Competence" by David J. Davis, soon to be published, which has already appeared in experimental form as part of an Evaluation and Adjustment Series.

During his career as a writer of tests his procedure usually followed a uniform pattern. First, he had to be convinced that a test for some specific purpose should be written; he listed very clearly the objectives that he wished to accomplish; he wrote an experimental edition which was published and which he submitted to a number of competent critics; the test was then revised and standardized. Norms were established by data obtained by a wide geographical distribution. Performance of pupils in many types of school systems were used in standardizing the tests; there was a sampling of scores made by children in cities of various sizes and of scores made by pupils in rural areas, collecting the latter being a time consuming task. As might be expected from Mr. Schorling's interest in scientific methods, before 1928 when business computing machines were still a curiosity, the reliability of two forms of the Schorling-Clark-Potter Arithmetic Test was computed on one of these rare machines which had been loaned to the University of Michigan.

The unreliability, difficulty of grading, narrow sampling, and general failure of the essay type test in geometry was a challenge to mathematics teachers, which had been met by only one pioneer new-type examination, the Minnick-Geometry

Test. Young Mr. Schorling, then of the Lincoln School, accepted this challenge and began work on a new-type power test in geometry in about 1920. Details of its construction and standardization are presented by his collaborator, Miss Vera Sanford, in THE MATHEMATICS TEACHER of January, 1925 p. 22–36, from which we quote:

"The original idea of the test was this: to make an examination which would be comprehensive in the ground covered, which would require a minimum of writing, which would be objective and therefore quick to score, and which could be taken in a short time.

"In its original form the test was in seven divisions which may best be described by the type of question each contained. These were: completion sentences; true-false of conclusions from given data; true-false of converse statements; matching reasons against conclusions; drawing valid conclusions from given data; computation; analyzing constructions.

"In order to get a measure of both the very good and the very poor pupils, it was planned that each part of the test should contain questions of graded difficulty and to keep the test within a short time limit, it was proposed that the questions of each unit be arranged in order of difficulty. Ideally, then, a pupil's progress through each unit should be an indication of his ability and if he failed on the fourth question of a group, he might reasonably be expected to fail on the later ones."

Additional discussion and specimen questions from that test are given in the National Committee report of 1923, *The Reorganization of Mathematics in Secondary Education*, pp. 380, 390–395 in the chapter by Clifford Upton on Standardized Tests in Mathematics for Secondary Schools. Further revisions of the examination were made by Miss Sanford and Mr. Schorling which included a new feature, the thought-provoking true-false-cannot tell. This test in two forms was subsequently published by the Bureau of Publication at Teachers College, Columbia University under the title The Schorling-Sanford Achievement Test in Geometry.

Soon after the completion of the geometry examination Mr. Schorling, in cooperation with John R. Clark and Selma Lindell, began work on the first of the instructional tests which was published in 1925 by the World Book Company under the title "Instructional Tests in Algebra, Adjusted for Pupils of Varying Abilities." These tests introduced the unique feature of including both tests and practice material. If a student did well enough on the first trial on some of these tests, he was permitted to skip certain tests. If he did not reach a desirable goal on practice material, he took the same test again to improve his skill by the necessary practice. This remedial work was organized to conform with Mr. Schorling's list of principles in the psychology of drill which he discussed in his President's Address, previously referred to, which is printed in the *First Year Book* of the National Council of Teachers of Mathematics on pages 58 to 105. This philosophy deeply influenced his writings and since the number of copies of the *First Year Book* is limited, these principles of drill are quoted below.

"1. Drill to be effective must be individual.

2. In general there should be much practice for a few skills rather than a little drill on each of many things.

3. A drill exercise should be specific.

4. A drill exercise must provide a scoring technique so that the pupil may watch his daily growth.

5. A drill exercise should be standardized.

6. Drill material should be constructed so as to make possible the diagnosis of individual abilities.

7. In the early stages in the fixing of a bond, progress should be relatively deliberate.

8. Drill should be organized so as to prevent the use of 'crutches.'

9. Not all bonds should be given practice until high skill is obtained.

10. The goal of a drill exercise should be a reasonable one.

11. *Right* practice makes perfect.

12. Errors should be corrected before habits become fixed.

13. Everything else being equal, a skill that is fixed in its natural setting will need less repetition.

14. In establishing a skill try to avoid the stage of diminishing returns.

15. School life should be staged so that all desirable activities will have pleasurable outcomes and all undesirable activities will eventuate in unpleasant results.

16. In fixing a habit a pupil must be given an attitude in which he becomes a student of his own growth.

17. Habits must be formed in the psychological order.

18. Practice should be distributed in diminishing amounts and at increasing intervals.

19. The more interesting aspects that enter into a skill should be taught early.

20. The less difficult elements should be given before the more difficult."

A revision of the "Instructional Tests in Algebra" was one of Mr. Schorling's last publications and bears a 1949 copyright date.

"Messrs. John R. Clark, Raleigh Schorling, and Harold Rugg, of the Lincoln School of Teachers College, Columbia University" to again quote the 1923 Report, p. 306–7, "found themselves handicapped in carrying on certain experimental work to improve the teaching of mathematics in the junior high school unless they could determine with a fair degree of certainty the knowledge which the children were supposed to have as a result of their study in the first six grades. Accordingly they devised an Inventory Test in Arithmetic to be given to children at the beginning of the seventh grade as a measure of their knowledge of arithmetic obtained in the first six grades. The Inventory Test consists of two four-page pamphlets containing in all one hundred twenty-five questions."

Since there was need for a power test to measure progress in the upper grades, in 1927 this examination was revised by gearing it to the more difficult phases of arithmetic taught in grades 7 and 8 and omitting, of necessity, most of the simple skills usually acquired in the lower grades whose acquisition can be measured by other tests. The revised examination was published under the title "Schorling-Clark-Potter Arithmetic Test." A second revision with another standardization appeared in 1942 and was called "Hundred-Problem Arithmetic Test."

Instructional Tests in Arithmetic booklets for each of grades 5, 6, 7 and 8 written by Mr. Schorling in collaboration with John R. Clark and Mary A. Potter was a natural sequel to "Instructional Tests in Algebra." This series, based upon the same pattern and conforming to the same psychological principles of drill, was first published for general sale in 1928, experimental work on the tests went back to 1923. Experiments with several thousand children, whose performances on individual tests were plotted, furnished the passing and failing goals for diagnostic tests as well as the x, y, and z goals for the practice material.

Learning to Compute, Books One and Two by the same authors and Carroll F. Deady was a revision of the 7th and 8th grade books on Instructional Tests, changed to agree with the more recent knowledge of the science of education. *Learning to Compute, Book Two* was adapted by the United States Armed Forces Institute and published as *Review Arithmetic* in two pamphlets with enough blank pages to be used in preparing the lessons.

A more recent power test in mathematics, written by Judson W. Foust and sponsored by Mr. Schorling, is called the "Foust-Schorling Test of Functional Thinking." This new-type mathematics test of 80 items was devised for use in high

school or college and is designed to measure ability to think in the terms of the symbols and concepts of mathematics, independent of ability to compute.

Although testing in mathematics was not the main educational interest of Raleigh Schorling, his influence in that field was enormous due to his keenness in analyzing the problems of testing, the resourcefulness he used in meeting those problems, his originality in techniques of test construction, his common sense use of test results, and his enormous energy in doing tasks that he felt should be done.

Tests published include:

An Inventory Test in Arithmetic (with Clark and Rugg). New York City: The Lincoln School of Teachers College, 1920.

Survey Tests in Arithmetic (with Clark). Forms A and B. Ann Arbor, Michigan: George Wahr, 1923.

Practice Exercises for Accuracy and Speed in the Fundamentals of Arithmetic (preliminary edition) (with Clark). Yonkers, New York: Gazette Press, 1923. Pp. 31.

Achievement Test in Plane Geometry—Forms A and B (with Sanford). New York: Teachers College Bureau of Publications, 1926. Pp. 11.

Schorling-Clark-Potter Arithmetic Test (with Clark and Potter). Forms A and B. Yonkers, New York: World Book Company, 1928.

Instructional Tests in Algebra with Goals for Pupils of Varying Ability (with Clark and Lindell). Yonkers, New York: World Book Company, 1926. Pp. vii+72.

Instructional Tests in Arithmetic, Fifth Grade (with Clark and Potter). Yonkers, New York: World Book Company, 1928. Pp. vi+57.

Instructional Tests in Arithmetic, Sixth Grade (with Clark and Potter). Yonkers, New York: World Book Company, 1928. Pp. vi+66.

Instructional Tests in Arithmetic, Seventh Grade (with Clark and Potter). Yonkers, New York: World Book Company, 1928. Pp. vi+65.

Instructional Tests in Arithmetic, Eighth Grade (with Clark and Potter). Yonkers, New York: World Book Company, 1928. Pp. vi+78.

Instructional Tests in Arithmetic, Teacher's Manual (with Clark and Potter). Yonkers, New York: World Book Company, 1928. Pp. iii+24.

Practice Exercises for Accuracy in the Fundamentals of Arithmetic Adjusted to Students at Three Levels of Ability (with Clark and Potter). Ann Arbor, Michigan: George Wahr, 1928. Pp. iii+80.

Standardized Check List for the Organization of Senior High School Grades (with O. I. Frederick). Ann Arbor, Michigan: Ann Arbor Press, 1933. Pp. 16.

Standardized Check List for the Organization of Junior High School Grades (with O. I. Frederick). Ann Arbor, Michigan: The Ann Arbor Press, 1933. Pp. 16.

Hundred Problem Arithmetic Test, Form V (with Clark and Potter). Yonkers, New York: World Book Company, 1938.

Learning to Compute, Book One (with Clark, Potter, and Deady). Yonkers, New York: World Book Company, 1940. Pp. viii+104.

Learning to Compute, Book Two (with Clark, Potter, and Deady). Yonkers, New York: World Book Company, 1940. Pp. viii+104.

Hundred Problem Test, Form W (with Clark and Potter). Yonkers, New York: World Book Company, 1942.

Test of Functional Thinking in Mathematics (with Foust). Yonkers, New York: World Book Company, 1942.

Workbook in Arithmetic, Grade Seven (with Clark, Smith, and Lankford). Yonkers, New York: World Book Company, 1947. Pp. 160.

Workbook in General Mathematics, Basic Course (with Clark and Lankford). Yonkers, New York: World Book Company, 1947. Pp. 160.

Workbook in Arithmetic, Grade Eight (with Clark, Smith, and Lankford). Yonkers, New York: World Book Company, 1948. Pp. 160.

Chapter Tests—Algebra—First Course (with Smith and Clark). Yonkers, New York: World Book Company, 1949. Pp. 29.

Instructional Tests in Algebra, Revised Edition. Yonkers, New York: World Book Company, 1949. Pp. viii+88.

". . . Provision for growth in the mastery of arithmetic should be continuous throughout the elementary and secondary schools. . . . In the past two decades, professional courses for teachers dealing with the teaching of arithmetic practically disappeared from our teachers colleges, with the result that most beginning teachers do not know how to teach arithmetic. In fact, some actually fear it and escape the task by excessive attention to other activities in which they feel more confident. To make matters worse, many are tempted to teach the little they do teach by the incidental method—presumably the ideal method, but also the most difficult which only a few out of a hundred teachers could manage even if they had good training. The teaching of arithmetic can be and must be improved. Here is a problem that is not trivial, and one on which we need help."—SCHORLING, RALEIGH. "The Need for Cooperative Action in Mathematical Education," *The American Mathematical Monthly*, Vol. 52, April, 1945, p. 196.

The Reports of the Post-War Planning Commission

By ROLLAND R. SMITH
Public Schools, Springfield, Massachusetts

A MAN who was intimately associated with Raleigh Schorling for the last thirty years wrote me just last week, "Schorling's cooperation on committee work was solicited because of his professional competence and willingness to work, to work hard." The purpose of this article is to record briefly Schorling's part on two important committees, the committee that went to Washington during the war to prepare the report called Pre-Induction Courses in Mathematics and the Postwar Commission that published three outstanding reports.

After several months of seemingly futile correspondence with the United States Office of Education and the Civilian Pre-Induction Training Branch of the War Department, I got a telephone call from the U. S. Office of Education. This was on Thursday, Dec. 10, 1942. The call was urgent. "Can you be in Washington on Monday morning with a committee ready to work?" was the question. There was no time to think. I had to have men seasoned in that kind of thing. I called Mallory, Reeve, and Schorling and to my relief they all accepted. They met me in Washington on Monday morning.

Our job was to discover, by means of conferences with men in the training departments of the armed forces and by reading pamphlets and books used by them as instruction material, what mathematics was needed by our youth before they were inducted into the service. We worked hard but we worked more efficiently because Schorling had been working in Washington before and knew the places to go and the men to see.

We went to Washington prepared to stay a month, but we gathered our data and then went home in a week, especially because Schorling had to attend his daughter's wedding. He was devoted to his family. No report could take precedence over it.

A week later we all gathered in New York and worked in Dr. Reeve's office. We worked for days before the report was ready. Schorling was unable to attend because he had developed pneumonia and was confined to his house. In lieu of his presence, he sent in a completed report for us to consider. Always a quick thinker and a facile writer, he had finished a report on his way home to the wedding. The committee did not accept his writing in its entirety but it did incorporate much of what he had written.

The report was published in THE MATHEMATICS TEACHER of March 1943 under the title Pre-Induction Courses in Mathematics. It was also printed separately by the United States Office of Education and distributed by the thousands. It did much to bring order out of the chaos prevailing in the objectives of secondary school teaching of mathematics as preparation for the war effort. This one quotation will suffice to show how sanity prevailed in the writing of the report in spite of the advice from here, there and everywhere that every boy and girl should study mathematics of the academic type. "The four-year sequence in mathematics, including trigonometry and solid geometry should be taken by those students who have a real interest in mathematics, who are capable of mastery of the subject, or are likely to use mathematics in their further training and ultimate occupation. Counselors must realize the need for certain mathematical skills and understandings. On the other hand it is wasteful to put pupils into courses if their aptitudes

and abilities indicate that they cannot obtain a secure mastery of the material they are studying. It should be remembered in every case that unless the mathematics taught is thoroughly mastered, it will neither be of practical use nor will it serve the other purposes of mathematical study."

One of the men on that committee wrote me, "Dr. Schorling's enthusiasm and unbounded energy contributed in no small degree to his committee work. His zeal for high school courses in Consumer Mathematics is evident in all of the reports. His fund of stories lightened the arduous committee sessions."

When I retired as president of The National Council of Teachers of Mathematics in February 1944 the time had come to think about mathematics in the postwar era. We hoped we would not lose the ground we had gained during the war. I discussed the possibility of a Postwar Commission with the board of directors and the active past presidents and was directed to appoint such a commission. Schorling was the man I wanted for chairman because he had the "know how," the contacts, and the reputation for hard work. He accepted the chairmanship and then followed what I believe to be his greatest contribution to the work of The National Council.

We had a small committee at first, only four members besides the chairman. The First Report came out in May 1944 only a short time after the commission was appointed. It was a preliminary report asking for suggestions. It was general in nature but it had in it the germs of the more influential Second Report. Even a casual reading will show Schorling's great interest in a new kind of mathematics for what he believed to be the majority of students. One of its statements was, "We should differentiate on the basis of needs, without stigmatizing any group, and we should provide new and better courses for a high fraction of the school's population whose mathematical needs are not so well met in the traditional sequential courses."

Then the commission was gradually increased to give better representation of various parts of the country. At the time of the Second Report there were eleven members. When the Guidance Pamphlet was published there were thirteen members. The commission held meetings whenever The National Council met and several times in between. Work was assigned. Manuscripts went back and forth. Finally the Second Report came out in May 1945.

This report was very emphatic in its recommendation for mathematics for functional competence. The check list of twenty-eight items, Schorling's idea, has given the greatest impetus to general mathematics. Almost every new book that comes out in this field makes reference to this check list. The work on the reorganization of mathematics begun in 1916 and reported in 1923 finds its latest development in this report.

The Guidance Pamphlet, the last work of the Postwar Commission, was also Schorling's idea. There were grand plans for it. The Commission wanted to have it illustrated in color and have it filled with cartoons to insure its acceptability by the young folks. We even got as far as to write part of it in that vernacular used by the present day high school pupil. This was distributed for criticism but a number of readers feared that this form would make some educators discredit the scientific value of the pamphlet. The proposed plan for illustrating the text was dropped on account of lack of funds. Even without these added attractions it has proved to be vitally helpful. Over forty thousand copies have been distributed and have been used by counselors and teachers to show the need of mathematics on various levels. A striking attribute of the pamphlet was that it took into account the fact that all students did not need the academic mathematics courses so long as they were not the only courses given.

When I was asked to write this article

Schorling's work on The National
l's committees, I immediately wrote
the views of the other members.
of the replies are expressed so much
than I could do that I am quoting
erbatim.

eigh was an excellent chairman for
mmission. This position gave full
the fine qualities I had known in
er a period of years. He was an
it presiding officer, never hesitat-
state his own views on important
ut at the same time never insisting
ose views be accepted. Every one
chance to say what he wanted to
that the Commission, composed
unlike personalities and represent-
te hostile conceptions, nevertheless
together with a minimum of fric-

had an amazing capacity for simul-
sly carrying on a multitude of ac-
. Many times he brought us up to
a current research projects in re-
reas. He was able to maintain our
; in the work at a sufficient level
e ball kept rolling, in spite of the
at each of us was overloaded with
uties. He must have done an enor-
mount of work himself. He was
tionably the driving force in the
ssion and had a good grasp of the
n at hand as well as ideas concern-
; logical solution. I did not feel,
r, that he imposed his will upon
rather that he sought our opinions
s careful to honor them. The fin-
roduct was a composite of our ef-
ther than a tempered statement of
ideas. I doubt if I shall ever forget
his sense of humor and his ability to tell a story. The twinkle which lit his eyes and the accompanying smile were well-nigh irresistible."

"He was energetic and persistent in carrying through this program in the face of inadequate funds and related difficulties. The Guidance Pamphlet which was evolved under his leadership will continue to influence the teaching of secondary mathematics for years to come. In particular, the formulation of the Check List in that pamphlet was primarily his contribution."

"Be sure to include in your article his gentle courtesy, humility regarding his own ideas, willingness to listen and when convinced to change his mind. He was firm when he needed to be, an excellent moderator when moments could be tense."

"When I look back upon the many meetings that I attended these thoughts and impressions come to my mind: Schorling's unusual resourcefulness and breadth of vision when undertaking a project or problem. His professional courage and vision. His fairmindedness and tolerance. His keen sense of humor. His energy and drive, never sparing himself. His capacity to get the right people to contribute to a joint undertaking."

All the members of the Commission have stated that it was a pleasure to work with him.

The last years of his life were saddened by the loss of his son. Just when he and Mrs. Schorling were rejoicing at the end of the war, they learned, the day afterward, that their son had been killed in action.

1951 SUMMER MEETING
National Council of Teachers of Mathematics
St. Olaf College, Northfield, Minnesota
August 20-23, 1951
See the March Mathematics Teacher for further details

The Reorganization Report of 1923

By CHARLES H. BUTLER
Western Michigan College, Kalamazoo, Michigan

AN APPRECIATIVE understanding of the position and the program of mathematics in the modern American scheme of secondary education can best be had by viewing it against the backdrop of history. Its evolution from the stereotyped arithmetic of colonial days to the comprehensive and varied offering of today represents a continuing effort to make mathematics contribute all it could toward the achievement of the broad aims of prevailing educational philosophies, and many influences have been operative in shaping its course. The story of the evolving program of secondary mathematics has been fully and well recounted in numerous books and articles. It is not the purpose of this paper to tell the whole story again, but merely to indicate something of the contribution of one important committee, and especially of one of its members, to the development of the program in mathematics in the United States in the past quarter of a century. This committee was the National Committee on Mathematical Requirements, and the member of it to whom reference was made was the late Professor Raleigh Schorling, to whose memory this issue of THE MATHEMATICS TEACHER is dedicated.

The so-called "reorganization movement" in mathematical education may be thought of as beginning at about the turn of the century, though it may have had its real genesis a little earlier than that. In the ensuing quarter of a century there was a rather widespread awakening of interest in the matter of reorganizing the mathematical courses and of improving the work in mathematics in the secondary schools. Notable impetus was given to this movement by such events as the presidential address of Professor E. H. Moore to the American Mathematical Society in 1902, by the appearance of reports of numerous committees and commissions, including, among others, those of the Committee on College Entrance Requirements and the College Entrance Examination Board, the International Commission on the Teaching of Mathematics, and the National Committee on Mathematical Requirements, by the unprecedented growth of the high school, and by the emergence and rapid expansion of the junior high school as an accepted part of the secondary school. An admirable account of the significant changes and improvements in secondary school mathematics in the first quarter of the twentieth century was prepared twenty-five years ago by David Eugene Smith. Under the title "A General Survey of the Progress of Mathematics in Our High Schools in the Last Twenty-Five Years" this illuminating article was published in 1926. Appropriately, it forms the opening chapter in the First Yearbook of the National Council of Teachers of Mathematics.

It was near the end of that quarter-century of progress that the classic report on The Reorganization of Mathematics in Secondary Education made its appearance. First published in 1923, this report was destined to have what was probably a more powerful effect in shaping the course of mathematical education than any other report published either before or since that time.

The committee which prepared this report was known as the National Committee on Mathematical Requirements. The nucleus of this committee consisted of six distinguished mathematicians, all filling eminent positions in leading universities, who were appointed in 1916 by the then president of the Mathematical Association of America. With its membership subsequently expanded to fourteen, to include representatives of high school

ics as well as representatives of and university mathematics, ittee worked for seven years senting its final report to the ius it is clear that the report d the combined and mature of a representative group of competent people.

sk assigned to this committee mmarized broadly by quoting m the Preface to the report:

ational Committee on Mathe-equirements was organized . . . rpose of giving national expres-e movement for reform in the of mathematics, which had nsiderable headway in various he country, but which lacked that coördination and united e could give. . . . This [original] was instructed to add to its ip so as to secure adequate tion of secondary school inter-hen to undertake a comprehen-of the whole problem concerned improvement of mathematical and to cover the field of second-llegiate mathematics."[1]

ort was published by the Mathe-ssociation of America, under pices the committee was estab-ie original edition contained 1 650 printed pages. Several copies of this monumental re-printed, and through the finan-ance of the General Education se were distributed gratis to individuals, schools, and librar-

ort was in two parts. Part I of eight chapters in which were the general principles that the felt should be observed in efforts to improve the courses and the work in mathematics, and the committee's recommendations for bringing about improvements in the courses at the different grade levels. This part of the report includes able and lucid statements of the committee's ideas and recommendations respecting the aims of mathematical instruction, the content and arrangement of the subject matter for the junior high school grades and the senior high school grades, specific proposals for the topical content and the arrangement of the courses in algebra and demonstrative geometry, suggestions looking toward more uniformity in the usage of terms and symbols and toward increased emphasis on the function concept, and proposals for the redefinition of college entrance requirements.

Part II consisted also of eight chapters in which were reported descriptions and results of a number of special investigations which were conducted for the committee. Of special significance among these were extensive investigations of the status of disciplinary values, of experimental courses in mathematics, of standardized tests in mathematics, and of the training of teachers of mathematics.

Even in its early stages the work of the Committee on Mathematical Requirements began to exert a marked influence upon the materials of the courses and upon the viewpoint governing instruction. As far back as 1926 one of the most competent and critical observers of mathematical education wrote, "It is not too much to say that the advance in the teaching of mathematics in the last decade has been due in large part to the work of this committee." By changing the word "decade" to "thirty-five years" the same statement could have been made without other modification and with equal assurance in 1951.

It is fortunate that the Reorganization Report came when it did. It came at a time when educational experimentation and curriculum and course revision were

al Committee on Mathematical Re-
"The Reorganization of Mathe-Secondary Education," (Hanover, Dartmouth Press, 1923), p. vii.
iginal edition has long been out of I of the report was subsequently re-y Houghton Mifflin Company.

in the air anyway; when the whole educational atmosphere was becoming charged with unrest and uncertainty both as to ends and means to those ends; when in view of this it was inevitable that the mathematical program would be subjected to many conflicting claims and counterclaims, and that its objectives, subject matter, and methods would be sharply challenged. At such a time it was well indeed that mathematics had taken such an adequate step toward setting its own house in order.

For a quarter of a century now, that report has stood as a powerful beacon and a bulwark for the mathematics program of the secondary school. It has defined the aims with vision and clarity. Its recommendations have provided guiding principles for a sensibly forward-looking program of course revision and instruction, and at the same time they have provided a stabilizing influence which has been of great value.

The effects of the report have been clearly discernible in every course and at every grade level in the secondary school. Its recommendations for the work of the seventh and eighth grades long ago set the main pattern for the work in these grades. In ninth grade algebra it made recommendations regarding such things as emphasis upon the function concept, the meaning, notation, and use of the formula, the technique and interpretation of graphical representation of functions, the types of exercises appropriate to this level of instruction in such topics as directed numbers, equations, factoring, operations with fractions, and very simple numerical trigonometry. These recommendations have been widely reflected in textbooks and courses of study almost ever since the report first appeared. In like manner, many of the modifications and shifts of emphasis which have been made in courses in plane and solid geometry, trigonometry, and algebra beyond the ninth grade were foreshadowed by the recommendations made in the Reorganization Report.

Of course the mere fact that one phenomenon is followed by a second phenomenon does not prove that the second is caused by the first. *Post hoc, ergo propter hoc* is an all-too-common pitfall for the unwary. But there is much additional evidence that such improvements as have been made in the teaching of mathematics in the past twenty-five years are in large part traceable to the Reorganization Report. It is probable that in that period not a single course of study in mathematics, which has reached the stage of being printed or mimeographed, has been promulgated without reference to the Reorganization Report for guiding principles and recommendations. It is certain that practically every book dealing with the teaching of secondary school mathematics which has been published in the past twenty-five years has emphasized the importance of this report, as have hundreds of articles which have appeared in periodicals. But perhaps the most conclusive evidence of all is to be found in prefatory statements and in the actual contents and arrangement of textbooks in high school mathematics which have been published since 1923. In the prefaces of numerous ones of these books, statements are found to the effect that "This textbook fulfills the recommendations of the National Committee on Mathematical Requirements." Prefaces, of course, are addressed to the consuming public, and statements of this nature made in the prefaces of textbooks give a pretty clear indication not only that the authors of the books have recognized the importance of the report, but that the committee's recommendations, through their influence upon the textbooks, have had far-reaching effects upon the actual classroom work in mathematics throughout the past twenty-five years.

* * *

Raleigh Schorling became a member of the National Committee on Mathematical Requirements a short time after the committee was first established. He remained

(*Continued on page 96*)

Mathematics in the Education of the Consumer

By FRANCIS G. LANKFORD, JR.
University of Virginia, Charlottesville, Virginia

PROFESSOR SCHORLING was deeply interested in the consumer education movement in our schools. He had keen insight into its implications for the contribution which mathematics may make to general education. This insight is clearly exhibited in two publications that bear his name as co-author. One of these is *The Role of Mathematics in Consumer Education*,[1] a report of a committee of the National Council of Teachers of Mathematics; the other is *Mathematics For The Consumer*,[2] a textbook for high school students. It, therefore, is entirely appropriate in this number of THE MATHEMATICS TEACHER dedicated to the memory of Professor Schorling that the topic "Mathematics in the education of the consumer" should receive attention.

To anyone seeking to improve the mathematics curriculum in our schools, consumer education offers many possibilities. First, everyone is a consumer and, therefore, the education of the consumer must be the concern of general education. Moreover, in many of the problems of the consumer, elements of mathematics are conspicuously present. Hence, it should be helpful in curriculum planning to attempt an answer to the question: what mathematical needs does the consumer have?

First, I would list the need to see meaning in numbers and number relations. I mean, for example, ability to sense that something is wrong when a decimal has been wrongly placed in a newspaper advertisement of a sale. I mean also ability to test the reasonableness of computed answers by making estimates. Surely ability to test the reasonableness of computed answers will help young consumers to answer such questions as: How much will my annual income be increased by an hourly wage increase of 25 cents or 13 percent? How much of this will be "take home pay" and how much will be withheld in income taxes? How much will I have left if I buy an automobile, making monthly payments of $59.25?

If pupils are taught to see meaning in numbers and number relations, they will also be able to make acceptable estimates of sizes and distances. The young housewife will not buy too few yards of curtain material after measuring her windows. She will have some idea whether a tempting bargain 9×12 foot rug will look right on her living room floor. She will not just take the salesman's advice of a gallon of paint to a room, but she will estimate the surface to be covered and the amount of paint needed. Then she will decide whether to buy a gallon anyway, when she really needs only three quarts, because she can buy a gallon for $3.49 and quarts cost $1.17 each.

Second, I would suggest that our young consumers will need to be able to interpret data in order to check the conclusions of others. Here they will certainly need to understand the relationship of assumptions to conclusions as they react to the claims made in the press and on the radio, as well as television.

Such a question as: "Shall we get an overdrive with our new car?" should be decided after considering the cost in relationship to possible economy. The arithmetical problem which the consumer needs to be able to solve is:

Data Given:
$90 = cost of overdrive.
Miles per gallon of gasoline without overdrive = 18

[1] *The Role of Mathematics in Consumer Education*, The Consumer Education Study, 1201 Sixteenth Street, Washington, D. C., 1945.

[2] Schorling, Raleigh, Clark, John R., and Lankford, Francis G., *Mathematics For the Consumer*. The World Book Company, Yonkers, N. Y., 1947.

Miles per gallon of gasoline with overdrive = 22
Price of gasoline = 30¢ per gallon.
Problem: How many miles will one have to drive before he saves enough on gasoline to pay for an overdrive?

Third, they will need to know how to collect and to interpret data needed to answer a problem. Many young people should consider seriously such questions as: Can I afford a television set? or, Should we continue to rent or try to buy a home? They are undoubtedly deciding these questions without careful consideration of the facts. The result is often an excessive debt burden that may prove tragic. The high school has some responsibility to teach these people how to use numerical data in solving such personal problems, lest they continue to be the prey of persons who gain by their lack of competence.

Fourth, there is the need to make approximate computations mentally. I do not mean "short cuts" that often confuse pupils more than the longer operations they are intended to replace. Rather I mean the practice of rounding off numbers in order to make estimates. The farmer's acre is 4,900 square yards instead of 4,840. With this approximation he can compare any surface he wishes to express in acres to a square 70 yards on a side. The well-known 6 per cent method of computing interest uses a 360-day year in order to make approximate computations easy.

In many high school courses we teach pupils a cumbersome method of computing the cost of installment loans and purchases. Yet we know they will not use this operation unless we also teach them how to estimate such costs without the aid of pencil and paper.

Fifth, there is the need which maybe we should have listed as Number One: to have a *mastery* of operations with whole numbers, common and decimal fractions.

Unfortunately, our record in meeting this need is not enviable. For a generation we have known that pupils were leaving school deficient in the most basic elements of mathematical literacy. Look back over the published reports of the last twenty to thirty years and you may be surprised how long we have had extensive evidence of serious mathematical deficiencies on the part of great numbers of those who have attended our schools.

Sixth, there is the need to have confidence in and satisfaction from dealing with mathematical situations. When pupils leave our mathematics courses with a "never again" attitude, not much has been accomplished. We must, by making mathematics instruction satisfying, refute the idea that it takes a "mathematical mind" to work with such topics as common fractions and per cent in everyday affairs. It is true that a person may get along in daily life with very meager knowledge and understanding of mathematics. Many persons, lacking confidence in dealing with mathematical situations, make all kinds of clumsy substitutions for mathematical solutions. These people are the first to claim the uselessness of mathematics in everyday living. The person who has confidence in his ability to deal with relatively simple mathematics understands how his daily living is thus enriched.

Seventh, consumers need to understand and respect the nature of mathematical proof in order to use it as a daily tool. I have mentioned the importance of understanding the relationship of assumptions to conclusions in deductive reasoning. More teachers need to recognize that logical reasoning is the essence of mathematics. They need to give their pupils this contribution which mathematics has to make to their general education as well as the ability to handle operations. "Remember that the intelligent reader is a consumer of ideas as well as of food, shelter, clothing, and services."[3]

[3] *The Role of Mathematics in Consumer Education*, The Consumer Education Study, 1201 Sixteenth Street, N. W., Washington 6, D. C.

Eighth, there is the need for a thorough understanding of common mathematical concepts. Such concepts as ratio, percentage, average, discount, and rate are encountered frequently by the consumer and constitute a large part of "mathematical literacy."

At this point one may very well ask, "Aren't these also the needs which a person has for mathematics as a homemaker, as a citizen, as a wage earner, or as one who operates on a high level in any aspect of his daily living?" And the answer is "Yes." The important point for curriculum reform, however, is that problems of the consumer provide an excellent medium, though not the only one, through which these mathematical competencies may be taught, so that young people can *use them*.We have long since learned the futility of trying to develop these universally needed competencies in mathematics *before* they are used, i.e. in a kind of academic vacuum. We have much better luck when we teach them *as* they are used in solving such first hand problems as those of the consumer.

Related to the question of what are the mathematical needs of the consumer? are the questions of grade placement of the mathematical topics of consumer education and how this content shall be organized for teaching. We know that there has been much consumer education content in the general mathematics courses of grades seven, eight, and nine. In the *Role of Mathematics in Consumer Education*, Professor Schorling and his committee expressed the view that

> "Our greatest mistake has been trying to teach the mathematical topics of consumer education completely, once and for all, much too early—in the seventh and eighth grades, long before pupils were ready for them. Generally speaking the study of investments, taxation, insurance, and the like is hard to motivate for immature pupils. It is true that by special effort and laboratory techniques, the gifted and resourceful teacher can make these topics as interesting as any other unit in mathematics. However, thirteen and fourteen-year old boys and girls do not have the mathematical maturity to enable the typical teacher to do a respectable job, to say nothing of making the problem challenging. Unfortunately some curriculum specialists (including enthusiasts for consumer education) do not know that arithmetic problems may be practical and very interesting to adults and yet be deadly boring to youngsters in the seventh and eighth grades, and remote from their interests and needs."

It was proposed, therefore, that at the junior high school level we emphasize the "information" or "appreciation" aspects of the mathematical topics of consumer education and give scant attention to the computational aspects. These would be left for a *separate* course in consumer mathematics at the senior high school level when pupils will be more mature and sufficiently close to the consumer problems studied to be genuinely interested in studying them. For example, at this level many pupils will be paying income taxes on their own earnings. Many will have social security cards and many will be contemplating soon establishing a home of their own.

Recently, I visited a senior class in consumer mathematics as they dealt with the problem, "Can a young couple afford to get married on $40 a week?" Enthusiasm was high because the problem was now a very real one.

Concerning the question of how the curriculum should be organized to include the mathematical topics of consumer education much might be said. Professor Schorling and his committee have already been quoted as advocating a separate course in mathematics at the senior high school level. This was also a recommendation of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics in their 1940 report entitled *The Place of Mathematics in Secondary Education*. The more recent *Second Report of the Commission on Post War Plans* which appeared in THE MATHEMATICS TEACHER for May, 1945, contained a similar recommendation in these words.

"New and better courses should be provided in the high schools for a large frac-

tion of the schools' population whose mathematical needs are not well met in the traditional sequential courses."

"Every pupil is potentially both citizen and consumer; hence all pupils should be given some understanding of the persistent problems that confront most of our families; viz., social security, taxation, insurance, against the numerous hazards of life, and material comforts and values with a given income."

There are curriculum specialists who disagree with these recommendations for a separate course in the senior high school dealing with the mathematical topics of consumer education. Instead they think such topics should become a part of a core curriculum or common learnings program and they decry the tendency, long prevalent, to reform the high school curriculum by adding courses. There is much to be said in support of this view. A realistic position on this matter, however, must leave us concluding that the recommendation of a *separate* course in consumer mathematics is the *only* way the mathematical topics of consumer education will be included extensively in the high school curriculum for many years ahead. Indeed it may be reported that an increasing number of schools today are offering consumer mathematics courses in the senior high school years. They are demonstrating the soundness of Professor Schorling's insight into the possibilities for using mathematics for the effective education of the consumer and for improving basic mathematical competencies of *all* high school students at the same time.

Reorganization Report

(Continued from page 92)

a very active member until its final report was published in 1923. The first eight chapters (Part I) of the report were prepared, criticized, and eventually approved by the committee as a whole. No statement is available to indicate what individuals actually wrote these chapters, but some characteristics of style suggest that Professor Schorling may at least have had a hand in it. In any case it is certain that his work and influence had much to do with the recommendations contained in the report. Indeed, his was a double contribution. Not only did he assume his full share of responsibility and work in connection with the deliberations of the committee, but in addition to that, he conducted and reported one of the special investigations which the committee wished to have made. This investigation is published as Chapter XII of the Reorganization Report, under the title "Experimental Courses in Secondary School Mathematics."

This was no casual report. One has only to read a few of the 102 closely printed pages of Chapter XII to appreciate the immense amount of time and work and the vision and competence which went into its preparation. The penetrating insight, the ability to get immediately to key issues, and the forthright, clear-cut writing, which have come to be such well-known characteristics of Professor Schorling's work, are all in evidence. This special report which he prepared contains, in addition to two preliminary sections, separate and meticulously detailed descriptive accounts of experimental work in the teaching of mathematics then being carried on in fifteen schools in different sections of the country. There can be little doubt that this special contribution of Professor Schorling to the Reorganization Report has provided the ideas and the impetus for much of the experimental work in secondary school mathematics which has been done since that time.

A Laboratory Course in Making Visual Aids for Mathematics Education

By RALEIGH SCHORLING
Edited by Phillip S. Jones

Preface: Early in the fall of 1948 Dr. Schorling conceived and began the planning of the laboratory course of which his description is found below. He enlisted the cooperation of Phillip Jones in organizing and administering the course, of Allene Archer as the "kick-off" speaker, demonstrator, and laboratory director for the first two weeks of the six weeks' summer session, of S. Katharine Hill as a general assistant, purchasing agent, and "greaser-of-the-wheels." Dr. Schorling secured funds for the staff and for the purchase of tools, equipment, and supplies. He also secured administrative permission to charge a five-dollar laboratory fee to be used to replace expendable materials or to buy new kinds of supplies for which a demand might develop.

The course was offered in the summer of 1949, in the spring semester of 1950, and in the summer session of 1950.

Dr. Schorling and Dr. Jones had discussed writing jointly a description of the program for the *Bulletin* of the University of Michigan School of Education. The following paragraphs bearing the date of April 3, 1950 were found in Dr. Schorling's file.

A NOVEL course is being given at the University of Michigan for prospective teachers of mathematics. It provides an opportunity to make aids that illustrate basic concepts and principles of mathematics. Although teachers of mathematics have long used such learning aids as blackboards, graphs, models, etc., the fact is that most mathematics classrooms are too barren of things—of aids that illustrate concepts and principles. The student teacher completing this course may be far too busy when he starts carrying a heavy load on his first job to make learning aids himself. However, with the information gained he should be able to so manage that each semester a few ingenious high school pupils will make them for use by the class, and thus contribute effective learning aids to the permanent equipment of the mathematics classroom.

The course was offered for the first time at the University of Michigan for experienced teachers in the summer session of 1949. Some work of this type, of course, has been done in short institutes and in several in-service programs. However, a course in the making of visual aids probably was never offered before for college credit—certainly not at the graduate level.

Each student in this course makes from five to twenty learning aids. He takes these with him and exhibits them in his classroom. Moreover, he takes with him photographs of such learning aids as are constructed by his fellow students but which he didn't have time to make. Letters from former students indicate that some classrooms have been given a "new look" by the teachers who took the laboratory course last summer.

The course is strictly a *laboratory course* from the first day to the last. There are more helpful reading materials than some teachers realize. For example, there is, of course, the Eighteenth Yearbook of the National Council of Teachers of Mathematics entitled "Multi-Sensory Aids in the Teaching of Mathematics." In addition the students spend short periods evaluating the output and discussing the utilization of each aid.

During the school year of the University of Michigan the course is offered primarily for student teachers, and they seem to find it interesting. The course meets on Saturday morning for a double laboratory period. It has one specific feature that is proving very valuable. On a given Saturday morning the teachers of mathematics from a nearby city are given a special invitation to visit the class and go to work side by side with our student teachers. On the following Saturday a group of mathematics teachers from another city

is invited, and so on throughout the semester. This working together of student teachers and teachers driving back to the University from the field is a fine experience for our student teachers.

Like any laboratory course the new course is very expensive. There are five instructors for only about twenty-five students. This semester the teachers are Raleigh Schorling, Alice Hach, S. Katharine Hill, Phillip S. Jones and Russell Schneider. Then, too, the course requires much in the way of materials and tools.

The main idea of this novel course is not too new. In fact, Thomas Jefferson stated it very neatly in a letter to George Wythe when he wrote as follows:

"I have reflected on your idea of wooden or ivory diagrams for the geometrical demonstrations. I should think wood as good as ivory; and that in this case it might add to the improvement of the young gentlemen that they should make the figures themselves."*

The following four paragraphs had been deleted by Dr. Schorling from an earlier version of the above article. They would have somewhat destroyed its unity, but we are printing them here as of value and interest in themselves and as embodying some idea of the breadth of his thinking and interests as well as of his philosophy of education.

Our experience suggests that the day may come very soon when a *general* course in audio-visual aids may be a required prerequisite for student teaching. Of course a unit of visual aids is very common as a part of student teaching or some prerequisite course, but I am disposed to think that something far more important is emerging. The right kind of course in audio-visual aids has a concomitant outcome that may be far more important than its stated objectives. A course in visual aids prerequisite for student teaching could be designed so as to be a novel and effective approach to basic problems of the curriculum and method and be nicely geared to the stage of development in which most student teachers find themselves.

In the normal school of earlier days they required us to do observation for credit as a prerequisite to student teaching. The purpose was altogether good. The trouble was that it couldn't be achieved. Our supervising teachers wanted us to have some laboratory experiences before reporting for student teaching so we were sent to see what was going on in the classrooms of the campus school. But we couldn't see very much—that is, really understand—until we had had real experiences.

Some of the reasons why observations meant so little to us is that we had to do our own interpretation and we had no valid scale of values for evaluation. Some of the things that we saw we now recognize as having been very bad for children, but at the time they looked good to us because they were so much better than the situations in which we had been pupils.

However, pedagogical maturity cannot be achieved by real experiences alone. Some of it—and a very high fraction—has to be gained by vicarious experiences; and let no one underestimate the importance of vicarious experiences. It is the only way that we can eat our cake without having it! Good literature is good literature if, as vicarious experiences, it is so vivid that it becomes a part of us. Visual aids are of course vicarious experiences; but they are of a very special kind. They are not only vivid but they can be so managed by ingenious methods that evaluation and interpretation are provided on the spot. So it may turn out that visual aids will make one of the finest contributions to teacher education, especially in providing pedagogical maturity for the beginning student teacher.

In addition to persons mentioned earlier, John Schacht, M. H. Ahrendt, and Ava Mae Seedorff each served as a visiting lecturer for a day this past summer.

Some evaluations of the success of the course and of the *incidental* values which accompanied

* Smith, D. E., "Thomas Jefferson and Mathematics." *Scripta Mathematica*, Vol. I (Sept., 1932), p. 5.

it and at times seemed among its greatest values should be noted. In 1949 this course along with all other Education School courses was rated by the students on the questions of "Value of Assignments," "Personal Value," "Would You Recommend It to a Friend?" On the scales used to represent these ratings the averages for all courses offered in the School were .65, .66, .86 respectively. The ratings of the "Laboratory Course in the Construction of Mathematics Teaching Aids" were .83, .91, and .93 respectively. Among the subjective evaluative comments made by the students some themes occurred repeatedly though in different phraseology. Examples are: "I was much impressed by the friendly and cooperative student relations and the informal class procedure." "I have profited from the exchange of teaching ideas and experiences with my classmates." "It has given me a better professional outlook on mathematics." "I have found a clarification of some mathematical principles in my own mind and have been stimulated to want to know more mathematics." "Fellowship." "Sheer enjoyment." "I have a feeling of security (in the construction and use of aids) and I plan on using the laboratory method of teaching as a result of this work." "It will strengthen the position of mathematics in our school on exhibits." "I have some ideas for giving meaning to mathematics."

Some of these comments seem to imply that informal and laboratory procedures may rate a higher place in our teacher-training program than they now receive. Perhaps we need to listen to and to respect our own words of advice about democratic procedures, laboratory techniques, and teaching for meaning which we so freely give to our student teachers.

As Others Saw Him

(*Continued from page 80*)

"Great as Dr. Schorling was in the field of education, we never were made to feel the weight of that greatness in our association with him. We were working with him rather than under him. We always felt free to go to him with our problems, knowing he would give us advice willingly and help generously. Any professional success we had was a pleasure to him, and his confidence in our ability was a support and a challenge.

"Quick to help us professionally, he was just as quick to help us personally, to rejoice with us, to sorrow with us, and to be interested in our well-being. Shortly after I came to University High School, Dr. Schorling came into my room while I was grading papers. 'What are you doing indoors when the sun is shining like this? Those papers can wait. Get outdoors!' And he waited to see that I went. His consideration for us was shown over and over again, in many ways, in the years to come.

"Those of us who are old enough to remember Dr. Schorling as he taught a junior high school class, cannot forget his understanding of children and the skillful teaching that made the pupils—oblivious of the observers around them—live in a fascinating world of mathematics. He never lost that interest in young people. The students became accustomed to having him step into a classroom to observe for a few moments. He enjoyed watching them in the corridors, was an enthusiastic supporter at athletic events, attended the school plays and operettas whenever possible, and many Friday mornings found him slipping into the back of the auditorium to listen to the student assemblies.

"The seniors in their yearbook ended their tribute to him with ' . . . but above all he was the warm and loyal friend of all the teachers and pupils in our school.' Perhaps that is the tribute which he would have prized most highly."

Raleigh Schorling returned to the University of Michigan in 1924 to organize and to set up its University High School and to serve as its first principal. Mrs. Edith L. Hoyle Craig was a member of the original faculty which he assembled for the school. The following "reminiscences" of "R. S." and the early days of the University High School are hers.

(*Continued on page 107*)

An Evolving Bill of Rights for Teachers*

By RALEIGH SCHORLING

IN RECENT months newspapers, magazines, and government reports have printed convincing facts regarding the shortage of qualified teachers. The prospects of getting enough good teachers in the next decade are poor. The fact is that many able young persons now in training, who under proper conditions would prefer to teach, are rejecting teaching as a career. Salary is an issue, but there are about a dozen additional reasons why young people turn away from teaching. Most of these relate to the *working conditions* of teachers.

These various reasons why teaching fails to attract as many able young people as the schools need are listed in the following pages in a Bill of Rights for Teachers. It is hoped that a valid bill of rights will emerge from study groups in order that the public may come to realize that we will never get enough good teachers, nor will the competent teachers now in service ever be able to do reasonably efficient work until certain working conditions are improved.

Inasmuch as every boy and girl of school age is entitled to good teachers, in a very real sense the proposal is a bill of rights for youth. However, a direct attack on the problem is forthright and presumably is more likely to be understood by the public.

THE RIGHTS OF A CLASSROOM TEACHER

1. *The right to teach classes that are not too large—in general, from ten to twenty pupils.*

The average class size in the schools of the armed forces for more than twelve million men and women was less than one-half of the actual class size of civilian schools in our metropolitan areas. Individual attention and proper guidance cannot be given by a teacher with *oversized classes.* Experts insist that crime and delinquency which now cost us, annually, at least six times what the nation pays for its schools, could be sharply reduced if teachers knew enough about their pupils and did the right things for them. A policy of small classes is economical in money and, above all, in human resources.

2. *The right to have time in the school day for planning.*

In general, the instructors in the schools of the armed forces had at least one hour to plan and to prepare for each hour of teaching. Teachers need to plan with their pupils, and with supervisors, parents, and other teachers. Planning is not possible if there is little or no time for planning, and if the people concerned cannot find the time to meet. It is not sensible for a teacher to operate hour after hour without a plan. Moreover, it is a waste of public funds. Wise observers of high school instruction have stated time and again that more than half of the pupil's time is wasted. Surely the public will want to allow a teacher of 100 pupils three hours a day for planning if by so doing fifty hours of his pupils' time can be put to better use.

3. *The right of a 45-hour week.*

In general, the teacher's week should include (a) 15 hours of teaching, (b) 15 hours of planning and pupil guidance, and (c) 15 hours devoted to sponsoring extra-

* This article appeared first in the *University of Michigan School of Education Bulletin* for May, 1946 and was later reprinted in *The Phi Delta Kappan* for January, 1947 and *Education*, January, 1947. It received favorable notice in several editorials and merits continued wide circulation amongst thinking educators. Here, in particular, it is further thought of as depicting in his own words and in a summary fashion, much of Raleigh Schorling's philosophy of education.

curricular activities, participating in community activities, and grading pupils' written work. It is extremely important for teachers to supervise student activities and to engage in community projects, but it is impossible for a teacher to maintain a high level of efficiency in his classroom if at the same time he is expected to be a social worker, a director of activities, and a guidance official. A teacher's extra work—grading papers, planning, visits to homes, etc.—cannot be left on the desk at the end of the school day. No one knows the average number of hours per week for all teachers, but it is probably much closer to 70 hours a week than it is to 45 hours.

4. *The right to an adequate amount of helpful and constructive supervision.*

The valid purposes of the right kind of supervision are (1) to help the teacher plan, (2) to aid in providing good materials and effective methods, and (3) to insure that the teacher grows on the job. In many of the schools of the armed forces an instructor was supervised from 40 to 60 per cent of the time that he taught. It is uneconomical to operate schools without good supervision. An occasional brief visit, inspectional in character, is in many cases unfair to teachers. The teacher who, day after day, does a good job, finds it difficult to maintain morale and to avoid devastating frustration when no one ever comes to see his work or to discuss his plans with him. The right kind of supervision in adequate amounts is deeply appreciated by teachers.

5. *The right to adequate compensation for the full year of fifty-two weeks.*

The average annual salary of public school teachers, principals, supervisors, and other instructional staff for the year 1944–45 is estimated at $1786, which is more than a $1000 less than the figures used for bargaining purposes in strikes that threatened the nation's economy. The teaching profession includes an astonishing number per hundred who labor with high competence and missionary zeal regardless of compensation. For that we are grateful and proud, but a million persons do not, year after year, give themselves to any calling without appropriate salary. What really happens is that many of our best prospects for teaching are attracted by better-paid jobs in industry, in commerce, and in the other professions. Witness the fact that in the three years preceding World War II the University of Michigan, a school of more than 10,000 students, recommended for certification a total of only eight teachers in chemistry and two in physics, whereas many times this number went from this university into industrial research. Society is unwittingly paying a devastating penalty when it employs persons who never should teach children merely because they are cheaper than good teachers.

6. *The right to have good materials and enough of them.*

The disposition of society toward the cost of educating for living in a peaceful world presents a sharp contrast with the attitude toward the expense of training for combat. The Army and Navy operated schools at a cost per student several times as large as the average spent per pupil in civilian schools. Thousands of mechanics were trained at an annual cost per man greater than the annual salary of the industrial arts teacher in the home school. Some of the schools of the armed forces had magnificent classrooms and laboratories such as teachers in civilian schools can scarcely imagine. Too often a teacher is expected to perform miracles, without the materials needed for the tasks. Such a teacher may have nothing in the classroom beyond a textbook—and that may be hopelessly out-of-date. School boards and administrators should each year check to insure that teachers have at least the minimum materials needed for effective work. Of especial importance are such audio-visual aids as are genuinely useful and readily available.

7. *The right to work in a room that, with the help of the students, can be made pleasant and appropriate to the tasks to be learned.*

Some industries have demonstrated that efficiency can be increased by the right working conditions. Too many classrooms give the impression of being places of detention, with little to suggest what the group is trying to do. The work of the teacher of English or mathematics, or Spanish will be easier and more effective if the atmosphere of his classroom contributes to the activities. Whenever possible the teacher should have a workroom of his own where he can arrange the settings to make them appropriate to the units of instruction. In the larger schools it is possible to avoid the wearying task of shifting disorganized materials from room to room throughout the school day.

8. *The right to the same personal liberties which other respectable citizens assume for themselves as a matter of course.*

Many of our best prospects for the teaching profession are rejecting teaching as a career because of the petty restrictions and prohibitions that many communities inflict on their teachers. These trivial but annoying requirements in personal conduct vary from community to community. Thus in one community the prohibition may be against teachers playing cards; in another, dancing on school nights; and in still another, smoking. Communities that object to a teacher attending movies are rare, but they still exist. However, the beginning teacher not uncommonly finds himself in a town in which the lawyer, doctor, businessman, priest, and minister can drink any form of liquor any time anywhere and still be held in high regard, whereas a teacher may not have his contract renewed if he drinks a single glass of beer in public. The truly professional teacher will, as a matter of course, respect the mores of his community and maintain at least as high standards as other respected citizens. A teacher with high professional zeal probably would *want* to do very few forbidden things that other citizens can do without being censored, but he certainly should have the *right* to do them. A community may well expect decency and idealism of its teachers, but it has no right to scrutinize every petty detail of their personal lives. No group of intelligent citizens in other occupations and professions would want every detail of their personal lives supervised. Teachers are only human beings and should be treated as such. At any rate it is unwise for parents to annoy teachers with trivialities. By doing so, they turn colorful vigorous personalities away from the teaching profession and thus deprive their own children of teachers who might provide strong leadership for high ideals.

9. *The right to an externship.*

No institution of teacher education can turn out a finished product. Yet seldom is the beginning teacher given a light assignment and an adequate amount of supervision. Indeed he is lucky if he does not draw the heaviest load and the most disagreeable tasks.

The solution to the problem of the beginning teacher is the idea of externship. This idea, so vital in medical education, has long been advocated in teacher education and is generally approved, but seldom found in practice. The nature and character of the period of externship is suggested by the following provision: (1) a light teaching load in the first yer of teaching —perhaps half-time; (2) salary appropriate to the load, ideally a living wage for a single person; (3) experience in a great variety of tasks, as for example extracurricular, administrative, and teaching; (4) adequate and competent supervision; (5) an opportunity to study the school and community as a "whole" before being limited to the teaching of a single subject or grade; (6) an arrangement truly professional which guarantees that the externship concept will not be used as a means of hiring a cheap teacher to replace a more expensive but experienced teacher; and (7)

the use of gifted teachers with special interest in professional problems to supervise beginning teachers, allowing them time in the school day to do the job and extra compensation for this valuable and technical service.

10. *The right to a realistic program of in-service education.*

By in-service education we mean training on the job. The experienced teacher from time to time needs to revise materials and methods. The in-service program should be geared to a competent department of research that will keep materials and techniques up to date without too much grief and waste of effort. Precious time and energy are now wasted by groups of tired teachers who, after school at the end of a long and weary day, are expected to revise the curriculum. Witness the fact that 85,000 courses of study, created largely by scissors and paste techniques, have been filed in the curriculum collection at Teachers College, Columbia University, and that only a few can be rated as helpful. Boards of education must come to realize that keeping the curriculum up to date is a difficult and technical task which, if attempted at all by teachers, must be done in the school hours of the work day and not in overtime periods without compensation.

11. *The right to participate in modifying the curriculum and methods, and in formulating school policies.*

Sound administration of schools will utilize the constructive ideas of all teachers to make sure that the service of the school to its pupils may be made as good as possible, and to insure that a teacher will grow on the job.

12. *The right to keep from being lost in the profession.*

Many excellent teachers are lost in the vast numbers who, with relatively little ability, training, and experience, come and go. There is no systematic provision for continued recognition of growth in the service. All are teachers! There is little differentiation that recognizes competence or length of service except by meager annual increments of increase in salary. To no small degree the gifted teacher is always in competition with all newcomers, however incompetent. Even the members of a strong school board may not be aware that they have especially fine teachers in their schools. Mere financial rewards are not sufficient to maintain morale. A good teacher has the right to be identified by professional recognition that will strengthen his hand in dealing with the public and the pupil. Such machinery for differentiation between the professional worker and the mere transient does not now exist. However, gifted teachers may reasonably look to their professional organizations for the design of a system of identification that is long overdue.

Raleigh Schorling

"If Professor Schorling's proposals, and similar ones by other educators, are approved, the teachers will have an adequate Bill of Rights to cover their profession. It is time that the teaching profession received its own Magna Carta."—FINE, BENJAMIN, "Teaching Career Entails Handicaps," *New York Times*, February 13, 1947, p. 27.

Raleigh Schorling—A Bibliography

Books

OF THE fifty-nine books bearing Raleigh Schorling's name, forty-four are secondary school mathematics texts, seven deal with the teaching of mathematics, five are concerned with student teaching in general and three with still more general educational questions.

With three exceptions the mathematics texts were written with John R. Clark (and others) and were published by the World Book Company in the interval from 1922 through 1950. The three exceptions are his first two books:

A Review of High School Mathematics (with Reeve). Chicago: The University of Chicago Press, 1915. pp. x plus 70.

General Mathematics (with Reeve). Boston: Ginn and Company, 1919. pp. xvi plus 488.

and the text which he prepared for the Navy during World War II:

Mathematics (Prepared for Standards and Curriculum Division, Bureau of Naval Personnel). U. S. Government Printing Office, Washington, D. C., 1945. pp. vii plus 527.

His World War II service to our government also included:

Review Arithmetic, Practice Books 1 and 2 (with Clark, Potter, Deady). Army Institute Bulletins No. 510.3, 511.4. Yonkers, New York: World Book Company, 1943. pp. 44 and 60.

In the field of the teaching of mathematics the following titles may be listed:

A Contemporary Guide in the Teaching of Arithmetic (edited by Raleigh Schorling, J. R. Overman, and W. O. Shriner). Ann Arbor, Michigan: Edwards Brothers, 1933. pp. v plus 126.

A Contemporary Guide in the Teaching of Junior High School Mathematics (edited by Raleigh Schorling, John P. Everett, and Walter O. Shriner). Ann Arbor, Michigan: Edwards Brothers, 1933. pp. viii plus 122.

Contemporary Guide in the Teaching of Arithmetic, Grades One to Four (edited by Raleigh Schorling, Walter O. Shriner, and Olive Tilton). Second Experimental Edition. Ann Arbor, Michigan: Edwards Brothers, 1934. pp. 169.

Contemporary Guide in the Teaching of Arithmetic, Grades Four to Eight (edited by Raleigh Schorling, Walter O. Shriner, and Olive Tilton). Second Experimental Edition. Ann Arbor, Michigan: Edwards Brothers, 1934. pp. 130.

Contemporary Guide in the Teaching of Arithmetic, Grades One to Eight (edited by Raleigh Schorling, Walter O. Shriner, and Olive Tilton). Second Experimental Edition, Ann Arbor, Michigan: Edwards Brothers, 1934. pp. 169.

Contemporary Guide in the Teaching of Junior High School Mathematics (edited by Raleigh Schorling, John P. Everett, and Walter O. Shriner). Second Experimental Edition. Ann Arbor, Michigan: Edwards Brothers, 1935. pp. 121.

The Teaching of Mathematics—A Source Book and Guide. Ann Arbor, Michigan: The Ann Arbor Press, 1936. pp. viii plus 247.

In addition to the latter book and the one prepared for the Navy, the third book to bear his name alone, and the book for which he was probably best known in fields outside of mathematics was:

Student Teaching. New York: McGraw-Hill Book Company, 1940. pp. xiii plus 329.

A second edition of this work was published in 1949, and it was in a sense supplemented and completed by the book which came off the press after his death, the last book to bear his name; namely,

Elementary School Student Teaching (with G. Max Wingo). New York: McGraw-Hill Book Co., Inc., 1950.

Motion Pictures

The five outstanding educational sound movies which he designed and helped to produce were planned to accompany and supplement *Student Teaching*. They were:

Learning to Understand Children Part I and slide film, distributed by McGraw-Hill Publishing Company, and Loew's International Incorporated, New York, 1947.

Learning to Understand Children Part II and slide film, distributed by McGraw-Hill Publishing Company, and Loew's International Incorporated, New York, 1947.

Maintaining Classroom Discipline and slide film, distributed by McGraw-Hill Publishing Company, and Loew's International Incorporated, New York, 1947.

Broader Concept of Method Part I and slide film, distributed by McGraw-Hill Publishing Company, and Loew's International Incorporated, New York, 1947.

Broader Concept of Method Part II and slide film, distributed by McGraw-Hill Publishing Company, and Loew's International Incorporated, New York, 1947.

Monographs

Seven of the twelve monographs with the production of which he was closely associated dealt with some phase of the teaching of mathematics. These were:

Reorganization of Mathematics in Secondary Education (with Young, *et al.*). A Report by the National Committee on Mathematical Requirements. Published by the Mathematical Association of America, Inc., 1923. pp. x plus 652.

A Tentative List of Objectives in the Teaching of Junior High School Mathematics—With Investigations for the Determining of their Validity. Ann Arbor, Michigan: George Wahr, 1925. pp. vii plus 137.

"First Report of the Commission on Postwar Plans" (with Wren, *et al.*). THE MATHEMATICS TEACHER, May, 1944. pp. 226–232.

"Second Report of the Commission on Postwar Plans" (with Wren, *et al.*). THE MATHEMATICS TEACHER, May, 1945. pp. 195–221.

The Role of Mathematics in Consumer Education. Washington, D. C.: The Consumer Education Study, 1945. pp. 23.

Manpower for Research, Volume Four of *Science and Public Policy*, A Report to the President by John R. Steelman, Chairman, President's Scientific Research Board, 1947. U. S. Government Printing Office. pp. 47–150.

Guidance Pamphlet in Mathematics for High School Students (with members of the Commission on Postwar Plans). THE MATHEMATICS TEACHER, November, 1947.

Research Studies

In research Raleigh Schorling's great interest in and contribution to mathematics education is evidenced by the following twelve studies. (He also produced a more general study of "The Techniques of Textbook Authors" and one dealing with procedures to be used with dull-normal children.)

"The Problem of Individual Differences in the Teaching of Secondary School Mathematics," *School Review*, XXIII (October and December, 1915), 535–49, 649–64.

"Course of Study in Secondary Mathematics in the University High School, The University of Chicago" (with Breslich and others), *School Review*, XXIV (November, 1916), 648–74.

"Terms and Symbols in Elementary Mathematics" (with Young and others), THE MATHEMATICS TEACHER, XIV (March, 1921), 107–18.

"A Program of Investigation and Cooperative Experimentation in the Mathematics of the Seventh, Eighth, and Ninth School Years" (with Clark), THE MATHEMATICS TEACHER, XIV (May, 1921), 264–75.

"Experimental Courses in Secondary School Mathematics," THE MATHEMATICS TEACHER, XV (February, 1922), 63–78.

"Experimental Courses in Secondary School Mathematics," *Reorganization of Mathematics in Secondary Education*, Chapter XII. Mathematical Association of America, Inc., 1923.

"An Investigation in the Teaching of the Skills of Ninth Grade Algebra" (with Lindell), *School Science and Mathematics*, XXV (November, 1925), 813–16.

"Report of the Sub-Committee on Junior High School Mathematics," *North Central Association Quarterly*, II (March, 1928), 396–419.

"The Need for Being Definite with Respect to Achievement Standards," THE MATHEMATICS TEACHER, XXIV (May, 1931), 311–29.

"Preliminary Report of the Committee on Individual Differences," THE MATHEMATICS TEACHER, XXV (November, 1932), 420–26.

"Report of the Committee on Individual Differences," THE MATHEMATICS TEACHER, XXVI (October, 1933), 350–65.

"Mathematics," The Curriculum, Chapter J., *Review of Educational Research*, II (April, 1934), 168–71.

Journal Articles

Dr. Schorling's complete bibliography has eighty-one entries under this heading. The thirty-five which dealt with mathematics education are listed here. The others dealt with many educational topics but with particular emphasis on the curriculum and on teacher training.

"A Mathematics Contest—Its Relation to the General Problem of Individual Differences," *School Science and Mathematics*, XV (December, 1915), 794–97.

"The Place of Mathematics in the 'Secondary Schools of Tomorrow'" (with others), *School Science and Mathematics*, XVI (October, 1916), 608–16.

"Significant Movements in Secondary School Mathematics," *Teachers College Record*, XVIII (November, 1917), 438–57.

"Elective Courses in Mathematics for Secondary Schools" (with Young and others), THE MATHEMATICS TEACHER, XIV (April, 1921), 161–70.

"College Entrance Requirements in Mathematics" (with Young and others), THE MATHEMATICS TEACHER, XIV (May, 1921), 224–45.

"The First Yearbook of the National Council of Mathematics Teachers—An Announcement," *School Science and Mathematics*, XXV (November, 1925), 871–72.

"Suggestions for the Solution of an Impor-

tant Problem That Has Arisen in the Last Quarter of a Century." *First Yearbook of the National Council of Teachers of Mathematics*, 1926. pp. 58–105.

"A Discussion of General Mathematics," *The Junior High School Curriculum*, pp. 185–91. Fifth Yearbook of the Department of Superintendence. Washington: National Education Association, 1927.

"General Mathematics," THE MATHEMATICS TEACHER, XX (February, 1927), 65–81.

"The Story of Geometry," *Plane Geometry* by Joseph P. McCormack, pp. 359–66. New York: D. Appleton and Company, 1928.

"Notes on the Status of Standardized Tests in Demonstrative Geometry," *Journal of the Michigan Schoolmasters' Club*, 1928. University of Michigan Official Publication, XXX, No. 22, pp. 154–62.

"Research in High School Mathematics" (with Breslich and Johnston). *The Development of the High School Curriculum*, pp. 328–43. Sixth Yearbook of the Department of Superintendence. Washington: National Education Association, 1928.

"How Shall the Objectives of Mathematics for the Secondary Schools Be Determined?" *Junior-Senior High School Clearing House*, V (February, 1931), 336–40.

"Report of the Committee on Individual Differences," THE MATHEMATICS TEACHER, (October, 1933). pp. 350–65.

"The Contribution of Mathematics to Sensible Integration," *University of Michigan School of Education Bulletin*, IX (May, 1938), 118–21.

"Social Mathematics for the Senior High School," *University of Michigan School of Education Bulletin*, X (March, 1939), 85–87.

"Social Mathematics for the Senior High School," *The Oklahoma Teacher*, XXI (September, 1939), 13–14.

"Mathematics in General Education," *School Science and Mathematics*, XL (January, 1940), 14–26.

"Social Mathematics for the Senior High School," *Bulletin of the Kansas Association of Teachers of Mathematics*, XLI (December, 1941), 25–27.

"Vitalizing Mathematics with Applications from Aviation," *University of Michigan School of Education Bulletin*, XIV (November, 1942), 21–24.

"Trends in Junior High School Mathematics," THE MATHEMATICS TEACHER, XXXV (December, 1942), 339–48.

"High School Science and Mathematics in Relation to the Manpower Problem" (with Havighurst, *et al.*), A Report of the Cooperative Committee on Science Teaching. *School Science and Mathematics*, XLIII (February, 1945), 126–56.

"Pre-Induction Courses in Mathematics," Report of the National Commission. THE MATHEMATICS TEACHER, XXXVI (March, 1943), 114–24.

"The Need for Cooperative Action in Mathematical Education." *American Mathematical Monthly*, LII, 4 (April, 1945), 194–201.

"What's Ahead in Secondary School Mathematics?" *Looking Ahead in Education*, Ginn and Company (1945), 115–23.

"The Preparation of High School Science and Mathematics Teachers," Report No. 4. The AAAS Cooperative Committee on Science Teaching. *School Science and Mathematics*, (February, 1946), 107–18.

"The Crisis in Science and Mathematics Teaching." *School Science and Mathematics* XLVII, 5 (March, 1947), 413–20.

"Mathematics—Grades One to Twelve." *The Science Teacher*, (April 1948), 59–60.

"What's Going on in Your School?—A Mathematics Inquiry." THE MATHEMATICS TEACHER, XLI (April, 1948), 147–53.

"A Program for Improving the Teaching of Science and Mathematics." *The American Mathematical Monthly*, Vol. 55 (April, 1948), 221–37.

"Let's Come to Grips with the Guidance Problem in Mathematics." THE MATHEMATICS TEACHER, XLII (January, 1949).

"Course in Arithmetic—Should all High School Freshmen be Required to Take a Course in Arithmetic?" *Nation's Schools*, Vol. 43, No. 4 (April, 1949).

"Mathematics—Grades One to Twelve." *The American Biology Teacher*, Vol. 11 (January, 1949), 16–18.

"Mathematics in General Education." *School Science and Mathematics*, XLIX, 4 (April, 1949), 296–301.

"Experimental Mathematics Program for College Preparatory Students." The California Mathematics Council Bulletin, VII, 2 (November, 1949), 10–13.

Tests, Check Lists, and Workbooks

A total of thirty-two items of the above nature, twenty-nine of which were related to mathematics education, were produced by Dr. Schorling in collaboration with Clark, Rugg, Potter and a few others.

Pamphlets

In addition to the five items listed below, Dr. Schorling helped produce two non-mathematical Lincoln School pamphlets, seven unit pamphlets under the general heading of *Mathematics in Life*, and four others bearing the general title *Social Mathematics* published by the World Book Company.

Illustrated Mathematical Talks by Pupils of the Lincoln School (with Woliung and others). New York City: The Lincoln School of Teachers College, 1919. pp. 44.

The Reorganization of the First Courses in Secondary School Mathematics (with Young and

others). A Preliminary Report by the National Committee on Mathematical Requirements. U. S. Bureau of Education Secondary School Circular, No. 5, February, 1920. Washington: Government Printing Office. pp. 11.

Junior High School Mathematics (with Young and others). A Preliminary Report by the National Committee on Mathematical Requirements. U. S. Bureau of Education Secondary School Circular, No. 6, July, 1920. Washington: Government Printing Office. pp. 10.

The Problem of Mathematics in Secondary Education (with Kilpatrick and others). A Report of the Commission on the Reorganization of Secondary Education Appointed by the National Education Association. U. S. Bureau of Education Bulletin, 1920, No. 1. Washington: Government Printing Office. pp. 24.

An Outline of Essentials in Junior High School Mathematics (Mathematics Department, University High School). Bureau of Educational Reference and Research, University of Michigan. April, 1929.

As Others Saw Him

(Continued from page 99)

"It was a great challenge to be a member of the University High School Faculty at Ann Arbor, Michigan when the school opened with Raleigh Schorling as its principal. We spent long hours under his leadership arguing, rejecting, recasting, and finally formulating and accepting ideals, standards, and philosophies for the new school. During those long sessions he challenged us constantly with such questions as these: 'If we do it that way will it make this school a good place for girls and boys to be?' 'Even if it is all right for this once, will it be a good tradition to set up?' 'What will its long time effect be on the individual pupil?' 'Will it be a worthwhile educational experience?' He never dictated but slowly and in a democratic way he led us to formulate our plans for the 'education of the whole child.'

"Dr. Schorling also showed great faith in the pupils. Time and again he called an 'Assembly of the Whole' to discuss some problem with them. They were given the opportunity and the educational experience of thinking the problem through and trying to find a solution that would be best for all concerned.

"His friends and fellow workers can testify to his quick and genuine sympathy for one who was in trouble. We will always remember the many kind and thoughtful things he did for us. If we were ill or bereaved he always urged the extra day or week to 'get really strong again.'

"Perhaps one of the best things that he did for his pupils and teachers was to expect a much higher type of work of them than they knew that they could do. Then when the task was accomplished he never failed to offer quick words of praise. One of my most satisfactory moments in University High School was when I found a note in my box at the conclusion of our first Christmas program. I had helped to write and had directed the play given that day. The note said, 'You have written yourself deeply into the heart of University High School today,' and it was signed 'R. S.'

"He always held that children and young people should not be sheltered from knowledge of and contact with those who were suffering from some misfortune. He said that poverty, sickness, death, race problems, mental ineptitude, and social maladjustments were all part of life's problems, and working and playing with those who were afflicted by them should be part of the educational plan. Therefore, during the school's first year, he accepted a girl who was subject to epilepsy, a rather bad spastic case, a Negro girl, three German refugees from World War I, several maladjusted pupils, as many children as he could get from the low economic group, and one overgrown seventeen year old boy with an I.Q. of about 80, but who also had a large diamond ring, a Chrysler

(Continued on page 134)

THE PRESIDENT'S PAGE

ONE of the most important activities of the National Council is arranging national meetings and providing good programs for them. These meetings cannot succeed without the support of local teachers in and around the place of meeting. Making local arrangements for these meetings is a huge task for local committees, all of whom serve without pay and little enough praise for the work they do. Although participants on these programs donate their services and take care of their own expenses, our programs are highlighted by outstanding leaders from our own and related fields, from school administration and supervision, and from business and industry. We are very proud of the fact that our meetings attract these leaders and that we are able to secure their services. In addition, the meetings are planned to give the classroom teacher a prominent place on the program and also to provide places for promising younger people. We aim to make these meetings of value to all teachers of mathematics from elementary through college, also to supervisors and administrators. Providing good meetings is one of the most valuable services of the Council.

As you know, we have several meetings of the National Council each year in various parts of the country, in places where we are invited and are assured of the necessary local support. Our annual meeting is held in the early spring, usually in April. The twenty-ninth *annual* meeting will be in Pittsburgh on March 28 to 31. The complete program of this meeting is in this issue of THE MATHEMATICS TEACHER. The eleventh Christmas meeting was held last December on the campus of the University of Florida. The 1951 Christmas meeting will be at Oklahoma A. and M. College. The place of the 1952 annual meeting has not yet been decided. Since the National Council has become a department of the NEA, we conduct a one-day meeting as a part of their annual convention which is held during the first week in July. The 1950 meeting with the NEA was held in St. Louis, the 1951 meeting will be in San Francisco.

While we have conducted only one Christmas meeting and one summer meeting each year, other than the one-day meeting with the NEA in July, I see no reason why we could not have more, providing these meetings are so located as not to conflict with one another. Any region desiring a summer meeting or a Christmas meeting of the National Council should make this known to us. If proper local support for these meetings can be assured, it might be well to have them. However, these same regions might prefer to have a mathematics institute or workshop. I look with great favor on mathematics institutes. A few very successful ones have been going for years and several new ones have been recently started. The idea of the mathematics institute has been growing and I think it should. If your region would be interested in having a mathematics institute, we would be glad to hear from you. Also, we would be glad to hear about mathematics institutes and workshops that are now in progress or are being planned.

We invite your suggestions on how to improve our National Council meetings. Note that we are instituting *continuity sections and discussions* at the Pittsburgh meeting. This has been done because of the demand for "more continuity" in our programs. The National Council meetings must serve the needs and desires of the mathematics teachers; giving them inspiration as well as information, affording them the opportunity of hearing from the leaders in the field, and allowing them the opportunity of exchanging ideas on teaching practices. The primary purpose of our meetings must be to help promote better teaching of mathematics.

H. W. CHARLESWORTH, *President*

Revision of By-Laws

National Council of Teachers of Mathematics (Incorporated)

CHANGES in the plan for the editing and business management of THE MATHEMATICS TEACHER recently adopted by the Board of Directors, the affiliation of the Council with the National Education Association, and the formation of a delegate assembly made changes in the By-Laws of the Council advisable. The Board of Directors therefore authorized a committee to study the By-Laws and propose changes to be voted on at the business meeting of the Council to be held in Pittsburgh this spring.

Members of the committee are George Hawkins, Lyons Township High School and Junior College, La Grange, Illinois; Mary Potter, Board of Education, Racine, Wisconsin; and Marie S. Wilcox, Chairman, George Washington High School, Indianapolis, Indiana.

The present By-Laws may be found in the October, 1949 issue of THE MATHEMATICS TEACHER. Printed below are the revised By-Laws as recommended by the committee.

PROPOSED BY-LAWS

ARTICLE I

NAME, PURPOSE, AND CORPORATE SEAL

1. This organization shall be known as the National Council of Teachers of Mathematics (Incorporated).

2. Its object shall be to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields, by holding meetings for the presentation and discussion of papers, by conducting investigations for the purpose of improving the teaching of mathematics, by the publication of papers, journals, books, and reports, thus to vitalize and coordinate the work of many local organizations of teachers of mathematics and to bring the interests of mathematics to the attention and consideration of the educational world.

3. This organization shall be a department of the National Education Association.

4. The corporate seal of the Council shall have inscribed thereon the name of the organization and the words "Corporate Seal—Illinois."

ARTICLE II

MEMBERSHIP

1. Any person who is interested in the field of mathematics shall be eligible to membership in the Council.

2. The Board of Directors may establish various types of membership in the Council and determine the fee for each. Each member is entitled to receive THE MATHEMATICS TEACHER.

ARTICLE III

OFFICERS, BOARD OF DIRECTORS, COMMITTEES

1. Officers. The officers of the Council shall be a president, four vice-presidents, a recording secretary, an executive secretary, and an editor.

2. Qualifications. A nominee for president must have been a member of the Board of Directors within the preceding five years.

The four vice-presidents shall represent the Elementary, Junior High School, Senior High School, and College fields.

3. Election, tenure of office, compensation. The president and vice-presidents shall be elected by members of the Council.

The president shall serve for two years and shall not be eligible for re-election.

Two vice-presidents shall be elected each year. Each shall serve for two years and shall not be eligible for re-election.

The Executive Secretary, Recording Secretary, and Editor shall be appointed by the Board of Directors. The Executive Secretary shall be appointed for a term of three years; he may be reappointed. The Editor shall be appointed for a term of three years; he may be reappointed but shall not serve for more than two consecutive terms. The Recording Secretary shall be appointed for a term of one year; he may be reappointed.

4. Compensation. Compensation, if any, for officers shall be fixed by the Board of Directors.

5. Duties of officers.

(1) President. The President shall be the executive officer of the Council. He shall preside at all meetings of the Board of Directors and at the annual business meeting of the Council. He shall have the usual duties pertaining to his office and such other duties as may from time to time be assigned him by the Board of Directors.

(2) Vice-Presidents. A vice-president shall, in the president's absence or inability to serve, have and exercise the powers of the president. Of the two vice-presidents who are serving their second year, the one having received the larger number of votes at the time of election shall assume the responsibilities of the president.

(3) Recording Secretary. The Recording

Secretary shall keep a record of the proceedings of all business meetings of the Council and of the Board of Directors.

(4) Executive Secretary. The Executive Secretary shall, working under the direction of the President, endeavor to carry out the policies formulated by the Council, the Board of Directors, and the Executive Committee. He shall serve as treasurer and business manager of the Council.

(5) Editor. The Editor shall be responsible for the official journal in all phases except that of business management.

(6) Board of Directors. The Board of Directors shall consist of 14 voting members: the President, four Vice-Presidents, and nine Directors. The immediate past president shall be a non-voting member of the Board.

The nine Directors shall be elected by members of the Council according to a geographic plan determined by the Board. Three shall be elected each year and shall serve for a term of three years or until their successors are elected. They shall be eligible for re-election but shall not serve for more than two consecutive terms.

The Board of Directors shall have general supervision of the activities of the Council, shall supervise the expenditure of funds, fix salaries and bonds of officers, and fill vacancies ad interim in any office.

7. Executive Committee. The Executive Committee shall consist of the president and two members of the Board which shall be appointed by the President with the approval of the Board.

This committee shall act for the Board of Directors when so instructed by that body and shall act in an advisory capacity to the President and the Executive Secretary.

8. Standing Committees. The standing committees shall be Auditing, Budget, Nominations and Elections, Year Book Planning, Supplementary Publications, and such other committees as shall be authorized from time to time by the Board of Directors. These committees shall be appointed by the President subject to the approval of the Board.

9. Duties of Committees.

(1) The Auditing Committee shall make an annual audit of the books and accounts of the Executive Secretary.

(2) The Budget Committee shall prepare an annual budget for the Council to be submitted to the Board of Directors at the time of the annual meeting.

(3) The Nominations and Elections Committee shall cause an announcement to be published in the official journal at least five months before the annual meeting inviting members of the Council to suggest nominees for elective offices. It shall place in nomination at least two persons for each elective office and shall instruct the Executive Secretary to mail ballots to all members at least one month before the annual meeting. A date for the closing of the balloting shall appear on the printed ballot.

(4) The Year Book Planning Committee shall serve as a planning committee for all Year Books. This committee shall recommend to the Board of Directors, at least two years in advance of the anticipated publication date, a topic and a board of editors for the publication of each book.

(5) The Supplementary Publications Committee shall supervise the editing and publication of all publications other than Year Books and the official journal.

10. Editorial Board. The Editorial Board shall consist of the Editor and six associate editors. The associate editors shall be appointed for a term of three years by the Board of Directors on recommendation of the Editor. They shall be eligible for reappointment but shall not serve for more than two consecutive terms.

ARTICLE IV

Meetings, Quorums, Rules of Order

1. The business meeting of the Council shall be held annually, at such time and place as the Board may direct. Special meetings of the Council may be called by the President under the authorization of a majority of the members of the Board of Directors.

2. The Board of Directors shall hold at least two meetings at the time of the annual meeting, one preceding and one following the annual business meeting of the Council. Additional meetings of the Board may be held at the call of the president under the authorization of a majority of the members of the Board.

3. Notice of any meeting of members of the Council shall be given by the Executive Secretary at least thirty days prior to the date set for said meeting. Notice of any meeting of the Board other than regular meetings herein provided shall be given to each member of the Board at least fifteen days prior to the date set therefor.

4. At any meeting of the Council a quorum shall consist of twenty-five members.

5. Eight members of the Board of Directors shall constitute a quorum for the transaction of business.

6. Roberts Rules of Order Revised shall govern the conduct of all business meetings of the Executive Committee, Board of Directors, and the Council.

ARTICLE V

Affiliated Groups

1. Any organized group of teachers of mathematics may petition the Board of Directors to become affiliated with the Council. The Board shall specify the conditions under which such affiliation may take place.

2. An affiliated group shall be entitled to send a delegate to the Delegate Assembly.

3. The Delegate Assembly shall be composed of representatives from affiliated groups. The Assembly shall hold a meeting at the time of

(Continued on page 114)

Candidates for N.C.T.M. Offices—1951 Ballot

THE 1951 Nominating Committee presents the persons listed below as candidates for the designated offices on the Board of Directors along with a brief biographical sketch of each. Two candidates are presented for each office to be filled. The names are listed in alphabetical order.

Every name which was submitted in time was considered by every member of the Committee. Many excellent suggestions were received but the offices were so few that not nearly all of the names could be used. The present Nominating Committee will turn all this material over to next year's Committee for their consideration.

On behalf of the Nominating Committee, I would like to thank the many members of the Council who sent in recommendations for the ballot.

ONA KRAFT
Collinwood High School
Cleveland, Ohio

Vice President—College Mathematics

KARNES, HOUSTON T., Louisiana State University, Baton Rouge, La. A.B., Vanderbilt

HOUSTON T. KARNES

U., A.M.; Ph.D., Peabody Coll.; summer sessions, U. of Michigan and U. of Wisconsin. Prof. math. and biol., Northwestern Junior Coll., Orange City, Ia., 1929–35; prof. and head of math. dept., Harding Coll., 1935–36; tchr. and dept. head, Nashville, Tenn. City High Schools, 1936–38; tchg. fellow, Peabody Coll., summers 1937, 1938, and 1939; assoc. prof. of math., Louisiana State U., 1938–. Member: N.C.T.M.; Math. Assn. of Am.; Am. Math. Soc.; N.E.A.; La. E.A.; Pi M.E.; K.M.E.; P.D.K.; K.D.Pi; O.D.K.; Lambdi Chi Alpha, International Pres.; Bd. of Trustees, Harding Coll.; A.A.U.P.; appeared on programs of N.C.T.M.; chm. NCTM Com. on Contests and Scholarships; member of M.A.A. Comm. on Math. Institutes; listed in American Men of Science. Contrib. to Math. Tchr., Am. Math. Monthly, Nat. Math. Mag., Lousiana Schools, including "Junior College Mathematics in View of the President's Report," Math. Tchr., Apr. 1950; "Education vs. Legislation," Am. Math. Monthly, Oct. 1950.

ZANT, JAMES H., Oklahoma A. and M. College, Stillwater, Okla. A.B., Southern Metho-

JAMES H. ZANT

dist U.; A.M. Columbia U., Ph.D. Assoc. prof. of math., Southeastern S.T.C., Durant, Okla., 1923–30; prof. of math., Oklahoma A. and M. Coll., 1930–, act. hd. 1942–46, prof. and asst. hd., 1946–; Dir. of Instruction Okmulgee Br., Okla. A. and M. Coll., 1946–48. Member: N.C.T.M. (Board of Dir. 1948–), Math. Assn. of Am., Am. Soc. of Eng. Edn., A.A.U.P., Okla. Acad. of Sci., P.D.K., K.D.Pi., Pi M.E., S.A.E., listed in Am. Men of Sci. and Who's Who in Central States. Author: "Elem. Math. Concepts," 1940; "Teaching Plan for Unit of Work in Math.," 1935; "Basic Mathematics, A Workbook" (with M. W. Keller); "Coll. Al. and Trig." 1949; "Papers on Vocational Opportunities for Students of Mathematics," Bull. Okla. A. and M. Coll. Apr.1946; "The Engineering Staff's Responsibilities and Opportunities in the Improvement of Learning and Teaching Mathematics in the Secondary School," Math. Tchr., Mar., 1948; "Professional Engineers and Engineering Education," Okla.

Prof. Eng., Nov., 1947; "Guidance Report of the Commission on Post-War Plans," Math. Tchr., Nov., 1947; "What Are the Mathematical Needs of the High School Student?", Math. Tchr., Feb., 1948; "A Program for Determining the Mathematical Needs of Engineering Students," Math. Tchr., Mar., 1950; "The United States Armed Forces Institute Examinations in Mathematics as a Means of Determining Credit in Engineering Mathematics," Jour. of Eng. Educ., Nov., 1945; "The Next Step in Planning for Post-War Mathematics," Math. Tchr., Oct., 1945; "Importance of Mathematics in Post-War Planning," Okla. Tchr., Feb., 1945; "Need to Improve Courses in Mathematics," Okla. Tchr., Nov., 1945; "Differentiating Courses in Secondary Mathematics According to Pupil Needs," Bull. of the Kans. Assn. of Tchrs. of Math., Dec., 1945; "Vocations Available for College Students of Mathematics," Proc. Okla. Acad. Sci., 1945; with J. Vincent Robison, "The Pre-Engineering Inventory Tests for Engineering Students of the Okla. A. and M. Coll.," Proc. Okla. Acad. Sci., 1945. "Guidance in the Field of Mathematics for High School and College Students," The Pentagon, Spring, 1946; "The Improvement of High School Mathematics Courses as Recommended by the Commission on Post-War Plans," Math. Tchr., Oct., 1946.

Additional Members of Board of Directors
Three to be elected

GAGER, WILLIAM A. University of Florida, Gainesville, Fla., B.S., M.S., Penn State Coll.;

WILLIAM A. GAGER

Ph.D., Peabody Coll. for Tchrs. Instr. in Eng., Penn. State Coll., 1919–23; tchr. of math. and sc., St. Petersburg H.S., 1926–27; head of math. dept., St. Petersburg Jr. Coll., 1927–42; summer tchg., Peabody Coll., 1936–39; assoc. and full prof. of math. at U. of Florida, 1942–; math. consult. Fla. St. Dept. of Educ., 1940, 42, 48, 49, 50; chairman of Fla. Curr. Com. 1947–50; chairman of math. clinic for state supervisors conference, 1948; dir. of state dept. study of math. in sec. schools of Fla., 1947–49; editor-in-chief of Duke Math. Inst. High Lights, 1948, 1949, 1950; ed. and dir. of State Dept. Bull. No. 40 and Bull. No. 36; assoc. ed. of Math. Tchr. 1950–; dir. of Fla. Council of Tchrs. of Math., 1950. Member: N.C.T.M., C.A.S.M.T., Math. Assn. of Am., Fla. Acad. of Sc., Fla. Ed. Assoc., Fla. Public Health Assoc., A.A.U.P., P.D.K., K.D.Pi, P.K.P., T.B.Pi, Theta Chi, listed in Who's Who in Am. Ed., 1945–51, Am. Men of Sc., 1949–51. Author: "Terminal Business Math. in the Jr. Coll.," 1940; "Mathematics for the War Effort," 1942; "Proper Appreciation and Correct Understanding of Mathematics" F.E.A., 1935; "Computation with Approximate Numbers," S. S.M., 1947; "Mathematics for the Other Eighty-five Per cent," S.S.M., 1948; "Aiding Students in Acquiring Basic Concepts," Duke H.L., 1949; "A Functional Program for Secondary Mathematics," M.T., 1949; "Quality Control in Secondary Mathematics," Duke H.L., 1950; "Concepts for Certain Mathematics Courses," S.S.M., 1950.

HALL, LUCY E., Wichita High School North, Wichita, Kan. A.B., U. of Kan.; M.A., Tchrs. Coll., Columbia U.; Summer sessions at Denver U., Wichita U., and U. of Chicago; attended Inst. of Foreign Affairs at American U., 1948.

LUCY E. HALL

All teaching experience in Kansas, a few years in the elementary grades, a few years in junior high school but the greater part in senior high school, always in mathematics work; head of math. dept. Wichita H.S. North, 1943–. Pres. of Wichita Math. Assoc.; local co-chairman of 10th Christmas meeting of N.C.T.M. in Wichita, 1949; for ten years a director of Wichita City Tchrs. Cr. Union.

LYONS, CATHERINE A. V., Perry High School, Pittsburgh, Pa. B.A., U. of Pittsburgh, M.S., Ph.D.; attended summer sessions at universities in other parts of the country. All teaching experience in Pittsburgh: mathematics from 3rd gr. through 12th gr., four summers in Dem. Sch., formerly at U. of Pittsburgh, four years in Vet. Tr. Program, at present senior math. at

Perry H.S. Member: N.C.T.M. 1930–; former officer of Western Pa. Assoc. of Math. Tchrs. and State Round Table; chairman of organization of Pa. Council of Tchrs. of Math., 1950–; co-chairman of local com., 29th Ann. Meeting,

CATHERINE A. V. LYONS

N.C.T.M. in Pittsburgh, 1951; officer of Doctoral Assoc. of U. of Pittsburgh; officer of A.A.U.W.; Pi Lambda Theta; Business and Prof. Women; has appeared on N.C.T.M. programs. Only publications are short articles in local bulletins.

ROURKE, ROBERT E. K., Pickering College, Newmarket, Ontario. Graduated with honors,

ROBERT E. K. ROURKE

gold medal in math., and research scholarship, Queen's U., Kingston, Ont.; M.A. Harvard U.; also completed courses for the doctorate; twice appointed Shattuck Scholar. Joined staff of Pickering Coll. Jan. 1928, asst. headmaster, 1940, assoc. headmaster, 1942, headmaster, 1947. Member: N.C.T.M. 1935–, presently Ontario delegate and member of Yearbook Com.; past pres. Ontario Assoc. of Tchrs. of Math. and Physics; member of executive of Canadian Math. Congress. Co-author of: "An Advanced Course in Algebra"; "Plane Trigonometry and Statics"; "Mathematics for Canadians," Books I, II, and III; "Why Study Mathematics"; articles and papers on teaching of high school mathematics. Hobbies: Gilbert and Sullivan (has directed and acted in G. and S. for 18 yrs.); philately (stamps of early Canada); camping (dir. of Camp Mazinaw for boys).

VAN ENGEN, H., Iowa State Teachers College, Cedar Falls, Ia. B.A. Nebraska Wesleyan U.; Ph.D., U. of Michigan. Tchr. of math., Jr. and Sr. H.S., Holdrege, Neb., 1927–29 and 1930–31; Tappan Jr. H.S., Ann Arbor, Mich., 1934–35; instr. School of Educ., West-

H. VAN ENGEN

ern Reserve U., 1935; asst. prof. of math., Kansas St. Coll., 1936; head, dept. of math., Iowa St. Tchrs. Coll., 1937–. Member: N.C.T.M., C.A.S.M.T., Math. Assn. of Am., Am. Math. Soc., I.S.E.A., A.A.U.P., Ia. Assoc. of Math. Tchrs. (Member of Board of Dir.), P.B.K., P.K.P., Sigma Xi. Author: "Analysis of Meaning in Arithmetic," Elem. Sch. Jr., Feb.–Mar., 1949; "Place Value and the Number System," Arith. 1947; "Arithmetic; Teaching Fractions," Ed. Service Pub. V.1, No. 3, Iowa St. Tchrs. Coll.; "An Aspect of Meaning in Arithmetic," Elem. Sch. Jr., Jan. 1946; "Developing an Understanding of Place Value in Second and Third Grade," S.S.M., May 1945; "Triads and Tables," N.E.A. Jr., Feb., 1946; "Developing the Fraction Concept," Neb. Educ. Jr., Dec. 1946–Jan. 1947; "Handbook for Arithmetic," Mid. Sch. Jr., Sept. 1944; "Logical Approaches to $(-a)(-b)$," Math. Tchr., Apr. 1947; "Unifying Ideas in Arithmetic," Elem. Sch. Jr., Dec. 1941; "Function of a Mathematics Department in a Teachers Institution." Sch. and Soc., Dec. 1938.

WAYNE, ALAN, Williamsburgh Vocational High School, New York, N. Y. B.S., Coll. of City of New York, M.S. in Ed.; grad. work at Tchrs. Coll., Columbia U., New York U., and in pure math. at Columbia U., 1939–49.

Instructor in math. and sc., Rhodes, School (private H.S.) 1930–40; tchr. of math., Jr. H.S. 120, Manhattan, N.Y.C., 1940–45; tchr.

ALAN WAYNE

of math., Brooklyn H.S. of Automotive Trades Annex, N.Y.C., 1945–48, chmn. of related and tech. subjects, 1948–50; chmn. of related and tech. subjects and chmn. of program and organization, 1950–; part-time inst. in ed. at N.Y.U., 1950–51; part-time inst. in math., Cooper Union Sch. of Eng., N.Y.C., 1949–51; co-ordinator and tchr. of in-service courses, N.Y.C. Bd. of Ed., 1950. Member: N.C.T.M., 1940–; C.A.S.M.T., 1939–; Math. Assn. of Am., 1941–; Nat. Assn. of Biol. Tchrs., 1942–50; Am. Math. Soc., 1941–; Assoc. of Tchrs. of Math., 1933–; pres. 1948–50, Mem. Ex. Bd. 1943–51, V. P. for Jr. H.S. 1945, editorial board of Bulletin, 1948–49; Assoc. for Sup. and Cur. Dev. 1949–; New York Cipher Soc., 1941–, charter member; Am. Voc. Assoc.; N.Y. St. Voc. and Pract. Arts Assoc., 1950–; N.Y. Riddlers Club, pres. 1939; member and subcommittee chmn. of N.Y.C. Supt. of Schools Com. to study the teaching loads in high schools, 1950; member of standing com. on math. of the H.S. division, N.Y.C., 1949–51. Co-author of laboratory manual "Directed Experiments in Biology" with Robbins; co-constructor (with 4 others) of N.Y. Arith. Comp. Test and the N.Y. Arith. Judgments Test. Articles in S.S.M. 1945; Am. Biol. Tchr. 1947; Bull. of Assoc. of Tchrs. of Math. of N.Y.C., 1945, 1946; Am. Math. Monthly, 1944, 1949; Scripta Mathematica, 1946.

Revision of By-Laws

(*Continued from page 110*)

the annual meeting of the Council and shall serve as a forum which may make recommendations to the Board of Directors concerning activities and policies of the Council.

ARTICLE VI

AMENDMENTS

1. Amendments to these By-Laws may be made at any annual business meeting of the Council or special meeting called for that purpose, providing that due notice concerning such amendments shall have been printed in the official journal or mailed to each member at least one month before the date of such meeting.

2. No changes in the Certificate of Organization or the By-Laws shall have legal effect until a certificate thereof, verified by oath of the President and under the Seal of the Council, attested by the agent of the Council, shall be filed in the office of the Secretary of State of the State of Illinois and recorded in the office of the Recorder of Deeds for McDonough County, Illinois.

Knox College Science Prize Competition, 1951-52

Two mathematics scholarships, one for $1000 and one for $450, will be awarded this spring in a contest to be held at Knox College at Galesburg, Illinois on Saturday, March 10, 1951 at 10 A.M. The examination in mathematics is primarily designed to measure ability and understanding in mathematics and presupposes a knowledge of algebra and plane geometry. Each high school may enter three contestants and only those students are eligible who rank in the highest quarter of their high school class and are recommended by a high school science teacher. Applications should be forwarded by the recommending teacher so as to reach the Committee on Scholarships, Knox College, Galesburg, Illinois not later than March 3, 1951.

NOTES ON AFFILIATED GROUPS

JOHN R. MAYOR
Chairman, Committee on Affiliated Groups

DELEGATE ASSEMBLY

SUGGESTIONS received from representatives of Affiliated Groups continue to emphasize the need for and desirability of increased co-ordination of the activities of the Affiliated Groups and the National Council. The Group representatives and other interested Council members continue to send in excellent suggestions for new services that the National Council might render the Affiliated Groups and individual Council members. The suggestions also have given consideration to the possibilities for the Affiliated Groups to help the National Council in rendering these services. For example, the Affiliated Groups could carry a major part of the responsibility in studies of certificate requirements in the various states, preparation of a bibliography of state and local courses of study, in the organization of plans for exchange of mathematics teachers within the country.

It has been pointed out that Affiliated Groups are prepared to appoint, or already have appointed, Co-ordinating Committees as proposed by the First Delegate Assembly, but that the functions of these Committees are not clear to the Affiliated Groups. This is a question of real importance for the Second Delegate Assembly. The delegates of the Groups can be instrumental in determining an organization which might supervise and correlate the work of these Committees. Some have expressed an interest in reconsideration of a few of the recommendations of the First Delegate Assembly.

Some unfavorable reaction has resulted from the question raised about the possibility of Affiliated Groups assuming, by a plan of rotation, editorship of the Newsletter of Affiliated Groups. Those making this suggestion in the first place did so only with the idea that the expense of publication under such a plan would be paid by the National Council from the annual renewal of affiliation dues. Affiliated Groups can be sure that the Committee on Affiliated Groups is strongly opposed to any suggestions which would increase the cost of affiliation for the various Groups.

A report of suggestions from Affiliated Groups will be published again in this section in the March issue of *The Mathematics Teacher*. While there will be little time then before the Delegate Assembly for your Group and your delegate to consider these suggestions, it is, nevertheless, important to give whatever opportunity is possible for widespread consideration of proposals of the various Groups.

Be sure that your Group is prepared to take advantage of the Second Delegate Assembly as a means of expressing your opinion on obligations and advantages of Affiliation and to make suggestions for the general welfare of the National Council and for the improvement of mathematics teaching everywhere!

ADVANTAGES OF AFFILIATION

In answer to rather frequent requests from groups, considering affiliation, for a statement of advantages of affiliation, the following list was prepared. Statements from readers which might be added to this list will be greatly appreciated and will be published in this section. What has affiliation meant to your Group? What advantages of affiliation do you think should be emphasized for those considering application for affiliation?

Some of the advantages which may come to your organization of teachers of mathematics from affiliation with the National Council of Teachers of Mathematics are:

1. Affiliation is a declaration of your support of the only national organization devoted to the improvement of the teaching of mathematics at all levels of instruction, and your affiliation strengthens this national organization.
2. Affiliation provides your group direct association not only with the national organization of mathematics teachers but also with other state and regional organizations with purposes and programs like your own.
3. Affiliation gives to you an opportunity to take part in discussions of the problems and activities of the National Council, especially in relation to Affiliated Groups, through the Delegate Assembly.
4. The National Council has a Committee on Affiliated Groups which works with designated officers in your group, if affiliated, to co-ordinate your program with that of other state and regional groups and with the National Council. This Committee can be of assistance in calling to your attention activities of the National Council which should be of especial interest and value to your group. The Committee will help you work out a plan of co-operation with other groups in your area for research and study of common problems and for regional meetings.
5. Affiliated Groups maintain an exchange list of publications so that the editor of your journal may receive copies of publications of other Affiliated Groups.
6. Materials prepared by Committees of the Council may more readily reach your members through the affiliated relationship.
7. There is a Newsletter of Affiliated Groups in which are reports of activities of other Groups similar to your own, announcements of activities of the National Council of special interest to state and regional groups, and statements of the program of the Committee on Affiliated Groups.
8. The Committee on Affiliated Groups is planning the organization of a Speakers Bureau to serve the member Groups.
9. Affiliated Groups have been influential in bringing meetings of the National Council to their part of the country and they are essential to the National Council in carrying out regional planning and arrangements.
10. An effective national professional organization must be able to reach all areas of the country and to reach individual mathematics teachers everywhere. The Affiliated Groups can serve their own best interests by making sure that the National Council is effective in this sense.

New Groups

At October and early November meetings four regional and state mathematics organizations voted to make application for affiliation with the National Council. These groups and the officers who sent requests for the application form are:

Arkansas Council of Mathematics Teachers: Mary Lee Foster, Henderson State Teachers College, Arkadelphia

Oklahoma City Mathematics Council: Virginia C. Shike, Jackson Junior High School, Oklahoma City

Mathematics Section, South Dakota Education Association: Florence Krieger, Rapid City High School, Rapid City

Teachers of Mathematics of the Western Division of the Tennessee Education Association: Rush W. Siler, East High School, Memphis, Tennessee

Kenneth R. Allred of Salt Lake City has written an enthusiastic letter about the progress of the Utah state organization of mathematics teachers and has asked for affiliation application blanks. It is anticipated that by the time of the publication of this report, all of these groups will have officially completed their affiliation and will have made plans to send delegates to the Second Delegate Assembly. Teachers in these areas who are not members of these groups may wish to communicate with the officers listed above about membership in and activities of these important regional groups.

Ona Kraft of Collinwood High School, Cleveland, has reported the organization early in November of the Ohio Council of Teachers of Mathematics. H. E. Grime of Cleveland has been elected president and plans are already underway for a meeting April 21, 1951, in Columbus.

Philip Jones of the University of Michigan has sent to the Committee on Affiliated Groups a very comprehensive report of a meeting of Michigan teachers in May.

Ontario Journal

H. E. Totton, Forest Hill Collegiate Institute, Toronto, has sent to the Committee on Affiliated Groups a copy of the fall issue of the Newsletter of The Ontario Association of Teachers of Mathematics and Physics. Mr. Totton is Publicity Representative of this Affiliated Group.

(Continued on page 127)

Program

The National Council of Teachers of Mathematics

Twenty-Ninth Annual Meeting

Hotel William Penn, Pittsburgh, Pa.
March 28, 29, 30, 31, 1951

WEDNESDAY, MARCH 28, 1951

9:00 A.M.–12:00 NOON. Meeting of the Board of Directors—Parlor G

2:00 P.M.–5:00 P.M. Meeting of the Board of Directors—Parlor G

4:00 P.M.–9:00 P.M. Registration—Corridor of 17th Floor

7:30 P.M.–10:30 P.M. Meeting of the Board of Directors—Parlor G

MORNING AND AFTERNOON. *Visiting Pittsburgh Schools.* (See announcement at end of program.)

THURSDAY, MARCH 29, 1951

8:00 A.M.–9:00 P.M. Registration—Corridor of 17th Floor

MORNING. *Visiting Pittsburgh Schools.* (See announcement at end of program.)

8:00 A.M.–10:00 A.M. Delegate Assembly (First Session)—Ft. Pitt Room
Meeting of the official delegates of the affiliated groups. (The second session is on Friday afternoon.)

9:00 A.M.–5:00 P.M. Exhibits, commercial and instructional—Urban Room

10:00 A.M.–11:00 A.M. Meeting of state representatives with the Board of Directors—Ft. Pitt Room
Presiding: KENNETH E. BROWN, University of Tennessee, Knoxville, Tennessee

10:00 A.M.–12:00 NOON. Mathematics Laboratory—Allegheny Room
Teaching Aids for Algebra,—Models, Curve Stitching, Paper Folding. FRANCES C. JOHNSON, Senior High School, Oneonta, New York

10:00 A.M.–1:00 P.M. Sightseeing Trip (See announcement at end of program.)

2:00 P.M.–3:15 P.M. *General Section*—Ft. Duquesne Room
Presiding: MARY A. POTTER, Racine Public Schools, Racine, Wisconsin
The Teacher as an Artist, W. W. RANKIN, Duke University, Durham, North Carolina

2:00 P.M.–3:15 P.M. *General Section*—Ft. Pitt Room
Presiding: HOUSTON T. KARNES, Louisiana State University, Baton Rouge, Louisiana
Number and Its Development on the Teacher Training Level, AARON BAKST New York University, New York City

2:00 P.M.–3:15 P.M. *Secondary Section*—Ball Room
Presiding: ONA KRAFT, Collinwood High School, Cleveland, Ohio
Targets for Teachers of High School Mathematics, ROBERT E. K. ROURKE, Pickering College, Newmarket, Ontario

2:00–3:15 P.M. *Teacher Education Section*—Monongahela Room
Presiding: WILLIAM A. GAGER, University of Florida, Gainesville, Florida
Topic: *Professional Laboratory Experiences in a Mathematics Methods Course*
Bases for Participation in Mathematics Classes, HAROLD FAWCETT, Ohio State University, Columbus, Ohio
Participation in the Public Schools, CLARENCE ETHEL HARDGROVE, Northern Illinois State Teachers College, De Kalb, Illinois
Participation in a Mathematics Clinic, NATHAN LAZAR, Ohio State University, Columbus, Ohio

2:00 P.M.–4:00 P.M. Mathematics Laboratory—Allegheny Room
Teaching Aids for Geometry,—Plastic Models, Demonstration Boards, AMELIA RICHARDSON, McKeesport High School, McKeesport, Pa.

3:30 P.M.–4:30 P.M. Business Meeting of NCTM (First Session)—Monongahela Room
Presiding: H. W. CHARLESWORTH, East High School, Denver, Colorado (The second business session is on Saturday morning.)

with. Admittance cards will be sent to those making requests before March 15. Admittance to these groups will be by admission card only.

Visiting Pittsburgh Schools

If you wish to visit the local schools on Wednesday (all day) or on Thursday morning, please send requests in advance to Dr. Dorothy Pickard, Beltzhoover School, Pittsburgh 10, Pa., or secure information at the Information Desk upon your arrival.

Mathematics Laboratory

This is a new feature on the program this year and is under the direction of Amelia Richardson, McKeesport High School, McKeesport, Pa. There will be three mathematics laboratory periods, each two hours in length. These will be on Thursday A.M. and P.M. and on Friday A.M. in the Allegheny Room. At the beginning of each period the Consultant in charge will give a thirty-minute talk and demonstration on the construction and use of models in the classroom. Then each member of the group will be given the opportunity of making one or two of these models. A minimum cost charge for materials used will be made. Refer to the program for further information. Use the Advanced Registration and Reservation Form supplied herewith.

Sightseeing Trip

Busses will leave Hotel William Penn at 10:00 A.M., Thursday, March 29, and return at 1:00 P.M. Points of interest will include the old Block House, a part of Fort Pitt, the Cathedral of Learning, the H. J. Heinz Plant, the Buhl Planetarium, the Golden Triangle of Pittsburgh, the city parks, and the steel mills. Stops will be made at all these points and an Incline Plane ride will be included. Price, $1.75. Make reservations for this trip before March 15. Use the Advanced Reservation Form supplied herewith.

Supplies and Equipment

Speakers and other participants in the program who need blackboards, projection equipment or other materials should communicate with Dr. Donald C. Steele, Baxter Junior High School, Pittsburgh 8, Pa., not later than March 15.

Commercial Exhibits

Textbooks and commercial teaching aids will be on exhibit in the Urban Room from Thursday morning at 9 o'clock to noon on Saturday. Inquiries for exhibit space should be addressed to L. McClure Lanning, University School, Howe Street, Pittsburgh, Pa.

School Exhibits

There will be an exhibit of mathematical models, instruments, teaching aids, and other classroom materials in the Urban Room from Thursday morning at 9 o'clock to Saturday noon. Inquiries should be addressed to Miss Helen Jenkins, McKeesport High School, McKeesport, Pa.

Meals for Special Groups

If any group wishes to arrange for a dinner on Thursday or a breakfast on Friday or Saturday, the local committee will help make these arrangements. Please write Dr. Catherine A. V. Lyons, 12 S. Fremont Avenue, Pittsburgh 2, Pa.

The Fawcett Ohio Staters will meet for luncheon at 12 o'clock, noon, on Friday, March 30. For reservations, write Mr. Oscar Schaaf, Department of Education, Ohio State University, Columbus 10, Ohio.

Location of Meeting Rooms

All rooms and parlors used for the meetings and discussions are on the 17th floor of Hotel William Penn, with the exception of the Pittsburgh Room which is in the basement of the hotel.

Mail and Telegrams

Mail and telegrams for those attending the convention should be addressed in care of The National Council of Teachers of Mathematics, Hotel William Penn, Pittsburgh 30, Pa. Mail may be obtained at the registration desk on the 17th floor.

Films and Filmstrips

These will be shown on Friday afternoon, Saturday morning and Saturday afternoon in the Allegheny Room.

Refunds on Reservations

No ticket refunds will be made later than three hours preceding the function for which reservations were made, i.e.,

sightseeing trip, the luncheon, and the banquet.

Certification of Attendance

If you care to take back to your school authorities a statement certifying your attendance at the convention, make request for it at the Registration Desk any time Saturday morning.

PROGRAM COMMITTEE

Chairman—H. W. Charlesworth, Washington, D. C.; Julia E. Adkins, Credo, W. Va.; Aaron Bakst, New York City; K. Eileen Beckett, Lebanon, Ind.; Lee E. Boyer, Millersville, Pa.; Charles H. Butler, Kalamazoo, Mich.; Dale Carpenter, Los Angeles, Calif.; Harold Fawcett, Columbus, Ohio; Howard F. Fehr, New York City; William A. Gager, Gainesville, Fla.; M. L. Hartung, Chicago, Ill.; Ida Mae Heard, Lafayette, La.; Agnes Herbert, Baltimore, Md.; E. H. C. Hildebrandt, Evanston, Ill.; Esther F. Jenssen, Chicago, Ill.; Lenore John, Chicago, Ill.; Donovan A. Johnson, Minneapolis, Minn.; John J. Kinsella, New York City; Catherine A. V. Lyons, Pittsburgh, Pa.; Virgil S. Mallory, Montclair, N. J.; J. R. Mayor, Madison, Wisc.; Amelia Richardson, McKeesport, Pa.; Mary C. Rogers, Westfield, N. J.; Robert E. K. Rourke, Newmarket, Ont.; Vera Sanford, Oneonta, N. Y.; Henry W. Syer, Boston, Mass.; Gilbert Ulmer, Lawrence, Kans.; Marie S. Wilcox, Indianapolis, Ind.; Annie John Williams, Durham, N.C.; James H. Zant, Stillwater, Okla.

LOCAL COMMITTEES

(NOTE: Members of committees are from the Pittsburgh public schools, unless otherwise indicated.)

Local Arrangements

General Co-Chairmen—Catherine A. V. Lyons and Amelia Richardson (McKeesport)

Secretary—Clementina George (Braddock)

Treasurer—L. McClure Lanning (University School, Pittsburgh)

Advisory Committee

A. W. Beattie, Superintendent, Allegheny County; Helen Calkins, Pennsylvania College for Women; E. A. Dimmick, Superintendent, Pittsburgh; James Lawson, Superintendent, McKeesport; Sister Michael, Mt. Mercy College; M. Ostrofsky, Duquesne University; Rev. Thomas Quigley, Superintendent, Parochial Schools; J. B. Rosenbach, Carnegie Institute of Technology; J. S. Taylor, University of Pittsburgh.

ADVANCED REGISTRATION AND RESERVATION FORM

Please fill out legibly and mail with remittance to Clementina George, 705 College Ave., Pittsburgh 32, Pa., before March 15, 1951. Checks or money orders should be made payable to L. McClure Lanning, Treasurer of the Pittsburgh meeting.

Name _____
Last First Initial

Home Address _____
Street and Number City Zone State

School Address _____
Name of School City State

Position _____

(Place an X before the address to which you wish your tickets sent.)

Registering as: Member of NCTM _____ **Member of MAA** _____ **Elem. Teacher** _____ **Student** _____ **Exhibitor** _____ **Non-Member** _____

Please check your field(s) of interest: Elem. _____ **J.H.S.** _____ **S.H.S.** _____ **College** _____ **Teacher Education** _____ **Supervisor** _____ **Other** _____

(Reverse side, please.)

Hospitality Committee

Co-Chairmen—Marjorie Harvey (Monessen) and Helen Williams, Lenora Allen, Maurice Arner, Agnes Bedell (McKeesport), Louise Di Prampero (Uniontown), Isabel Epley, Kathryn Hayes (Donora), Ella Koch, Grace Lemon (McKeesport), Mabel Milldollar (Harrison Township), Earl Whipkey (Mt. Lebanon), Kathryn Yurchich (Freeport).

Publicity Committee

Co-Chairmen—Chester Jelbart (Swissvale) and Charles E. Manwiller, Joseph Dennison (Swissvale), Ivo Kirsch, John Knipp (University of Pittsburgh), T. Leaman (Shadyside Academy), Carolyn Loeffler (East McKeesport), Andrew Miller, Frank Sanders, John Schaeffer, L. D. Shriver (Charleroi), Sister Mary (St. Rosalie's), Jane Walker (Clairton), Merle Washburn.

Supplies and Equipment Committee

Co-Chairmen—Joseph Pounds and Donald Steele, Q. D. Ellenberger (Bellevue), Robert Magill, James Mulligan, Eugene Peckman.

Films and Filmstrips Committee

Chairman—Oscar Belles, Andrew Bradac, Joseph Feree (Bridgeville), Robert Graham, J. C. Stuchel.

Usher Committee

Chairman—Helen Malter (Coropolis), Gertrude Artzberger (West View), Elmo Callaway, Mildred Clyde (Stowe Township), H. E. Felich (Scott Township), Mary Elizabeth Hughes (McKeesport), Margaret Sampson (Washington), Louise Spinelli (Homestead), Cindy Staggers (S. Fayette Township), Jean Teats (University of Pittsburgh), Martha Ann Verner (McKeesport), Louise Ward (Sewickley).

Luncheon Committee

Chairman—Marie Kruse, Isabelle Blyholder, Mary Lou Fisher (Slippery Rock), Ruth Fretts (Dormont), Walter Horner, Benjamin Kann, Helen McLain (Dormont), Earl McWilliams, Wesley Mills, Margaret Russell (Monessen), Gus Schnabel, Chester Sterling.

Banquet Committee

Chairman—Bertha Kirkpatrick, Julia

Registration Fee	$0.50 or $1.50 _____	Amount	_____
Sightseeing Trip	Number @ $1.75 _____	Amount	_____
Banquet	Number @ $4.00 _____	Amount	_____
Luncheon	Number @ $2.85 _____	Amount	_____
	Total Amount Enclosed		_____

Discussion Groups (Regular and Continuity)

First Choice: Group _____ Leader _____

Second Choice: Group _____ Leader _____

Third Choice: Group _____ Leader _____

Mathematics Laboratory (Indicate 1st and 2nd choices)

Thursday Morning _____ Thursday Afternoon _____ Friday Morning _____

Enrollments in the Discussion Groups and in the Mathematics Laboratory will be made as registrations come in and will be limited in number.

This advanced registration and reservation form should be used by one person only. If you need more forms, request these from the Secretary of the meeting, Clementina George, 705 College Avenue, Pittsburgh 32, Pa.

Brooke (Uniontown), W. C. Hulley, Jean McCrum (Aspinwall), L. E. Malvern (Carnegie Institute of Technology), George Phillips, Anne Rightmire (Mt. Lebanon), R. R. Sawhill (Carnegie), Ethel Turner, Floyd Wheelan, Edna Wintermute.

"Visit to Schools" Committee

Chairman—Dorothy Pickard, A. N. Addleman (Allegheny County), Sister Mary Louis (Parochial Schools).

Commercial Exhibits Committee

Co-Chairmen—L. McClure Lanning and W. J. Wagner.

School Exhibits Committee

Chairman—Helen Jenkins (McKeesport), George Anderson (Millersville), Lyda Hamilton (Latrobe), Oliver Henry, Florence Learzaf, Sarah Smith, I. J. Stright (Indiana State Teachers' College), Robert Trithart (Allison Park).

Sightseeing Committee

Co-Chairmen—T. R. Leaman (Shadyside Academy) and W. S. Luke (West View), Patricia Brown (Bridgeville), Rosemarie Kavanaugh (Aspinwall), Nellie Oliver (North Hills Joint Schools), Evelyn Schane.

Information Committee

Co-Chairmen—Lucy Soule (Ellis) and Sister Tarsicius (St. Peter's, McKeesport), Rosemary Geary, Margaret Hall (Ursuline Academy), Sister Hilary (St. Thomas', Braddock), Agnes James (Irwin), Lyda Johnston (Winchester-Thurston), Inez O'Donnell, Florence Reed (Washington), Sister Regis (St. Peter's, McKeesport), Stella Schiedhauer (Duquesne), Sister St. Bede (St. Veronica's, Ambridge).

Registration Committee

Chairman—Mabel Love Baker (Penn Township), Roy Beck (Har Brack), Grace Coulter (Swissvale), Freas Downing (Bradford), Harvey Goehring (Penn Township), Ann Hartman (West View), Vivian Kelley (Turtle Creek), Mary L. McBride (Latrobe), Lillian Rumbaugh (Forest Hills), Dorothy Shott (Oakmont), Leonard Smith (Ridley Township), Carl Streams (Mt. Lebanon), A. E. Van Kirk (Bentleyville).

Secretaries Committee

Chairman—Ida Price, Mrs. Campbell Coons (Brentwood), Marjorie Heimberger, Helen Hinsey (Uniontown), Margaret Hockenberry (Pitcairn), Lucille Johnson (Clairton), Mabel Milldollar (Harrison), Mary Ruth Sampson (Brentwood), Mrs. Vera Stauffer (Oakmont).

Signs and Printing Committee

Co-Chairmen—Harry F. McKee (Wilkinsburg) and C. L. Sterling, Harry L. Black, T. D. Bottenhorn (Oakmont), James S. Couch, Thomas L. Phipps (Wilkinsburg), John Skolnick (Clairton), John Spalic (Springdale), William L. Wiegman.

Special Projects Committee

Co-Chairmen—Elizabeth Brown and Mrs. Mary Smith (Sewickley), Grace Bargesser, Freda Becker, Edward Frankel.

Thursday Night Reception Committee

Chairman—L. McClure Lanning and chairmen of other local committees.

Affiliated Groups

(Continued from page 116)

The Ontario Newsletter is a lively publication which certainly is of much value to the members of the Ontario Association. In this particular issue were articles by J. A. Sonley, Ottawa, Past President of the Association; C. N. Shuster, Trenton, New Jersey, State Teachers College; and W. B. McLean, U.T.S., Toronto. In addition the journal carries a Question and Answer section with much useful information.

Membership Message

MARY C. ROGERS
Roosevelt Junior High School, Westfield, N. J.

WE ARE pleased to submit to you this third listing of Membership Honor Schools which we promised to you in our May 1950 membership message. We believe you will be proud of this report for it shows such a marked increase in Honor Schools over those listed in our first report just one year ago. Here are the figures.

	Dec. 1949	Dec. 1950	% of Change
100% Schools	26	41	58% increase
"All but One" Schools	14	12	14% decrease
Total Honor Schools	40	53	33% increase

Doubtless many more of you qualify for this recognition but were unable to submit your reports in the time limit given you. For you we have planned another listing in the May 1951 MATHEMATICS TEACHER. Present indications are that the May report will again announce a generous gain. Recent membership analyses by States show most commendable growth in many localities. It is quite probable that many schools in these States have all of their mathematics teachers members of National Council. May we hear from you to this effect not later than March 15, 1951. We shall be watching the mails for your letters. Simply complete the accompanying form and mail it to

Mary C. Rogers
462 North Avenue, East
Westfield, New Jersey

Please accept our sincere thanks for your interest and cooperation. Your enthusiastic support and generous assistance is greatly appreciated by the National Council. In return, we hope to give you increasingly effective service. We shall look forward to greeting a great many of you in person at the Annual Convention at Pittsburgh in March.

MEMBERSHIP REPORT TO THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

School _____

School Address _____

Number of Teachers of Mathematics in Your School _____

Number of National Council Members _____

Report Submitted by _____

Your Address _____

National Council of Teachers of Mathematics

Membership Record

100% Schools—as of December 10, 1950

1. Ocala, Florida	Ocala High School
2. Ruskin, Florida	Ruskin School
3. Carterville, Illinois	Carterville Community High School
4. Freeport, Illinois	Freeport High School
5. Maywood, Illinois	Proviso Township High School
6. Normal, Illinois	Illinois State Normal University

City, Missouri	Green City High School
Park, Missouri	Valley Park High School
ic City, New Jersey	Brighton Avenue Elementary School
ell, New Jersey	Mt. St. Dominic Academy
nt Station, New Jersey	College of St. Elizabeth
eth, New Jersey	Battin High School
wood, New Jersey	Englewood School for Boys
nd Park, New Jersey	Hamilton Junior High School
nd Park, New Jersey	Senior High School
City, New Jersey	State Teachers College
lair, New Jersey	State Teachers College
k, New Jersey	State Teachers College
Brunswick, New Jersey	Rutgers Preparatory School
ank, New Jersey	Senior High School
e, New Jersey	Harrison Avenue Elementary School
Amboy, New Jersey	Harold G. Hoffman High School
, New Jersey	Sussex High School
sboro, New Jersey	Swedesboro High School
n, New Jersey	State Teachers College
ngton, New Jersey	Washington High School
ksville, North Carolina	Jones County Training School
tte, North Carolina	Second Ward High School
ry, Pennsylvania	Hickory High School
ille, Tennessee	Central High School
n, Texas	Denton Junior High School
n, Texas	Denton Senior High School
n, Texas	North Texas State College Junior High School
am, Texas	Wortham High School
ond, Virginia	Thomas Jefferson High School
ngton, D. C.	Coolidge High School
ngton, D. C.	Eastern High School
ngton, D. C.	Eliot Junior High School
ngton, D. C.	Gordon Junior High School
ngton, D. C.	McKinley High School
ngton, D. C.	Roosevelt High School
ngton, D. C.	Stuart Junior High School
ngton, D. C.	Western High School
Bay, Wisconsin	West High School

"All but One" Schools—as of December 10, 1950

wood, Colorado	Englewood High School
ur, Illinois	Decatur Senior High School
ne, New Jersey	Accredited Evening Senior High School
gton, New Jersey	Burlington High School
ell, New Jersey	Caldwell College
lle, New Jersey	Memorial High School
olly, New Jersey	Regional High School
k, New Jersey	Barringer High School
c, New Jersey	Jefferson Junior High School
ank, New Jersey	River Street Junior High School
Edge, New Jersey	River Edge Junior High School
it, New Jersey	Kent Place School

If Your School Is Not a 100% School

how the announcement on page 137 to those mathematics teachers who are not members hem to take advantage of this offer today?

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

Guidance: The Case for Mathematics

TWENTY years ago one heard very little about "guidance" in high school mathematics. Many students took mathematics only because they were required to do so, or because they were told it was good for them. Other students took no mathematics—having been misguided by well-meaning adults. World War II temporarily usurped the role of guidance counsellor. There was little doubt as to what had to be done.

At the close of the war, the National Council of Teachers of Mathematics created the Commission on Post-War Plans. Under the able and stimulating leadership of its chairman, Raleigh Schorling, this Commission dealt with two major problems: (1) recommendations for the improvement of mathematics in Grades 1–14 (Second Report, 1945), and (2) the preparation of a guidance pamphlet in mathematics for high school students (Final Report, 1947). Schorling addressed himself to these tasks with enthusiasm and untiring effort. Both reports were well received. In particular, the guidance pamphlet filled a long-felt need; nothing of the sort had been available up to that time. Some 35,000 copies have been distributed since its first appearance, which attests not only the gap that needed to be filled, but also the contribution which it has made. Its success is due in no small measure to Schorling's imagination and sincerity; with characteristic patience and determination he welded together data submitted by a score of experts in various fields.

More recently there has appeared a similar pamphlet, sponsored by our Canadian colleagues, entitled *Why Study Mathematics?* An attractively illustrated brochure of 33 pages, addressed to teachers and students, it discusses careers which require an expert knowledge of mathematics, careers which require knowledge of specialized fields of mathematics, careers which require a substantial foundation of mathematical study, the mathematics needed in the education of a competent citizen, the contribution of mathematics to sound reasoning, and the intellectual and aesthetic satisfactions that may be derived from mathematical study.

Although considerable progress has already been made, much still remains to be done. The following would seem to warrant serious consideration:

(1) There should be prepared additional detailed material describing the specific kind and amount of mathematics required for various trades, vocations, industries, business activities, technical jobs, semi-professional and professional positions. This material should probably cover many more fields and occupations than have heretofore been given attention.

(2) Guidance should not be solely directed to students. Appropriate material should also be prepared for teachers of other subjects, as well as for mathematics teachers; for guidance counsellors; for school administrators; for businessmen and industrial leaders; and, of course, for parents. Such material should receive the widest possible dissemination.

(3) Mathematics teachers should become acquainted with the specific nature of the activities in many vocational areas, and particularly with the mathematics used in those areas. Vague or general allusions to "needed mathematics" often

mean little to the uninitiated; a few realistic problems or situations which might be encountered on the job are likely to strike home. The mathematics teacher of the future will himself have had some first hand experience in the machine shop, the drafting room, the laboratory, the office, the factory, or the bank; he will have used the instruments of the navigator, the surveyor, or the engineer.

(4) Overemphasis on the vocational uses of mathematics might well be avoided. Of equal or greater significance are the cultural values accruing from the proper study of mathematics, to say nothing of the importance of mathematical literacy or functional competence for "personal" and "consumer" purposes. In our enthusiasm we may be prone to think more about bread-and-butter values than of intellectual and spiritual values. A sound guidance program must also pay attention to the role of mathematics in the general education of all the students.

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As Others Saw Him

(*Continued from page 107*)

car, and a good sized checking account. He tried to find some special talent or ability in each pupil for which he could be praised and of which he could be proud; then he tried to lead faculty and pupils alike to respect that ability and to bring out all that was best in the afflicted one. That, he insisted, was one of the most important phases of education for life.

"Dr. Schorling was enthusiastic over all sports, although he excelled in only one—fishing. As the years went by and his committees, speeches, and multitudinous writings cut further and further into his leisure time, he dropped out of participation in games, but continued his support of University and High School athletics, and he did still take time off for an occasional fishing trip. It would be hard to guess whether he was happier catching fish or cooking and serving them over an outdoor fire. He was always the 'official cook' for all picnics.

"Another great pleasure of his later years was his vegetable garden. Started as a war-time activity, it became a real hobby.

"Probably he was never as happy as in *his own* home with his family, to whom he was passionately devoted, or sitting quietly in front of his fireplace visiting with one or two close friends. There one really sensed the depth of his kindliness, his high ideals, and his firm belief in the democratic principles of education."

MARY ANN TUMPERI recently completed her undergraduate work leading to a bachelor's degree and a teachers' certificate under Dr. Schorling's guidance. During part of her undergraduate days she also stayed in the Schorlings' home. Hence she could write:

"Knowing Dr. Schorling was a key to many wonderful friendships for me. I would meet a mathematics teacher or someone in education and the minute I mentioned knowing Dr. Schorling, we were on common ground and were off on a conversation that would last for hours. We would talk on about his wonderful sense of humor, his many accomplishments and of the many favors he did for his friends since he was one of those people who felt that if you were worthy, he would go all out to help you.

"As a member of his class you were very much aware of his keen mind. In fact it

(*Continued on page 148*)

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, New York

The Art of Reckoning

IV. ALGORISMS: COMPUTING WITH HINDU-ARABIC NUMERALS

THE introduction of Hindu-Arabic numerals in Europe was a slow process. The use in Europe of these numerals with their positional value and their symbol for zero dates from the latter part of the tenth century. The earliest certain use of these numerals in a manuscript is in a work written in Spain in 976. The earliest description of the use of these numerals in computation in Europe is in a manuscript of about 1130. The title of this work *Algoritmi de numero Indorum* suggests that it is a translation of the work on computation by the Arab scholar al-Khowârizmî (*c.* 825) which is no longer extant. Certainly the word "Algoritmi" is derived from the name al-Khowârizmî and this name in the form "algorism" has been applied to the small treatises sometimes in verse, sometimes in prose, telling how to use the new numerals—treatises that appeared in considerable number in the twelfth though the fifteenth centuries. These were written in Latin, in Hebrew, English, Italian, French, and even in Icelandic. As might be expected, these were often very similar. Among the most copied were the manuscript mentioned above which is variously attributed to Adelhard of Bath and to Robert of Chester, the *Algorismus Vulgaris* attributed to Sacrobosco (*c.* 1230), and the *Carmen de Algorismo* written by Alexandre de Villedieu (*c.*1225).

Some of the algorisms were very brief, giving the processes alone. Others coaxed the reader along, encouraging him, telling him how to check his work, congratulating him on successful accomplishment. Even at the risk of confusing the reader, a patchwork of citations from different algorisms gives a better idea of their content than would quotations from a single one. The quotations that follow are taken from the translations of two French Algorisms made by E.G.R. Waters, quoted by the kind permission of the editor of *Isis*, Dr. George Sarton.

A thirteenth century algorism begins with a statement of the importance of the subject and states that four of the Seven Liberal Arts, arithmetic, geometry, astronomy, and music, cannot be learned unless one knows how to reckon any more than the other three, grammar, rhetoric and logic, can be mastered if one does not know how to write.

A fifteenth century algorism has this beginning:

"Here follows the noble and subtle science which is called the calculation of algorism. . . .

"Algorism is a system of numbering. If one employs that system when reckoning rapidly, without doubt it is a form of reckoning which tells no lie on any exact point; it has a high reputation for accuracy, and is otherwise known as the system of arithmetic. It forms a part of mathematical science, as also does that branch which is seldom idle, called the very noble teachings of the abacus. The present branch teaches how to reckon and to calculate by means of the combination of the nine figures and the cypher, to multiply one set of figures by another without contestation, to split up, and to divide, thus separating one from the other. It is a very noble science. He who encloses it in his mind has forthwith speedily learned it, if he has acquired accuracy in the use of it. I tell you and inform you, then, that he who wishes to know algorism should listen carefully to this poem, and continue to do so until the end . . . for he who wishes to profit by it must know the whole of it. And if he will remember it well, he will be able to carry out many reckonings without a mistake, and to effect them with the greatest imaginable speed, as you shall see hereafter provided you do not relax your attention."

Thus admonished, the reader goes on to a description of the ten numerals and the idea of place value. But for that, we quote from the thirteenth century Algorism, where the numbers are classified as digits, articles (i.e., multiples of 10) and composites (two-digit numbers, an article and a digit).

"When you are able to recognize each figure according to its nature, and what each signifies, then know, and have no doubt, that each one when first in a number has just its own value without any other number. In the second place it is worth ten times as much as in the previous place. In the third place it embraces ten times as much as in the second place, and in the fourth place it contains ten times as much as in the preceding third place. You can see this in the following manner, which may seem easier to you: if you put a figure in the first place, it is worth units, in the second it is worth tens, in the third place hundreds, and in the fourth place it denotes thousands; if it goes beyond a thousand, on no account forget to always multiply by ten whatever you write in successive places. These figures . . . are not sufficient for reckoning without this one: 0, called naught, which is worth nothing, creates nothing but shadow, to such an extent does it encumber the ground, yet no article can ever by written in any work without it; of itself it can bear no fruit, but it gives significance to others."

The topic of addition as given in these works shows either that the addition combinations were known or that the computer had some method of deriving them as needed. The description of addition tells that the numbers are to be arranged with the smaller close below the larger, the first figure of the one beneath the first figure of the other and so on. The work begins on the right and the sum is written above the numbers. The reader is told that the sum of two digits will be another digit, a composite, or an article. The directions then read:

"If it (the sum) is a digit, one should remove the figure above it, and make in its place another figure worth as much as the two were worth which were there before. And if on adding it befalls you that you have to make an article, you must make a cipher in that place above, move your unit (which was worth ten) farther on, and add it to the figure next beside it; and if the figure is nine, make me a cipher in the proper manner, and you must join your unit with that which you find next after it. If further it befalls you on adding that you have to encounter a composite, then write a digit in the place above, and carry over the article alone to the next place. . . . When no article at all is left in the place where it was, your addition will be complete."

Subtraction has offered difficulties for many computers. The algorisms covered the subject in a short section.

"If you wish to take away one number from another, you must write your greater number over and above the smaller, in just the same way as you did in addition; for no one can by subtraction, whatever he may so subtract a great from a small number, but the great can very well compress itself and allow itself to be made smaller. And if you wish to subtract, begin on the right, and subtract towards the left, and take away the under from the upper figure in the proper manner. Sometimes you will not be able to take it away, when you see them equal; or you will find the upper figure much smaller than the lower. If it is equal, you must remove it and put your cipher there. If it is smaller, borrow a ten from the next figure, i.e., one, which will be worth ten. These ten must be added to the preceding figure—not, however, in writing, but you must set it down in your mind. When you have to borrow your ten, you must reduce the upper figure; you will afterward subtract the lower from it. Sometimes you will find a cipher there; if it does happen that you find it to contain nothing that you can borrow, you must pass beyond the cipher, and from the figure that you find next to it borrow a unit. And know for certain that no borrowed unit could ever be brought under a cipher when one is subtracting, without making it into a nine. When you have brought it into its place, then the figure below must be subtracted from the number that you have above, and you must leave the remainder there. You will know how to add and subtract if you can do as I tell you. And know that subtraction is proved by addition; and he who knows well how to do it proves addition by subtracting."

In many cases, multiplication was preceded by duplation or doubling and division was preceded by mediation or halving.

Multiplication is treated in a variety of ways. In one case, multiplication involves six rules. These cover the product of two digits, a digit by an article, a digit and a composite, two articles, an article and a composite, and two composites. Occasionally rules are given to help in finding the products. A device that appears for finding the product of two large numbers (digits greater than 5),

of finding the square of a number, is this: to find the square of a number, find the difference between 10 and the number and subtract this from the original number. This difference is the ten's digit of the square. The unit's digit is the product of the difference by itself. Thus to find the product of 8 by 8, since 10−8 is 2 and 8−2 is 6, the ten's digit of the square is 6 and the unit's is 2×2 or 4. To find the product of two digits, find the difference between 10 and each of the numbers. The ten's digit of the product is the number found when the difference between 10 and the smaller number is subtracted from the larger number. The unit's digit is the product of the two differences. Thus 8×9 has 2 and 1 for the two differences. The ten's digit of the product is 9−2 or 7, and the unit's digit is 2×1 or 2. Thus the product is 72.

Division is a complicated process. One definition of division is this:

"Division is when one can have two numbers, and it is necessary to find from these two a number which can be taken away from one's greater number as many times as there are units in the lesser. With less than two numbers the operation is impossible."

No rule seems to be given for determining the proper quotient figure.

Several of the algorisms also included sections on the sum of arithmetic progressions, and a few also include square root. The author of the thirteenth century algorism from which a number of quotations have been made, says that the square root requires close attention which is no exaggeration considering the way in which he went about it. The reader is cautioned not to be bewildered and the explanation ends with this statement

"If you wish to test whether you have calculated properly and your root is correct, multiply it forthwith by itself, and add to the resulting number what is left over from the quadration; on putting them together thus, you will see the original number. Then be joyful if you have extracted your root correctly."

Certain of the algorisms as for instance the anonymous Italian algorism in the possession of Columbia University contained puzzle problems as well as the directions for performing the operations of addition, subtraction etc. These problems in the Columbia Algorism are of the familiar type—a boatman takes a fox, a goose and a head of cabbage across a stream in a boat which will carry the boatman with one of these, but not two and where the fox left with the goose will eat the goose and the goose left with the cabbage will eat it. The Algorism also has the problem of the 8, 5, and 3 measure jars with 8 measures of liquid in the largest the problem being to measure out one unit of the liquid without using any other container.

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DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

THE CENTROID DEMONSTRATOR*

THE principal use of the device suggested in this article is to demonstrate that the medians of any triangle are concurrent. As will be shown later, however, the device can be used to illustrate a number of other interesting geometric principles. While the drawings included here may appear somewhat complicated at first sight, close inspection will reveal that the actual construction is not. Boys studying plane geometry will find building and manipulating this particular device both interesting and challenging because its mechanical features are somewhat different from the usual type.

The device consists essentially of four parts: sides AB and BC of triangle ABC, the base box, and the block which slides lengthwise inside the base box. (See Fig. 1.) The following materials are suggested for building the centroid demonstrator:

* Plans for the construction of the Centroid Demonstrator were developed in the Mathematics Laboratory, Monroe High School, St. Paul, Minnesota.

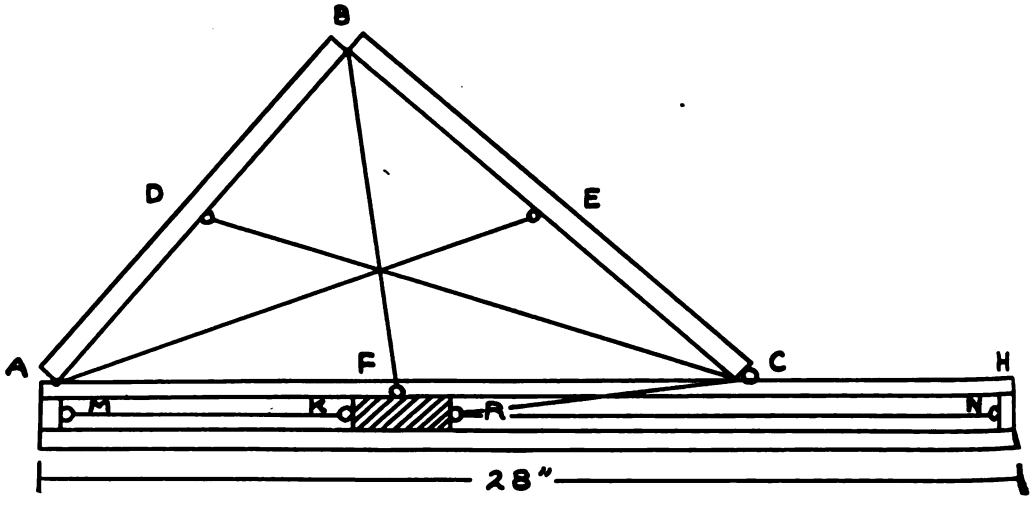


FIG. 1. Diagram of Centroid Demonstrator with one side of base box removed.

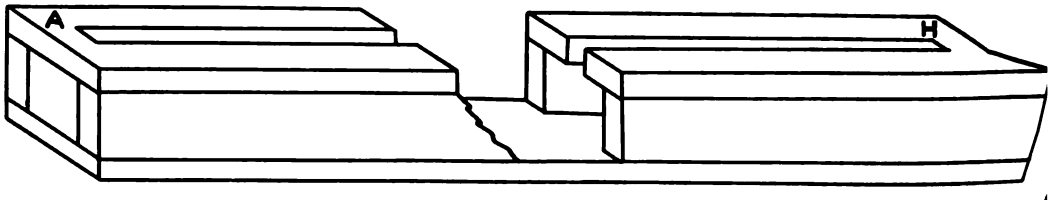


FIG. 2. Diagram of base box illustrating inside and cross section construction.

Side AB	1 piece of pine	$\frac{3}{4}'' \times \frac{3}{4}'' \times 12''$
Side BC	1 piece of pine	$\frac{3}{4}'' \times \frac{3}{4}'' \times 14''$
Base box	2 pieces of plywood	$\frac{1}{4}'' \times 2\frac{1}{4}'' \times 28''$
	2 pieces of pine	$\frac{3}{4}'' \times \frac{3}{4}'' \times 28''$
	2 end pieces	$\frac{3}{4}'' \times \frac{3}{4}'' \times \frac{3}{4}''$
Block KR	1 piece of pine	$\frac{3}{8}'' \times \frac{3}{8}'' \times 1\frac{1}{2}''$

Hardware and miscellaneous items which are needed include two small brass hinges ($\frac{3}{4}''$ wide), eight screw eyes, one piece of strong lightweight cord, and a supply of rubber binders.

Figure 1 shows how the device is assembled and how it operates. Side AB is fastened to the base box at A with a brass hinge. AB and BC are joined together at B also with a hinge. C at the end of BC is free to slide along AH.

The base of the device is in the form of a closed hollow rectangular box with a slit AH cut in the top side. (See Fig. 2.) This slit should be $\frac{1}{4}''$ wide and 26″ long. Actually the top of the box could be made from two separate strips each 15/16″ wide and 28″ long. The inside dimensions of the box must be such that the block ($\frac{3}{8}'' \times \frac{3}{8}'' \times 1\frac{1}{2}''$) will be able to slide freely from M to N. Fastened to the block are three screw eyes—one at K, one at R, and a third at F. The one at F should be turned sideways so that it may slide along the slit AH in the top of the box. Other screw eyes are located at D and E (midpoints of AB and BC respectively), at M and N on the inside ends of the base box, and at C on the lower end of BC. The one at C should be turned sideways so that it also may slide along the slit AH. Rubber binders are strung between the points A and E, D and C, B and F, and M and K. A piece of cord is tied to the eye at N, passed through the eye at R, and tied again to the eye at C. The exact length of cord needed can be determined by fixing the position of C along AH, and then pulling the cord through C until F is at the midpoint of AC. It will be noted that the rubber binder MK acts like a spring on the block in the direction KM, and that the screw eye at R serves the same purpose as a pulley. After the cord is tied at C, the eye to which it is tied may be moved along the slit AH. F will be at the midpoint of AC for any position of C along AH.

That the device may now be used to show that the medians of any triangle are concurrent is easily apparent. If C is located along AH so that $BC = AC$, then we have an isosceles triangle and the medians AE and BF are equal. As another suggestion, remove the rubber binders which represent the medians and stretch them between the points D and E, E and F and F and D. Now we have four congruent triangles, each similar to triangle ABC and one-fourth as large. By using the rubber binders appropriately many theorems relating to the parallelogram can also be demonstrated with this device.

Adjustable Volume Device

This is a visual aid that can be used advantageously on the junior high school level for teaching the concept of volume.

The device illustrated in Figure 3 consists of three main parts: a base, an upright block, and a movable shield. The base is a piece of board $\frac{3}{4}'' \times 6'' \times 12''$. It is marked off into fifty square units as indicated. Down the center of its length a groove is cut through wide enough to accommodate a 3/16″ bolt. The upright block is a piece of board $1'' \times 3'' \times 5''$. The marks indicating cubic inches appear on both sides of it and may either be painted on the surface or grooved with a saw. Brads to which rubber binders may be hooked are driven in at vertices A and B. In the device pictured here the block is fastened to the base with wood glue, but nails or screws may also be used.

The movable shield may be made of tin, plastic, or plywood, but tin is probably the most practicable material to use because it can be bent up at the bottom to form a lip with the vertical. (See Fig. 4.) Doing so will make the shield more rigid. The face of the shield must be 3″ high and 5″ wide, and it must be marked off into square units which correspond in number

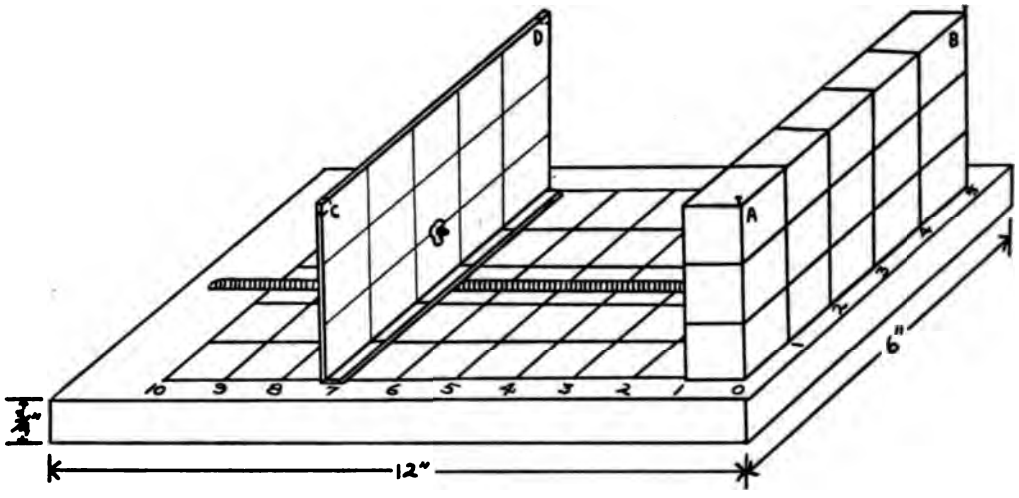


FIG. 3

and position to those on the face of the upright block. The corners at C and D are grooved so that rubber binders may be hooked between C and A, and D and B. On the back side of the shield is a 90° angle iron. Through its horizontal arm there are two 3/16″ bolts with wing nuts. The heads of these bolts extend below the groove in the base. Thus the shield may be moved back and forth and adjusted as desired.

The device is completed by stretching rubber binders between C and A, and D and B. Thus a rectangular box is clearly outlined. The device is so constructed that the concept of volume may be taught by the "layer method." Starting with the area formulas students may be led to visualize a cubic unit upon each square unit of the base. The upright block is to be taken as the first layer of cubic units. It is marked off in such a way that the number of cubic units in this layer may actually be counted. The markings on the

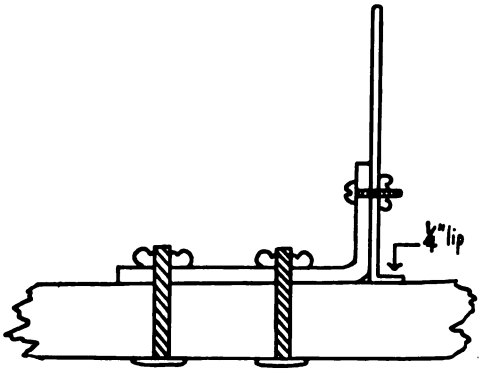


FIG. 4

bottom side of the box make it possible to count the number of layers needed to fill the box. The size of the box may be varied by shifting the position of the movable shield. In this way students may be given a variety of exercises in the calculation of changing volumes.

Marvin L. Johnson
Rochester Junior High School
Rochester, Minnesota

National Council Members

Have you read the suggested revisions of the N.C.T.M. By-Laws appearing on pages 109–110? Please study them carefully and send your comments to any member of the Committee within the next two weeks, or bring them with you to the Annual Business Meeting at Pittsburgh on Thursday, March 29, 1951.

APPLICATIONS

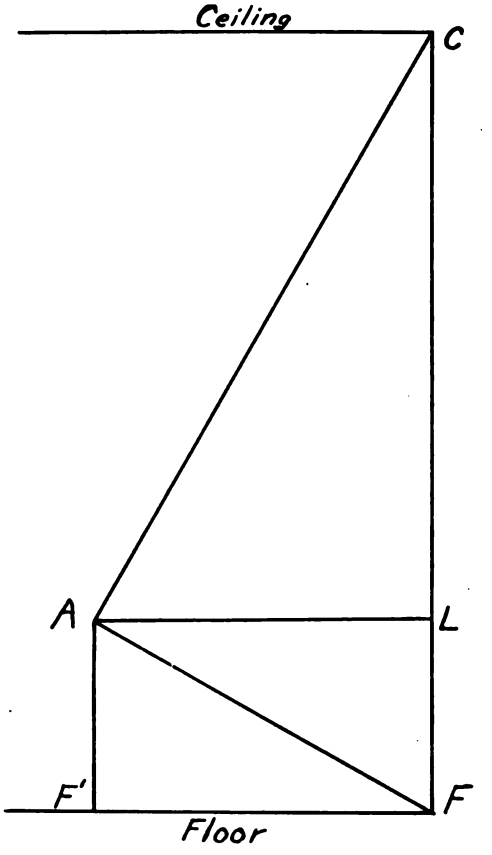
Edited by SHELDON S. MYERS
University School, Ohio State University, Columbus, Ohio

WE ARE pleased with the letters that are being received about "Applications." Both contributions and criticisms are welcome. Some letters which correct errors will be mentioned in the department, such as the one by M. H. Ahrendt of Anderson College, Anderson, Indiana, who discovered in T. 1 "Measuring the Earth's Circumference" that the length of the Kalamazoo shadow should be 56.99, not 53.99 inches, and the one by Paul J. Eby of Boston University, who refined the opening sentence in Ar. 2 "Aliquoting" as follows: "Ten grams of a substance are dissolved in a reagent to make 100 ml. (milliliters) of solution." This a more precise statement of actual practice. However, we shall try to answer all letters directly by mail.

P.-G. 2. Gr. 10–12. FINDING THE HEIGHT OF A ROOM WITH THE AID OF A PLANE GEOMETRY THEOREM

"The altitude on the hypotenuse of a right triangle is a mean proportion between the segments of the base" is a theorem of plane geometry which is usually presented because of its role in developing other theorems, rather than for any immediate practical use. It is possible to show the use of this theorem in a very effective classroom project. A measuring stick or tape measure and an angle mirror[1] are needed.

Briefly, the angle mirror locates the vertex of a right angle whose imaginary legs pass through two given points. In the drawing below, point C is on the ceiling, point F is on the floor, while point A is found with the angle mirror. Angle CAF is a right angle. It is convenient to make CF a wall.



In $\triangle AFC$, AL is the altitude on the hypotenuse, while FC is the hypotenuse. The above theorem gives us the following relationship:

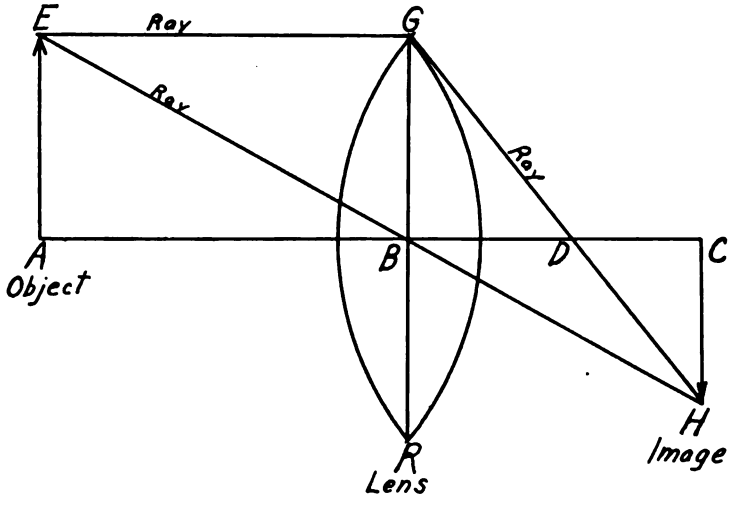
$$\frac{LC}{AL} = \frac{AL}{LF}.$$

AL is the distance of the angle mirror from the wall and AF' is the distance of the angle mirror from the floor, both of which can be measured easily. Since $AF' = LF$, the distance LC can be computed. The sum of LC and LF gives the height of the room. A 40-minute period is more than adequate for students to receive preliminary instructions, perform the measurements, and check results with a ladder.

[1] Berger, Emil. "Devices for a Mathematics Laboratory." THE MATHEMATICS TEACHER, 1950, vol. XLIII, 6: 287–288.

P.G. 3. Gr. 10–12. Use of Similar Triangles in Deriving the Law of Lenses

This application is admittedly for the more able student in mathematics and science, especially if he is asked to go on his own power from step one below to step six. The lens law is a formula in physics



relating the object distance, the image distance, and the focal length of any convex lens. The close relation between plane geometry and optics is well known, Euclid, himself, writing a book on geometrical optics.[2]

Given Data: AC (principal axis) bisector GR

$EA \perp AC$, $CH \perp AC$, $EG \parallel AC$

D is the principal focus

$O = AB$ distance of object from lens

$I = BC$ distance of image from lens

$F = BD$ focal length of lens

Problem: Find a relationship, if any, between I, O, and F. First prove $\triangle ABE \sim \triangle BCH$ and $\triangle BDG \sim \triangle DCH$.

(1) $\dfrac{CH}{AE} = \dfrac{BC}{AB}$ $\qquad$ $\dfrac{CH}{AE(BG)} = \dfrac{DC}{BD}$

(AE is substituted for BG)

(2) $\dfrac{BC}{AB} = \dfrac{DC}{BD}$

(3) $\dfrac{AB + BC}{AB} = \dfrac{BD + DC}{BD}$

(4) $\dfrac{AB}{AB} + \dfrac{BC}{AB} = \dfrac{BC}{BD}$

(5) $\dfrac{AB}{AB \cdot BC} + \dfrac{BC}{AB \cdot BC} = \dfrac{BC}{BD \cdot BC}$

(6) $\dfrac{1}{BC} + \dfrac{1}{AB} = \dfrac{1}{BD}$

(7) $\dfrac{1}{I} + \dfrac{1}{O} = \dfrac{1}{F}$ (The Lens Law).

Ar. 6. Gr. 7–9. Animal and Bird Spe

The following set of problems were u in 1937 by Dr. Charles C. Weidem now retired from The University Sc and Ohio State University. These p lems would obviously lead to many in esting discussions and comparisons junior high school pupils.

1. The sparrow averages 17 miles per ho _____ feet per minute.
2. The seagull at 30 miles per hour aver ½ mile per minute or _____ feet minute.
3. The crow averages 1 mile per minut _____ miles per hour.
4. The pigeon averages 67 miles per hou _____ feet per minute.

[2] Smith, D. E., *History of Mathematics*, New York: Ginn and Co., 1923, vol. I, p. 106, vol. II, p. 338.

The duck hawk averages 3½ miles per minute or _____ miles per hour.
An elephant can average 32 miles per hour or _____ feet per minute.
The greyhound can run ¼ mile in 25 seconds or _____ miles per hour.
The jackrabbit for a short distance can run at the rate of 1 mile in 1½ minutes or _____ miles per hour.
A race hourse has run at the rate of 59 miles per hour over a 2 furlong course, or _____ feet per minute.
The antelope runs at the rate of 62 miles per hour or _____ feet per minute.
The lion can run at the rate of 68 miles per hour or _____ feet per minute.
A duck hawk's speed is how many times that of a sparrow? _____
How do the speeds of a pigeon and a lion compare? _____
How do the speeds of a seagull and an elephant compare? _____
A greyhound on the average runs slower than a jackrabbit. Give three reasons which explain the fact that greyhounds catch rabbits.
How do speeds of crows and antelopes compare? _____
An elephant's speed is _____ as fast as a lion's speed.
Arrange the animals named above in the order of their speed.

3. 1. Gr. 11–12. Solid Geometry and the Law of Inverse Squares for Light

An important solid geometry theorem n be used to deduce the law of inverse uares for light radiation.

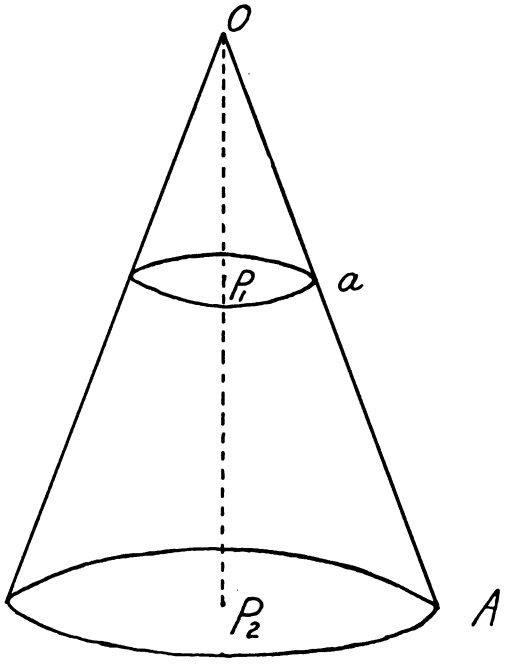
A flashlight at O is shining downward, st on a circular surface of area "a" at and distance "d" from O, and second a circular surface of area "A" at P_2 d distance D from O. These two surces form the bases of two right circular nes with O as the common vertex. By a lid geometry theorem it is known that e following relationship exists with spect to the two cones:

$$\frac{a}{A} = \frac{d^2}{D^2}.$$

It can be experimentally determined or uitively assumed that the light intenty varies inversely as the area over which is spread. The same amount of light from O falls on a bigger area at P_2 than at P_1. In this case one can assume that the bigger the area, the weaker the light. Therefore:

$$\frac{L}{l} = \frac{a}{A}.$$

Where L is the light intensity at P_2 and l is the light intensity at P_1.



It follows from these two relations that:

$$\frac{L}{l} = \frac{d^2}{D^2}.$$

The verbal statement of the relation is known as the "law of inverse squares for light" and is here given: The intensity of light varies inversely as the square of its distance from its source. Suppose that the distance at P_1 was 2 inches and at P_2 was 6 inches, or 3 times as much. Then:

$$\frac{L}{l} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}.$$

This means that the light at P_2 would ⅑ as strong as at P_1.

AIDS TO TEACHING

Edited by

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and

DONOVAN A. JOHNSON
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BOOKLETS

B.54—Let's Measure Things

Cornell Rural School Leaflet; New York State College of Agriculture, Cornell University, Ithaca, New York

Booklet; 9″×6″; 63 pages; 1948; $.35.

Description: One of the unique features of this pamphlet is the broad field of measurement and units of measure that it covers. The main topics are; "Measuring Space," "Measuring Areas," "Measuring Volume," "Time, What is Time?", "How Much Does It Weigh," "Measuring Numbers," "Measuring Position," "What is Temperature?", "Let's Measure Humidity," and "Let's Measure Speed." The first topic is especially elaborate for it includes, not only the common linear measurements, but also many meaningful items such as; "Measurement in Athletics," "Measurement of Interest to Hunters and Fishermen," and "Measurements in the Carpenter Shop." Interesting and helpful student activities include judging of distance, measuring heights and how to make a sundial. Very complete tables of units are given with all of the topics studied.

Appraisal: Here will be an invaluable aid to the teacher during the study of measurement and its practical use in other countries as well as ours. It is felt that more use could have been made of illustrations such as picto-graphs especially when studying speed. An index or table of contents would have helped when using this pamphlet as a classroom reference. Both student and teacher will be able to use this leaflet. The material covered and the method of presentation make it useful for intermediate and senior grades as well. (Reviewed by Lloyd W. Green, Central High School, St. Paul, Minn.)

B.55—Brain Resters and Testers

Cooperative Recreation Service, Delaware, Ohio.

Booklet; 25 pp.; 4″×7″; Single copies—$.25.

Description: This booklet contains thirty-six clever puzzles, tricks, and games. Most of the tricks are mathematical such as dividing 28 by 7 to get a quotient of 13 or how to construct a magic square. Others such as "Doublets" and "Mind Reading" are not mathematical. Answers are provided for those who get weary.

Appraisal: The mathematics teacher will find this inexpensive booklet a source of entertainment for her classes. Tricks such as how to tell the age of a person, stunts such as spelling card numbers as they turn up, problems such as how to make magic squares or games such as the Fifteen Game stir up interest from the dull to the bright. It's worth a quarter.

CHARTS

C.20—National Forum Social Studies Charts

National Forum Inc.; 407 S. Dearborn St., Chicago 5, Illinois

Charts; 17″×22½″; $17.00 per set (with masonite easel)

Description: Thirty-one charts are lithographed in very attractive colors on heavy white paper. The set is subdivided into eight sections according to the concepts

l: (1) Principles of Economics, hasing Power, (3) Money and t) Economic Organization, (5) gy, (6) Labor, (7) Farm, and ervation. Important current eco- ıta are pictured by means of rawings and pictographs. The m is used quite effectively to antitative comparisons, whereas er indicates economic relation-

; is bound within a heavy black l cover which serves as an easel splay.

ld drawings employed to visual- oncepts are large enough to be ı a distance and the easel facili- ılay of the charts.

eographed booklet contains two pertinent information to accom- ı of the charts. The information ences given are quite helpful in he teacher to direct student dis- ınd to prepare assignments.

sal: Widespread recent revisions ıior high school curriculum have ed the desirability of teaching mathematics at the lower grade her than the elementary pure tics which is taught primarily benefit of those students who take college preparatory courses hool.

gh the charts are designed for dies classes, they are very useful g the mathematics teacher to hose economic concepts which he new socialized nature of the In the light of present general , the social nature of mathe- n hardly be ignored. In fact, the of the newer junior high school drawn largely from current conditions.

nplicity of the charts enhances ity and the very fact that they -date and dramatic in presenta- contribute to the reduction of ess.

ıarts would probably be too expensive if their usefulness was limited to the teaching of mathematics; since they are exceptionally effective in teaching current affairs, however, their advantages may be utilized by both the mathematics and the social studies departments. This dual effectiveness would certainly make the high price less objectionable.

Other charts prepared by the National Forum Company are useful in teaching mathematics but cannot be purchased individually. The price of seventeen dollars for an entire set would be prohibitive if only a small fraction of the charts were to be used. An excellent example of such a situation arises in the case of a set called "Government." Six of the thirty-one charts are concerned with taxation: 24. "Sources of Revenue," 25. "Where the Tax Dollar Goes," 26. "The Federal Debt Load," 27. "Who Pays the Sales Tax?". The only justification for purchasing these would be for teaching social studies; they could then be borrowed for teaching mathematics. (Reviewed by Bernard Singer, Hyannis, Mass.)

EQUIPMENT

E.36—Visualizer

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Cabinet Table; 36″×36″×18″; 3″ plywood; $92.50.

Description: The Visualizer is a multipurpose cabinet, 36″ long, 36″ high and 18″ side, made of 3″ plywood in natural finish. It has two drawers, one 3″ and the other 7″ deep, and two shelves, all designed to hold smaller visual materials. The front panel is a square yard. Each of the sides is also a square yard when the doors are open. By folding back the hinged false top to rest on the doors, a cubic yard is formed. The lateral surfaces of the cubic yard are divided into square feet by yellow lines. A graph chart, ruled in inch squares, is glued to the under side of the front panel that is hinged at the

top. When raised to an upright position and fastened by a simple device to the partially lifted false top it is ready for use. The under side of the false top is heavily coated with black flock and when opened to position for use as a demonstration board, rests on a support concealed in the upper drawer. It is equipped with casters and handles to move it easily.

Appraisal: The Visualizer is a practical visual aid for developing concepts of linear, square and cubic measures. Higher than a regular desk, it provides a demonstration desk and also a solid table for the projector or other machines. It may be moved to any part of the room to secure proper lighting or to another room for use by the same or other teachers. The ever-ready solid-back graph board is easier to use than those that roll away in a case. It may serve as a display board for large charts, pictures, etc., by clipping them to the side or top of the board.

The flocked demonstration surface of the false top holds the plane geometric figures or other similar materials more securely than backgrounds of velour paper or flannel. Since the upper drawer is partly open when this demonstration board is in use, it is a convenient storage place for the small items, either commercial or teacher made, that are being used. The second drawer keeps the larger items orderly and convenient. There is space for the cubic foot at one end of the lower shelf and special places for additional materials needed in senior high school mathematics. The other shelf provides space for pictures and charts usually difficult to store. The Visualizer is intended for use with many smaller and less expensive visual aids and becomes increasingly more valuable as they are added. (Reviewed by Alice M. Phillipson, Mathematics Consultant, Los Angeles, Calif.)

E.37—Demonstration Case

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Case; 2″×19″×28″; $5.00.

Description: This demonstration case is made of plywood 2″×19″×28″. It has a handle on one side so that the case can be carried like a suit case. The lid is hinged at one side and has hooks to hold it firmly in place on the other sides. The under side of the lid, like the false top of the Visualizer, is flocked and when raised to the proper angle is supported by a long hook. The body of the case, like the upper drawer of the Visualizer, is the storage space for small disks or materials used on the demonstration board, as well as the circle, linkages, etc.

Appraisal: This small case does not have the storage space, graph board, etc., the Visualizer has. But it does provide a convenient flocked demonstration board and storage for the materials used with it such as the plane geometric figures, which are useless without such a board. It is a good substitute for those who cannot purchase the Visualizer. (Reviewed by Alice M. Phillipson, Mathematics Consultant, Los Angeles, Calif.)

INSTRUMENTS

I. 29—Blackboard Drawing Set (No. 1298)

J. L. Hammett Company, Kendall Square, Cambridge 42, Mass.

No. 1298; $7.80.

Description: The hardwood drawing set consists of four items—a twenty-four-inch T-square, thirty-six-inch straight edge, twenty-four-inch triangle, and a fifteen-and-one-half-inch protractor. The joint at the stem and cross-bar of the T-square is reinforced by a wooden brace. The long leg contains a two-foot ruler graduated in eighths of an inch.

The straight edge is a three-foot rule graduated in eighths of an inch. The thirty-degree right triangle also has a two-foot rule printed upon its longer leg. The protractor has its angular calibrations printed at one-degree intervals.

Appraisal: The entire drawing set is hardly necessary because the straight edge and protractor may be used to draw any

diagrams which could be drawn with the aid of the entire set. The instruments are well made but suffer from the fact that graduations are not sufficiently visible to be seen by the students during blackboard demonstrations; the visibility of markings upon the protractor is especially poor. Each instrument contains a small round knob which serves as a handle for positioning. Holes are provided in each tool for hanging against the wall when not in use. A straight edge and protractor are essential items in every mathematics classroom. Fortunately, the items in this set may be purchased individually. (Reviewed by Bernard Singer, Hyannis, Mass.)

MODELS

M. 16—Geometrical Solids and Surfaces

Central Scientific Co., 1700 Irving Park Rd., Chicago 13, Ill.

(Order #174110) $7.50 per set

Description: The set contains 24 unpainted wooden solids, 17 bristol board plane figures, and three wires bent to represent angles. The set is packed in a durable wooden box. The solids included are: triangular, square, hexagonal, and octagonal prisms; cylinders, cones, conic sections, sphere, hemisphere, ellipsoids, ovoids, and models to represent the intersection of a cone and cylinder. The plane figures represented are: acute, right, obtuse, isosceles and equilateral triangles; square, pentagon, hexagon, octagon, circles, semi-circles, ellipse, and oval. The bent wires illustrate acute, right, and obtuse angles.

The wooden models are well constructed and the light wood enables them to be seen from a distance regardless of the fact that they are only 1½″ to 3″ high. The bristol board surfaces are sufficiently durable to withstand much handling but the wire angles are small and easily bent out of shape.

Appraisal: The models should be quite useful for aiding students to visualize the figures whose characteristics they study. Such help is certainly needed for many students are weak in ability to "see" spatial relationships, even when a two-dimensional diagram is provided. The use of such concrete objects along with diagrams can contribute to the development of this ability. Likewise, when referring to an octagonal prism, the teacher should be able to eliminate misunderstandings by demonstrating a model which may be examined.

Teachers will benefit from similar use of the bristol board figures. The usefulness of the set for teachers of Algebra II is probably limited to use in explaining the reason why the parabola, hyperbola, ellipse, and circle are called "conic sections." Students of general mathematics may determine areas and volumes of the models.

The aids should be kept in a convenient place where they will be available whenever they might facilitate understanding. In such situations the appropriate model should be passed among the students for examination. Another effective use would be to display the models and have the students determine where these mathematical forms appear in nature and industry.

This set of models is quite expensive; many schools which have carpentry workshops may prefer to construct their own sets. (Reviewed by Bernard Singer, Hyannis, Mass.)

PICTURES

P. 10—Geometric Designs

Division of Photography, American Museum of Natural History, Central Park W. at 79th Street, New York 24, N. Y.

Photographs of geometric forms in nature and industry

Description: The museum's collection of 175,000 negatives includes many which illustrate geometric shapes in nature and industry. A list is available from the museum. The Division of Photography fulfills request for excellent prints of the nega-

tives; glossy contact prints are furnished unless the teacher request other sizes. Enlargements are made according to desire; reductions are made at a minimum cost of twenty-five cents. When ordering, it is necessary to state whether the prints are to be used for education, reproduction, advertising, etc. The size and price of *contact prints* are determined as indicated below:

1.	5″ × 7″	$ .25
2.	4″ × 5″	.10
3.	8″ ×10″	.50
4.	6½″× 8½″	.40
5.	11″ ×14″	1.00

Appraisal: The importance of geometric forms in nature and industry can certainly be overdone; yet, the fact that some teachers mistakenly give a place of major importance to the topic is not sufficient justification for eliminating its study. It is worthy of mention and may result in a greater appreciation of mathematical beauty and increased interest in geometry.

The entire set of pictures is hardly necessary for the attainment of the two general objectives mentioned above. The number of pictures purchased and those selected are dependent upon the funds available and the individual preferences of teachers. The prices of these prints are certainly reasonable. (Reviewed by Bernard Singer, Hyannis, Mass.)

As Others Saw Him

(*Continued from page 134*)

always kept you very alert and on your toes. In class he often showed his amazing ability to organize material and to summarize a long class discussion. I remember his telling us a story when he came back from Washington, D. C. He had worked there for a year on a navy training program. He attended a meeting along with some important navy officials. After a few hours of discussion he was called on to summarize. He did this and also pointed out, in all frankness, that they weren't achieving their goal. This is another trait which I admired in him—his frankness. You always knew exactly what he thought and where you stood.

"It always amazed me what a deep faith he had in people. When he told you that he thought you could do a job and do it well, he almost had you believing it too. This really means a lot to a person, to know that other people have faith in you.

"I still have not fully realized that we will not see him again. I know I will never feel that he is gone—there is still too much of him around. So many things that he helped plan—so many books he has written that sound like him talking. He liked to put the names of people he knew into his textbooks. Many a time a delighted youngster (maybe the family butcher's son) would beam and say that Dr. Schorling had put his name in a math book.

"He had one favorite story which he liked to tell about a teacher who happened to eat near him on a diner. They started talking about Schorling's *Student Teaching!* He got a lot of criticisms from her on the book. (All this without knowing who he was.) In fact he kept pointing out weak spots of the book and leading her on in the criticisms. When she finally got up to leave, he told her who he was and you can well imagine her reaction. He claimed that it was a good way to find out what people really thought of his writing.

"He liked to try new things. I remember his bringing home an expensive gadget that would make juice out of solids such as carrots. Mrs. Schorling wasn't very keen on the idea, for she didn't think the gadget was worth the money. So one afternoon he tried to make a concoction out of carrots, raisins, and other vegetables. He made the mistake of unscrewing the glass container which had no bottom except the beater. He should have just poured the fluid out. Anyway we did a rush job of cleaning up the floor before Mrs. Schorling got home."

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
School of Education, New York University, New York 3, N. Y.

IN THE January issue of THE MATHEMATICS TEACHER the following problem was proposed. "Divide the area of a circle into n (n, any integer) equal parts, but the division of the circumference of the circle is not permitted, nor is the drawing of concentric circles or of parallel chords permitted."

The solution of this problem is as follows. Since n is any integer, we shall take a specific case, say, $n=6$.

Let the radius of the circle be r. The area of the circle is πr^2. Divide the diameter AB of the circle into six equal segments. This division may be performed by means which are generally developed in the first course in high school geometry. Thus, we have that $AB_1=B_1B_2=B_2B_3=B_3B_4=B_4B_5=B_5B=\frac{1}{3}r$.

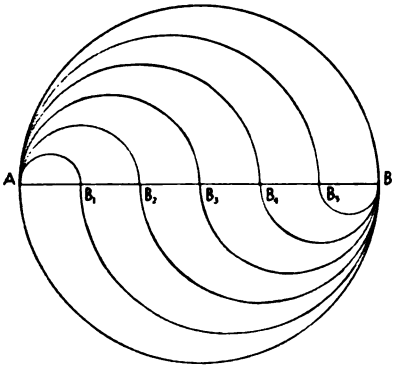

FIG. 1

Construct on AB_1 and B_1B as diameters semicircles so that the semicircle on AB_1 lies above the diameter AB and the semicircle on B_1B lies below the diameter AB.

Let us compute the following areas: the area of the semicircle on AB_1, the area of semicircle on AB, and the area of the semicircle on B_1B. Then we shall add the areas of the semicircles on AB and AB_1 we shall subtract from this sum the area of the semicircle on B_1B.

The area of the semicircle on AB is equal to $\frac{1}{2}\pi r^2$.

The area of the semicircle on AB_1 is equal to $\frac{1}{2}\pi(\frac{1}{3}r)^2$.

The area of the semicircle on B_1B is equal to $\frac{1}{2}\pi(\frac{2}{3}r)^2$.

Adding the first two areas and subtracting the third area we obtain

$$\tfrac{1}{2}\pi r^2(1+\tfrac{1}{3^2}-\tfrac{2^2}{3^2})=\tfrac{1}{6}\pi r^2$$

which is equal to one sixth of the area of the circle.

Construct on AB_2 and B_2B as diameters semicircles so that the semicircle on AB_2 lies above the diameter AB and the semicircle on B_2B lies below the diameter AB.

Let us compute the areas of the semicircles on AB_2 and B_2B. Then we shall add the areas of the semicircles on AB_2 and AB. From this sum we shall subtract the area of the semicircle on B_2B and the area which was computed above (that is, $\frac{1}{6}\pi r^2$).

The area of the semicircle on AB_2 is equal to $\frac{1}{2}\pi(\frac{1}{3}r)^2$.

The area of the semicircle on B_2B is equal to $\frac{1}{2}\pi(\frac{2}{3}r)^2$. We have then

$$\tfrac{1}{2}\pi r^2(1+\tfrac{1}{9}-\tfrac{4}{9}-\tfrac{1}{3})=\tfrac{1}{6}\pi r^2,$$

which is equal to one sixth of the area of the circle.

Construct on AB_3 and B_3B as diameters semicircles so that the semicircle on AB_3 lies above the diameter AB and the semicircle on B_3B lies below the diameter AB.

Let us compute the areas of the semicircles on AB_3 and B_3B. Then we shall add the areas of the semicircles on AB and AB_3. From this sum we shall subtract the area of the semicircle on B_3B and the sum of the two areas which were computed above (that is, twice the area equal to $\frac{1}{6}\pi r^2$, or $\frac{1}{3}\pi r^2$).

The area of the semicircle on AB_3 is equal to $\frac{1}{2}\pi(\frac{1}{2}r)^2$.

The area of the semicircle on B_3B is equal to $\frac{1}{2}\pi(\frac{1}{2}r)^2$. We have then

$$\tfrac{1}{2}\pi r^2(1 + \tfrac{1}{4} - \tfrac{1}{4} - \tfrac{2}{3}) = \tfrac{1}{6}\pi r^2,$$

which is equal to one sixth of the area of the circle.

The computation of the areas of the remaining three portions of the circle is identical with the computations stated above.

The development which is illustrated above may be varied by assigning to n any other values, such as 2, 3, 4, 5, 7, etc.

In a class of intermediate algebra this problem may be generalized so that the computation may be performed for the generalized symbolic treatment as if the division of the area of the circle were required to be performed into n equal parts.

It should be noted that this solution of the division of the area of the circle results in very a interesting design.

Incidentally, the national flag of the Republic of Korea (South) has on its national flag as the main design the division of the area of the circle into two equal parts as shown below.

FIG. 2

The department is still very anxious to know whether the material presented in its column is helpful in your classroom situations. This department is designed to serve and assist the classroom teacher. If you have any questions or comments please send them in. In you have any interesting problem please share them with other members of the National Council.

New Mathematics Kit

A new mathematics kit known as Computation Unit No. 123 has been issued by Science Service in cooperation with the National Council of Teachers of Mathematics. A supply of these has been ordered for the use of readers of THE MATHEMATICS TEACHER. This is the third kit in the series.

The new kit contains materials with which the reader can construct a slide-rule type of device for adding and subtracting signed numbers; a logarithmic slide rule for multiplying, dividing, and the like; and a simplified set of Napier's rods. In addition there is a commercially made plastic slide rule of a type that would cost 50¢ on the market. Also included is an explanatory leaflet which explains simply but in considerable detail both the theory and use of these different instruments. This kit should be useful to both teachers and students.

The supply of the first kit in the series, Geometric Models Unit No. 102 is completely exhausted, and no more orders for it can be filled. There is still a supply available of the second kit, Straight line Unit No. 113. All kits sell for 50¢ each or 3 for $1.00. Your order may be assorted, if you wish, among the 113 and 123 units. Send all orders with payment to the chairman of the kit committee, Professor M. H. Ahrendt, Anderson College, Anderson, Indiana.

What Is Going on in Your School?

By JOHN R. MAYOR
University of Wisconsin, Madison, Wisconsin

SECTION A of a study on trends in teaching mathematics was published in the November, 1950 number of THE MATHEMATICS TEACHER. A summary of replies to the questions in Section A, which was concerned with mathematics enrollments, will be reported in the April issue. This question-answer department is on an experimental basis. It is hoped that this plan will yield information on trends in teaching mathematics useful in study and research in teaching problems. Such a procedure was used with considerable success by the Commission on Post-War Plans. The title given to this section was first used by Raleigh Schorling in reporting on a similar kind of study made by the Commission in the April, 1948 number of THE MATHEMATICS TEACHER. The success of this project depends on the cooperation of the readers of THE MATHEMATICS TEACHER.

The topic chosen for study in Section B is one about which a great many questions are asked. Teachers everywhere are anxious to know what other schools are doing about "general mathematics." As the term is used in the questions below, general mathematics is a course which includes topics from arithmetic, algebra, geometry, and trigonometry, and which is planned as a course to meet the needs of students who may not elect algebra or who may be advised not to elect algebra because of interests, past achievement, or aptitude.

Please send the answers to these questions to John R. Mayor, North Hall, Madison 6, Wisconsin by March 10, 1950. A summary of replies will be published in the May number. We believe that this department can be most effective if replies are sent in promptly and promptly reported.

Readers of THE MATHEMATICS TEACHER are also invited to submit questions for publication in future issues.

In replying please give your name, the name and locality of your school, and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades 10–12; junior high school, grades 7–9; etc. The questions may be answered by numbers without use of a particular form or repetition of the questions. The names of teachers or schools will not be revealed in any of the reports.

B. GENERAL MATHEMATICS

1. Does your school offer both algebra and general mathematics (or a course which might be called general mathematics) in the ninth grade? If not, does your school offer general mathematics in a later grade?
2. What per cent of your students take a course in general mathematics as their first course in high school mathematics (grades 9–12)?
3. For placement of students in general mathematics do you use:
   a. An algebra aptitude test?
   b. An arithmetic achievement test?
   c. The student's grade in eighth grade mathematics?
   d. An intelligence test score?
   e. Written recommendations of teachers?
   f. Other?

   Please indicate which of these or any other placement procedures you use.
4. Upon completion of one year of general mathematics may your students elect:
   a. A second year course in general mathematics?
   b. Algebra?
   c. Plane geometry?
   d. Other courses in mathematics?
   e. None?

   If possible, indicate what per cent of these students enroll in each course.
5. Do the colleges which your graduates attend accept general mathematics on the same basis as they accept a course in algebra?

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York 3, New York

The Question: What is the place of mathematics in the elementary schools of other countries of the world?

The Study: U.N.E.S.C.O. & International Bureau of Education. *Introduction to Mathematics in Primary Schools.* Geneva, Switzerland, 1950. Six Swiss francs.

Comparisons are not always odious but, on the contrary, often suggestive of new ways of attacking problems. The content and method problems involved in teaching mathematics in the first six grades will probably always be with us. An opportunity to learn what our neighbors in the rest of the world are thinking and doing about these problem would hardly seem to be a waste of time.

Forty-six countries replied to a questionnaire transmitted to them by the Research Division of the International Bureau of Education. This instrument contained eighteen queries about time allotments, content, importance, aims, methods, teaching materials and reforms related to the teaching of elementary mathematics. Some of the more significant results of this investigation follow:

1. *At What Age Does Mathematics Education Begin?*

An introduction to mathematics is given in the first year except in one country. There were ten countries, however, including Belgium, Scotland and Yugoslavia, which had pre-school establishments where some notions of quantity were given at three years of age.

2. *How Much Time Is Given to Mathematics?*

An average of four to six hours a week is assigned. The amount of time is usually the same for each day with the lessons during the first three years being a little shorter or less numerous than in the last three years.

3. *What Are the Aims of Teaching Mathematics during the First Six Years?*

They are, in general, (a) to provide a useful tool and (b) to give a mental discipline. More specifically, they are to acquire a technique, form habits of work, acquire mental attitude, gain knowledge and interest, and to train the intellect.

4. *What Are Some of the Common Elements in the Curricular Pattern?*

(a) The notions about numbers and the operations on them are started early and spread over a period of years instead of being taught in concentrated fashion in a brief period. These early stages involve a considerable use of number in familiar school experiences, games and the manipulation of concrete objects.

(b) Geometry, interpreted as the measurement of magnitude and the study of space and its properties, is of concern from the third year onward. In many countries even the measurement of perimeter, areas and volumes is considered as well as the meaning of angle, parallel and perpendicular. Sequences vary greatly; in some countries teachers begin with the point and the line while in others solids and volumes are given first attention.

5. *What Can Be Said about the Methods of Teaching?*

In addition to providing an experiential base before arriving at the use of numbers in an abstract way teaching aids, readily available without cost in the immediate environment, are brought into the instruction. Some countries believe in the use of projects and incidental teaching while others oppose these procedures because of their interference with the systematic

teaching of mathematics. In one-third of the countries an inductive approach is advocated. By this is apparently meant the discovery of generalizations through experiences with the concrete and specific.

6. *When Are Addition and Subtraction of Whole Numbers Taught?*

The general tendency is to give addition and subtraction exercises on numbers up to ten in the first year, up to twenty in the second year and other important combinations in the third year. Except for three countries no mention is made of addition and subtraction tables. Exercises in putting together and taking apart groups of concrete objects precede the abstract work with numbers. Learning to "carry" is acquired either in the second or third year.

7. *What Are the Limits of the Multiplication Tables?*

These seem to be determined to some extent by the bases used for the system of weights and measures. This may mean twelve in some countries but in India and Pakistan children must learn up to the 16-times table because of the nature of the monetary units.

8. *When Are Fractions and Decimals Taught?*

Practices vary considerably, from teaching addition and subtraction of certain fractions in the first year in Nicaragua and Colombia to the initiation of fractional computations in the third year in Canada and South Africa with the multiplication of a fraction by an integer.

In those countries having the metric system operations on decimals reach a peak in the fourth year while fractions receive their major attention in the fifth and sixth. In the countries not having the metric system it is not unusual to delay the treatment of decimals until the beginning of secondary schooling.

KRAMER, MAX. *Mathematics in Meterology, with Suggestions for Teachers of Mathematics and Meteorology.* Ph.D. dissert. Teachers College, Columbia University. New York. 1949.

This is not only a contribution to the occupational information aspect of guidance but is a source of applications of mathematics to the study of weather, always a matter of interest to all of us.

LETTERS

Exchange Teachers

"I quite agree with Frances Lethlean of Glencoe, Illinois concerning exchange teaching. I tried to promote that locally and the administration countered that temporary replacements are hard to find. I endorse the proposal of a listing in the official organ."

ROSAMUND J. JONES
Ely, Minnesota

"About exchanges! Since they are approved and arranged primarily by School Departments I'm wondering how useful a list would be. Thought it would be interesting to see. *However* what about an article now and then *by* some exchange teacher. These could be *very* helpful, I should think. You could assign general topics or give them free rein! Just what do our visitors think of us? and what do our travelers think of the places they visit?"

MISS FRANCES K. PARRIS
Malden, Massachusetts

A member in Washington, D. C. calls attention to a recent government publication entitled "Teachers Abroad." This bulletin discusses the interchange of teachers between the United Kingdom and the United States since the end of World War II. Published in 1950, it contains 40 pages, costs 20¢, and its catalog number is No. FS5.3:950/10. Send orders to the Superintendent of Documents, Government Printing Office, Washington 25, D. C.

Order of Operations

"I am troubled by a contradiction in rules found in two algebra texts I have used or am now using. I never knew until this fall that there is no agreement among mathematicians on this subject of 'order of operations.' Surely something should be done about it. Please will you discuss the matter through the magazine, THE MATHEMATICS TEACHER?"

EDITH WOOD
Anchorage, Kentucky

BOOK SECTION

Edited by J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is available whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

ELEMENTARY SCHOOL

Experimenting With Numbers, Teachers Manual for Use with Beginners, by Catherine Stern, the Castle School, New York. Paper, 105 pages, 1950. Houghton Mifflin Company, 2 Park Street, Boston. $.60.

HIGH SCHOOL

Practical Use of the Slide Rule, College Outline Series, by Calvin C. Bishop. Paper, vii+147 pages, 1950. Barnes and Noble, Inc., 105 Fifth Avenue, New York. $.75.

COLLEGE

Analytic Geometry

An Outline of Analytic Geometry, College Outline Series, by C. O. Oakley, Haverford College. Paper, xviii+246 pages, 1950. Barnes and Noble, Inc., 105 Fifth Avenue, New York. $1.25.

Analytic Geometry and Calculus, by Lyman M. Kells, United States Naval Academy. Cloth, viii+623 pages, 1950. Prentice-Hall, Inc., 70 Fifth Avenue, New York, 11, N. Y. $4.75.

Statistics

Technological Applications of Statistics, by L. H. C. Tippett, Head of the Mechanical Processing Division, British Cotton Industry Research Association. Cloth, ix+189 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.50.

An Outline of Statistical Methods, Fourth Edition, Revised, College Outline Series, by Herbert Arkin, and Raymond R. Colton, College of the City of New York. Paper, xiii+224 +47 pages, 1950. Barnes and Noble, Inc., 105 Fifth Avenue, New York. $1.50.

Teaching of Mathematics

Measurement and Dimensionality by Aaron Bakst, School of Education, New York University. Paper, ii+70 pages, 1950. New York University Book Store, Washington Square, New York 3, N. Y. $.90.

MISCELLANEOUS

Mathematical Snapshots, Revised, by H. Steinhaus, the University and Polytechnicum. Wroclaw, Poland. Cloth, vi+266 pages, 1950. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. $4.50.

Sourcebook on Atomic Energy, by Samuel Glasstone, Consultant to the United States Atomic Energy Commission. Cloth, 546 pages, 1950. D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York 3, N. Y. $2.90.

REVIEWS

Mathematics to Use. Mary A. Potter, Flora M. Dunn, Emmy Huebner Allen, and John S. Goldwaithe. Boston, Ginn and Company, 1950. ix+500 pp., $2.40.

Mathematics to Use is intended primarily for the ninth grade but could be very helpful in courses in general mathematics at various high school levels. The authors have established the validity of the topics in the text by checking them with the Check List compiled by the Commission on Post-War Plans of The National Council of Teachers of Mathematics.

Arithmetic is emphasized throughout the book, but important topics from algebra and geometry are to be found. Special emphasis is placed on the simple equation so that the student may adequately manipulate the formula. The geometry presented is informal in nature. It is particularly noteworthy that the social applications of mathematics are stressed throughout the text.—WILLIAM H. NAULT, W. K. Kellogg Junior High School, Battle Creek, Michigan.

Fundamental Algebra with Practical Applications. Robert L. Erickson. New York, McGraw-Hill Book Company, 1949. xi+317 pp., $2.80.

This text, designed for a basic mathematics course which "reviews and integrates the fundamentals of arithmetic . . . and . . . algebra," is a radical departure from the conventional treatment of elementary mathematics—the arrangement of the material and the author's style being unique.

The first four chapters include a thorough treatment of the arithmetic of signed numbers. The remainder of the text treats of topics from elementary algebra and a brief introduction to trigonometry. The author avoids as much as

possible the presentation of formal rules and formulas, but relies on the student's reasoning ability. The book contains an enormous number of exercises, a large proportion of which are of a very practical nature.

Many teachers will find this a valuable source of material to supplement other textbooks. The book seems best adopted for vocationl courses, but might well be considered for use in courses in general mathematics.—H. D. LARSEN, Albion College, Albion, Michigan.

Arithmetical View Points: An Introduction to Mathematical Thinking. Sidney G. Hacker. Pullman, Washington, The State College of Washington, 1948. vii+144 pp.

The titles of these six lectures suggest their content: "The Art of Reckoning," "Mechanical Counting Devices," "Certain of the Foundations of Arithmetic," "Some Celebrated Theorems of Arithmetic," "The Rational Numbers," and "The Irrational Numbers." Starred subtitles in the Table of Contents indicate sections that might be omitted by "readers of limited mathematical experience." All illustrations are grouped into a single section at the end of the book.

These lectures were planned as a part of a course on Physical Sciences at the State College of Washington when time did not permit an entire course on the Mathematical Sciences. They are introductory in nature and some proofs are omitted; where such omissions are made, ample references are made (in Notes at the end of Chapter VI) to places where proofs may be found.

Although much of the material offered is not new, it is a departure from many "introductions to mathematics" in two respects: (1) the inclusion of some theorems not usually presented, and (2) in the content of the chapter on mechanical counting devices.—ELINOR B. FLAGG, Illinois State Normal University, Normal, Illinois.

Dynamic Plane Geometry. David Skolnik, with the editorial assistance of Miles C. Hartley. New York, D. Van Nostrand Company, Inc., 1950. xii+289 pp., $2.56.

The authors define *dynamic* geometry as "a study of the parts of space and their relation to one another while they are in motion and changing." They combine in this text the best of the traditional course in geometry with much that is new. The purpose is not only to make the subject more meaningful and understandable to the student, but also to aid him in clear thinking in other life situations.

Part One develops the meaning of proof. The first chapters give very clear explanations and varied applications of fundamental assumptions and definitions, followed by a thorough development of the meaning of a theorem. Congruent triangles are introduced only after an extensive study of geometric concepts and methods. Part One concludes with a study of constructions and quadrilaterals.

Part Two, called "Patterns of Thinking," is devoted to the development of different types of dynamic thinking through the study of the circle, area, proportion, similar triangles, and functional relationships in the triangle. The appendix contains a section on inequalities, a summary of fundamental theories, tables, and lesson plans.

Nearly all of the pages of this attractive, well-bound book have double columns of large print. It contains almost one thousand excellently drawn figures and twelve hundred exercises which are grouped into three levels of difficulty. The unusual arrangement of subject matter, the great number of theorems postulated, and the emphasis given to the application of clear thinking in other fields of knowledge are outstanding features of this book. These characteristics, along with the carefully worked out and gradual approach to sound reasoning, make it a very usable and purposeful text for the beginner in geometry.—DOROTHY RIDGWAY, Central High School, Fort Wayne, Indiana.

College Algebra. Harry A. Bender. New York, Pitman Publishing Corporation, 1950. xi+451 pp., $3.50.

The author states in the preface that it is his purpose to write a textbook which is self-directive as far as possible and also one in which the techniques used and the materials presented will be such as to develop the student's power of analysis as well as his algebraic skills. A careful study of the book would seem to indicate that the author has succeeded in these purposes to a large extent.

All conventional topics are included with exercises of sufficient difficulty to develop the student's skills and to challenge his ability as well. Historical accounts which lend interest to the topics are included with the material to which they refer. Review material is adequate (sixty pages of fundamental algebraic processes).

Unusual features of the book are: (1) A short discussion of our number system, (2) A discussion of functions and their graphs with the concept of slope and simple derivatives included, (3) the use of determinants for the solution of systems of linear equations is introduced early, and (4) systems of equations involving quadratics are delayed until Chapter XIII.

The physical make up of the text is such that it is very legible.

In the opinion of the reviewer this is one of the better college algebra texts and deserves careful consideration by those persons teaching the subject.—HERBERT HANNON, Western Michigan College of Education, Kalamazoo, Michigan.

NEWS NOTES

A Geometry Teaching Institute, sponsored by The School of Education with the cooperation of the Department of Mathematics was held at the University of Michigan on Saturday, January 13, 1951. An audience and panel discussion included brief reports from Josephine Montague of Central Michigan College of Education, Dorothy Noyes of Ann Arbor High School, Clara Mueller of Cass Technical High School in Detroit, Howard F. Beatty of Saginaw High School and Harold Fawcett of Ohio State University. Discussion and laboratory groups met in the morning and afternoon and were led by Russell Schneider of Lansing Eastern High School, Donald Marshall of Dearborn High School, Norman Anning of the University of Michigan, Gertrude Pratt of Central Michigan College of Education, Kenneth Leisenring of the University of Michigan and Lauren Woodby of the University High School. The principal address was delivered in the afternoon by Professor Fawcett and was entitled "The Interplay of Induction and Deduction in the Teaching of Geometry."

The Fourth Conference of Mathematics Teachers at the University of Virginia will be held February 23 and 24, 1951. Principal speakers and Forum Leaders include Professor John R. Clark of Columbia University, Z. T. Kyle of the Virginia State Department of Education, H. W. Charlesworth of the National Council of Teachers of Mathematics and Veryl Schult of Washington. Round table discussions will be led by Louise D. Rice of the Andrew Lewis High School, W. L. Lord, of the Woodberry Forest School, and Theodore Gibson of Roanoke College.

Dr. Donovan A. Johnson of the University of Minnesota will conduct a Workshop for Teachers of Mathematics at the University of Colorado during the first term of the summer session of 1951, from June 18th to July 20th. This Workshop will provide an opportunity for mathematics teachers to specialize in areas in which they have individual needs and interests.

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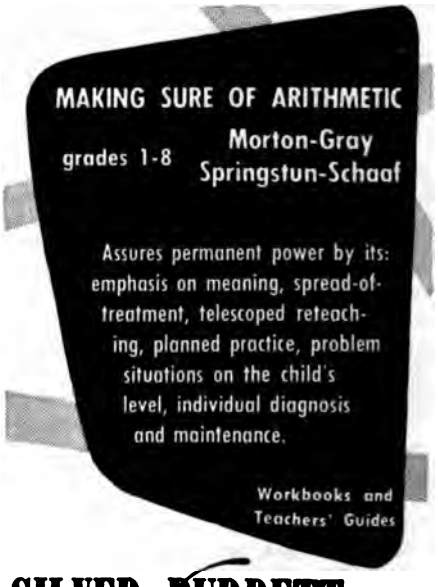
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THE MATHEMATICS TEACHER

Volume XLIV Number 3

What About the Structure of the Mathematics Curriculum?[1]

By F. LYNWOOD WREN

George Peabody College for Teachers, Nashville, Tennessee

SLIGHTLY more than three centuries ago the Latin Grammar School, the first secondary school in the United States, was established in Boston, Massachusetts. This school was organized primarily to prepare boys in Latin grammar and literature for entrance into college. Although mathematics found no place in this curriculum it later gained entrance under the guise of arithmetic and through the avenue of the writing schools. Once admitted, arithmetic continued to gain significance as an integral part of the curriculum.

From this one school with a few boys as pupils and one purpose in its curriculum the program of public secondary education has grown to include approximately 29,000 schools and 5,600,000 pupils of both sexes, not to mention the private schools with approximately one-half million boys and girls. Furthermore, in many quarters today the concept of secondary education has expanded to include the first two years of the college program. Concomitant with this great increase in size in the program of secondary education there has been a vast expansion of purpose. Modern educational problems are rooted in the context of mass education which seems to nurture extensive, and at times seemingly excessive, curriculum design. These problems will always remain complex; there is no way to simplify them. This complexity becomes even more aggravated when we extend our considerations to include the approximately 18,000,000 boys and girls enrolled in the public elementary schools of our country, and is only bounded in expanse when we restrict our thoughts to the contributions of one subject-matter field, such as mathematics, to a functional educational program. The only hope for an intelligent approach to the solution of these problems is through the co-operative effort of well-organized groups who are interested in the formulation and promotion of that program in our schools which will be most significant when evaluated in terms of the educational needs of a school population that is so highly heterogeneous as to interests and abilities.

In the early frontier days the need for mathematics and the motivation for its development were very slender. The

[1] An address delivered before the National Council of Teachers of Mathematics at its Ninth Summer Meeting at Denver, September 1, 1949.

farmer and the ordinary worker needed little beyond the ability to add and subtract. Those boys who aspired to enter some form of trade experience needed to know something of simple computation along with a slight knowledge of common measure, a few very simple fractions, and the use of English and other European monies. The sea-faring man needed to know the basic principles of navigation; the clergyman found astronomy necessary in fixing the dates of religious festivals; and the public official, when fixing territorial boundaries, found surest recourse in surveying. From this meager source flowed the shallow stream of mathematics in sixteenth-century America. Consequently, it is not surprising that mathematics received little attention in the early elementary schools. They were established for the primary purpose of teaching "writing and reading" with an occasional reference to "ciphering." In fact, one who could qualify as an "arithmeticker" was likely to be considered as especially endowed, although this sobriquet implied no more than that the individual was able to perform the simplest of computations.

The low degree of intellectual effort in these early days caused many of the leading statesmen of the period a great deal of concern. Some made distinct efforts to analyze the situation and to get at the causes. The early settlers were not interested in the works in literature and science produced by the masters of Europe. There was even opposition to public education "on the ground that it made boys lazy and dissatisfied with farm life, and led to religious skepticism."

It was not until the latter part of the nineteenth century that real progress began to take place in the development of mathematics in America. By this time there was a fair-sized group of native talent working in the field of mathematics, and there were major influences shaping up to promote significant development in mathematical research. As most important among these influences we might list: the vision of the presidents of certain outstanding universities and their appreciation of the need for promotion of interest in mathematics in this country; the founding of the American Mathematical Society; and a closer contact with European scholars. Many of the young men of that era, who were interested in mathematics, pursued advanced study and sought advanced degrees in mathematics, in European universities, in particular, those of Berlin, Göttingen, and Leipzig. Furthermore, many European mathematicians were imported to this country to teach in our universities, some temporarily and others on a permanent basis.

It was well that the leaders in the field of mathematics should have become so concerned with the promotion of mathematical research in this country. The research worker is the producer of mathematics. It is he who must stay out on the frontier in the expansion of the effectiveness, usefulness, and significance of the field of mathematics as an important aspect of our culture and social structure. The investments of effort and planning made by these early leaders have paid excellent dividends to the stockholders in American culture, for today the American corps of mathematicians occupy the vanguard of mathematical thought.

Thus, we see that during the course of approximately four centuries, vision and scholarly effort have produced a strong and impressive superstructure of enlightened research in the field of mathematics in our country. But what of the understructure of effective teaching and intelligent subject-matter planning at the elementary and secondary levels? Recently, in the city in which I live, crumbling cement was found on the supports of two important bridges across the Cumberland River. Although the superstructures of these bridges were still as strong and impressive as they had ever been, the city engineer immediately restricted travel

on both bridges and called in experts to help determine the full seriousness of the situation. If these experts are deserving of their titles, they will not be over-awed by the impressiveness of the superstructure, but they will be considerably concerned with the effectiveness of the understructure of each bridge. So it is in the field of mathematics. The events of recent years have created an international feeling of awe and admiration for the marvelous accomplishments of mathematical research, at the same time there have been many pointed expressions of concern over faulty teaching and deficient curricula in mathematics at the elementary and secondary levels. Who is the engineer to take charge, and who are the experts to be called in to determine the validity and seriousness of such charges? What should be done?

In 1903 the Central Association of Science and Mathematics Teachers was organized. One of its expressed purposes for organization was to promote better teaching of mathematics and the sciences. In 1915 the Mathematical Association of America was organized for the purpose of contributing to "the advancement of mathematical science by the stimulating of teaching." In 1920 the National Council of Teachers of Mathematics was organized to promote interest in mathematics, especially in the elementary and secondary fields. These three organizations have been instrumental in promoting constructive thought concerning better instruction in mathematics at the elementary, secondary, and collegiate levels of instruction. Furthermore, there are certain state groups and regional groups that have done some excellent work. The fact still remains that there is much to be done.

The history of mathematical development, with its important emphasis on research, prepares us to think at the college and university levels in the perspective of traditional topics and courses and a traditional way of teaching. Is it not likely that we have allowed such thoughts to exercise too strong an influence at the lower levels of instruction? The higher up the educational ladder we climb, the more selective becomes the educable clientele. In fact, at the university level it is desirable that our criteria of educability be shaped after the pattern of high selectivity. Not so at the lower levels! The American pattern of a democratic form of education for all American youth creates problems cut from cloth entirely different from that from which those at the university level are fashioned. At the university level the criteria for the adaptability of student and subject matter might well be summed up in one question. This question takes the form of a single criterion which emphasizes student adroitness. It is: "Is the student sufficiently mature in aptitude, interest, and preparation to enter into the study of a particular body of mathematical subject matter?" At the elementary level such criteria shape themselves into a somewhat converse question which takes the form of a single criterion emphasizing subject-matter adaptability. It is: "Is the subject matter adaptable in selection, organization, and method of presentation to the mathematical immaturity of the student?" In planning the content and teaching procedures of the mathematical program in our schools there should be a gradual transition from the emphasis and implications of the second question to those of the first. Throughout the elementary school and the junior high school the predominating emphasis should be on subject-matter adaptability. Beginning with the senior high school and extending probably through the student's first course in calculus there should be a balance of emphasis between subject-matter adaptability and student adroitness. In the subsequent years of college and university study the controlling criterion should be that of student maturity and adroitness.

The problems of the organization and presentation of mathematics at the ele-

mentary and secondary levels are closely interwoven with problems related to the training of teachers of mathematics. There does exist the problem of what should be done about the program of mathematics in the elementary and secondary schools before we can begin to formulate an intelligent program for the training of teachers of mathematics in these schools. A recent exchange of correspondence with an outstanding teacher of mathematics in one of the larger high schools of the middle west concerning some of the basic problems related to teacher training brought forth the following question from this teacher: "What kind of mathematics is a high school teacher supposed to teach?" This is a very pertinent question and might just as well have been: "What kind of mathematics is a high school teacher or an elementary school teacher supposed to teach?" A student in one of my classes recently posed this difficult question: "What are we supposed to do in the, Seventh Grade when we get pupils that do not know even the basic operational facts of arithmetic?" Is it stretching the imagination too far to suppose that, instead, this student might have asked these two questions: "What mathematical ability is a student supposed to have when he enters the Seventh Grade?" "Is there any real basis for an authoritative answer to this question?" Of course, we can refer the student to the 1923 report of the National Committee on Mathematical Requirements, or to the 1940 report of the Joint Commission. Each of these reports carry statements concerning the "mathematical equipment of the American pupil who has satisfactorily completed the work of the sixth grade." Furthermore, the Second Report of the Commission on Post War Plans, published in 1945, makes a similar declaration. These reports merit praise for the good influence they have exercised and continue to exercise in the interest of improved instruction in mathematics in *our schools. Would it* not be well, however, to subject to critical review their assumptions as to mathematical attainment and their bases for selection and organization of subject matter? Are they being validated through the test of classroom experience or is there evidence that the mathematical program in our schools is overcast by the shadow of traditional choice, grade-placement, and presentation of subject matter?

In its Second Report the Commission on Post War Plans presents a Check List which it uses to define functional competence in mathematics. This list contains twenty-eight items, and was later expanded in the Guidance Pamphlet to include twenty-nine. The Commission then makes this statement: "Although many of these ideas should be taught in earlier grades, this list of essentials for functional competence in mathematics should constitute the main goals of the work in grades 7 and 8. It is not to be inferred that all the essentials needed for functional competence in mathematics should be taught in grades seven and eight, nor that very many of these ideas can be completely taught to all pupils enrolled in these grades. However, grades 7 and 8 provide a golden opportunity for the mathematics teacher." Are these claims too ambitious? Can they be justified?

Who is to answer all these questions? Does not the principal responsibility for taking the lead in this great task lie in the hands of the Mathematical Association of America and the National Council of Teachers of Mathematics? These two organizations are national in nature and have declared openly in their articles of organization that they are interested in improved instruction in mathematics. The determination of intelligent answers to these questions, and many others that can and should be raised, is a herculean task that will require time, money, and effort. It will demand the maximum of co-operation on the part of these two organizations and should receive the sympathetic interest and active support

other groups with a vested interest
hematics. However, the great need
operation does not stop here. While
groups must take the lead, they
seek counsel from the benefits of
rative thinking with duly organized
ional groups whose responsibility
o study the entire school program
the points of view of curriculum
zation and instructional techniques
perspective of the modern demands
ss education. Some progress has
nade in this direction but, in view
t might be achieved, the co-opera-
the groups that should be jointly
ned over these problems is very
like the three men who met on the
corner one day. Tom and Sam had
ngaged in conversation for some
hen Bob approached. Bob recog-
Tom and called: "Hi, Tom! How're
tting along?" Tom replied: "Fine!
e you been?" He then motioned for
o join him and Sam. As Bob came
m shook his hand and asked him
estion: "Bob, do you know Sam?"
hich Bob replied: "Well, we've
d but we ain't shook." The above-
ned organizations have only "how-
in their efforts to co-operate in the
n of problems related to the edu-
l program in our schools. Why
ey not "shake" and get together
a table to discuss these important
ns, which should be considered as
n problems, and seek intelligent
s which would point toward the
g of a program of mathematics for
hools which all groups would be
to accept as authoritative and
to support actively and construc-
Maybe the Check List proposed
Commission on Post War Plans
serve as a starting point for the
ing of issues. Is the amount of time
an be allowed mathematics in the
d curriculum sufficient to reach the
rds set by this list? Are any of the
too difficult or too abstract for
understanding at the grade levels
ich they are proposed? These and
many other questions could be raised beneficially about the items of the Check-List and concerning other problems of grade-placement and organization of instructional materials.

We want the content of instruction to be such that it will lead to competent scholarship in mathematics. This scholarship will vary widely with different levels of learning, but the fundamental structure which gives it form is essentially the same for all levels of learning. This statement has just as significant implications for the shaping of the programs for the training of teachers of mathematics as it has for the construction of the mathematics curricula in our schools. Regardless of the level at which an individual might be engaged in teaching or the subject-matter content which constitutes his instructional medium, there are six major objectives which delineate very sharply his responsibilities in his efforts to direct the learning activities in which his pupils engage. They are: (1) Proficiency in fundamental skills; (2) Comprehension of basic concepts; (3) Appreciation of significant meanings; (4) Development of desirable attitudes; (5) Efficiency in making sound applications; and (6) Confidence in making intelligent and independent interpretations. Each pupil in school has a right to expect full opportunity for the maximum attainment of which he is capable in each of these six major objectives of learning. This expectancy on the part of the pupil defines a very specific responsibility on the part of the teacher. Each prospective teacher, in turn, has a right to expect that the training program will provide full opportunity for building up a backlog of information, appreciation, and competence in mathematics that will enable him to accept this responsibility confident that he is prepared to orient his subject properly in the perspective of the school curriculum and to deal with its properties with a feeling of security and assurance.

What is the subject matter content which gives substance to the instructional

program shaped around these six major objectives as its basic structure? The program for the training of teachers, as well as the program of instruction in our schools, finds its substance and significance in the answer to this question, for this answer will define a program for acquiring functional competence in mathematics at the various levels of instruction, and no teacher can hope to lead his students to a level of functional competence higher than that which he himself has attained. This is a basic point which, to my way of thinking, has been overlooked in previous efforts to set up a program for the preparation of teachers of mathematics. There have been several commendable efforts to construct such a program or to analyze the fundamental principles upon which such a program should be built. These studies used various methods of approach but, in the main, ended up at the same place, namely, the acceptance of the content substance of mathematics pretty much in its traditional form and then recommending a teacher-training program which consisted merely of an individualized arrangement of this traditionally organized material. This may be correct, but I wish to call attention to one very fundamental point. The evidence is rather strong that this acceptance was more nearly a tacit axiomatic acceptance than the result of thoughtful and intelligent analysis of the existent organization of mathematical subject matter in the prism of present-day demands. Why is it that we, who work in a field of thought which emphasizes the attitudes and procedures of critical thinking, should be inclined to postulate a pattern of subject-matter organization whose major cause for existence may be mere tradition? This question takes on more significance when we pause to realize that this tradition has its roots in an educational atmosphere which, at the very best, is only remotely akin to that in which our schools function today. Furthermore, its form has been fashioned largely by the counter ebb and flow of practical significance and vague abstraction. Ever since arithmetic, because of its useful techniques, was admitted to the curriculum of the early schools, these two forces have operated first to include mathematical concepts and techniques in the school program and then to modify them or exclude them entirely. There have been certain outstanding studies made of the curriculum in mathematics and the program for the training of teachers of mathematics. Among the most important of these studies are those which have been sponsored jointly or separately by the Association and the Council. There have been many changes in the mathematics program which have resulted from these studies. For example, the text which I used in 1908–09 as a high-school student of algebra did not have a single graph in it. Today graphs permeate the whole structure of algebra and arithmetic as well as other phases of mathematics. The text which I used in geometry started off with fourteen pages of definitions, postulates, and general statements, followed by Theorem I which presented the proof of the fact that all straight angles are equal. Truly an Iron Curtain of futility and "much ado about nothing" to an unsophisticated high-school junior. Today we find not only an intuitive and experimental treatment of geometry earlier in the mathematics program, but we find also that the formal study of geometry is approached through avenues which present more significant challenge and inspire a livelier interest on the part of the geometric tyro. Then there is the early use of numerical trigonometry and the recommendation for the double-track program in the early high-school years.

With the exception of the introduction of graphs and the suggestion of the double-track program, and maybe a few more such cases, all the recommendations for reform and modification of the mathematics program have been primarily within the framework of tradition. We

have seen some deletion of topics, some rearrangement of topics, the inclusion of a few new topics, and some variation in treatment. Permeating all this work there is evidence, however, of a preponderant influence of the postulates of traditional grade placement and traditional sequence. A good example of this fact is to be found in some of the considerations related to the double-track program. The literature is full of strong support for this program in mathematics, in fact there are urgent pleas for it. You will find these pleas are not for weaker standards in mathematics but for a different form of organization and presentation, one that recognizes that there are differences among people in interests and abilities. One of the major obstacles encountered in trying to give the non-sequential, non-college-preparatory track mathematical dignity is the inability to get the approval of fellow workers in the field of mathematics. This is true also at the college level. Courses that do not follow the traditional pattern of College Algebra, Trigonometry, Analytic Geometry, and the Calculus are seldom recognized for transfer value or credit. This happens frequently within the institution, or even the department, in which the course is offered. Why should this be true? What constitutes mathematical dignity? Traditional ancestry always constitutes a rather strong basis for a claim to dignity but this claim becomes very thin unless it is supported by intelligent activity and purposeful accomplishment in the present.

How do we *know* that the traditional sequence and the traditional treatment of subject matter in mathematics is the most significant possible in the perspective of problems of modern education? Maybe they are, but what authoritative evidence do we have for judging whether they are or not? When we attempt an answer to this question we must remember that with reference to any subject-matter area there are four categories into which people fall: (1) Those who are interested and have ability in the subject; (2) Those who are interested but have no ability; (3) Those who are not interested but have ability; and (4) Those who have neither interest nor ability. Furthermore, both interest and ability can range from absolute absence to very decided presence, so each of these four categories can be divided into several distinct sub-categories. In thinking about the mathematics program in our schools there is one group, a very small group, which probably should not cause us much concern; namely, those pupils who have absolutely no interest or ability in mathematics. We should feel concern for all other groups and should make sure that our thinking and planning are not prejudiced in favor of another comparatively small group; namely, those pupils who have very decided interest and ability in mathematics.

Three illustrations will suffice to give further point to the question just raised as to the significance of tradition in the arrangement and treatment of mathematical subject-matter. That these illustrations might be presented in their proper perspective, I wish to remind you that I am merely raising questions, not implying answers.

(1) At the arithmetic level of instruction attempts have been made to study grade placement of various concepts and skills. These attempts have been criticized rather severely, and there seems to be abundant reason for criticism. Nevertheless, they were efforts at an important problem and represented an approach that was willing to proceed on the assumption that maybe tradition can be in error. Why should not we who are interested in improved instruction in mathematics make some effort to study problems of this type in the same spirit of inquisitiveness? Why should not we be willing to proceed on the assumption that there might be stupidity in tradition as well as wisdom, and that it is our responsibility to winnow the chaff from the grain? It may be true that such experimentation has

brought about a "tragedy of postponement" in mathematical instruction in our schools. But how do we know, what authoritative evidence do we have for saying that one pattern of selection and organization of instructional materials is better than another? How do we *know*, for example, that the instructional emphasis on fractions should be centered in the elementary school rather than in high school, or even the college? Many of the aspects of fractions are quite abstract in nature. I have *not* been recommending that instruction in fractions be postponed until high school or college. I have been saying that there seems to be evidence that something is wrong in the present program of instruction in fractions and that we need to do something about it. It is in this context that I have raised the above questions. These same questions can be raised in connection with the grade and curriculum placement of all topics in mathematics.

(2) The literature has quite a bit to say on the pros and cons of the fusion of Plane and Solid Geometry. Careful analysis of these remarks will show that the proposals have been made and evaluated in the context of the traditional Euclidean sequence of geometrical subject matter. Maybe this is the correct organization for us to be following, but do we *know* that it is? There has been at least one interesting and significant departure from this pattern. Birkhoff and Beatley used a rather novel approach to the study of geometry in their text, *Basic Geometry*. Assuming the fundamental properties of the system of real numbers they were able to construct a two-dimensional geometry. with some incidental attention paid to intuitive three-dimensional geometry, which, to quote the authors, "although the reverse of Euclid's, is logically equivalent to it." Interestingly enough the authors make the claim that the differences between the two geometries "reflect the progress of mathematics since Euclid's time."

This work is still in the framework of the traditional separation of Plane and Solid Geometry. Maybe they should remain separated, but do we *know* whether it is possible or impossible, desirable or undesirable, from the point of view of functional learning to organize a course in geometry such that it would be an organization of materials which would present the important geometric concepts, techniques, theorems, and relationships in the more natural setting of the three-space in which we live along with their implications to the sub-spaces of one and two dimensions?

Furthermore, we might well raise the question as to whether the geometric content of the high-school mathematics curriculum is adapted to the mental and mathematical maturity of the high-school student. Euclidean geometry was formulated many centuries ago, and for mature men rather than teen-age boys and girls. Has it found its place in the high-school curriculum as a result of thoughtful educational design or accidental submergence in the fast-flowing stream of mathematical development? Questions such as these might also be asked concerning the algebraic and arithmetic content of the school program.

(3) Are we *sure* that the compartmentalized form of organization of mathematical subject matter is the most significant and efficient form of organization from the points of view of challenging instruction and effective learning? Maybe it is, but what authoritative evidence do we have for saying that it is? For example, what basis do we have for saying that it does make sense or does not make sense to say that there are at least three places in the mathematical instruction program where a general course in mathematics is desirable? Why does it not make sense to say that the beginning of the high school program (the ninth grade), the freshman year in college, and the first year in graduate school are places where the student should be given the chance to

see the significance of acquired information and skills in the perspective of the opportunities which lie ahead? A news item taken from the St. Louis *Star Times* of March 14, 1949, seems pertinent here.

> Albert Einstein, famous inventor, has poked a probing pen into education. . . . .
>
> Harking back into his teens, when he was a student at the Polytechnic Academy in Zurich, Switzerland, he writes:
>
> "In Zurich, I had excellent teachers . . . so that I really could have gotten a sound mathematical education. However, I worked most of the time in the physical laboratory, fascinated by the direct contact with experience. The balance of the time I used in the main in order to study at home.
>
> "The fact that I neglected mathematics to a certain extent had its cause not merely in my stronger interest in the natural sciences than in mathematics, but also in the following strange experience.
>
> "I saw that mathematics was split into numerous specialties, each of which could easily absorb the short lifetime granted to us. Consequently I saw myself in the position of Buridan's ass, which was unable to decide upon any specific bundle of hay. This was obviously due to the fact that my intuition was not strong enough in the field of mathematics in order to differentiate clearly the fundamentally important, that which is really basic, from the rest of the more or less dispensable erudition."

Would it not be well for the program of instruction to offer opportunities to make syntheses of previously acquired information as well as to have the opportunity to feel out the challenge of unexplored areas?

In the preceding remarks there have been no positive implications that tradition is bad and that the entire mathematical program in our schools is all wrong. Nor has there been any argument for "soft options or sagging standards." I have been making a plea for the need for setting up reasonable expectations of mathematical attainment in our democratic frame of education and within the specification of experimental evidence, and it is in that perspective that I have been trying merely to sound the caution that we should know why we are doing what we do and should be sure that we know where we are going rather than complacently following what might be merely the ruts of tradition. The spirit and intent of all remarks and questions raised have had their roots imbedded in these four pertinent questions: (1) Do we know where we are going? (2) Do we know where we want to go? (3) Do we know why we are doing what we do? (4) Do we know what we should be doing?

Is it without reason to raise the question as to the possibility that the entire mathematical program in our schools, along with the concomitant problems of the best program for the training of teachers of mathematics at each respective level of instruction, needs intelligent study by a co-operating group of educators composed of mathematicians and those representing the administrative officers and classroom teachers of our elementary schools, secondary schools, and colleges? The literature of recent years has been tragically eloquent in its appeal for such a study. The Association and Council, supported by the American Mathematical Society, should take the lead in promoting and directing this study. The study must be made in the perspective of the nature of our school population and the mathematical needs of the pupils which constitute this population. It should be a study projected on the basis of professional co-operation for the purpose of designing an educationally significant program in mathematics for our schools, a program patterned in the interest of children who are looking hopefully up the road of educational achievement rather than of specialists who are looking down this road from points safely beyond such pitfalls as indifference, conflicting interests, and learning difficulties.

Be Sure to Meet Your Friends and Make New Ones at
The Twenty-Ninth Annual Meeting of the National Council
Hotel William Penn, Pittsburgh, Pennsylvania, March 28-31, 1951

Concrete Representation of Geometric Progression
(With Illustrations from the Decimal and the Binary Number System)

By CATHERINE STERN
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ARITHMETIC is the science of numbers. It describes what happens if certain operations are carried out, and it prescribes through symbols and signs what is to be done in certain numerical situations. Laws and rules that govern arithmetic are expressed not in words but in symbolic language, like "$a+b=c$," or, if numerical values are substituted, $5+4=9$. Such an abstract statement may be applied to concrete objects which are counted if given as individual units or measured when presented in continuous magnitudes. Five and four apples are nine apples, and five plus four feet measure nine feet. The first statement may be verified by counting the apples, the second by measuring the feet with a yardstick.

A great many of us have been led to believe that verification of abstract arithmetical statements by actual operations with concrete objects is sufficient to make numbers and numerical operations "concrete" for children. We have overlooked the fact that applications in daily life which testify to the correctness of a statement do not necessarily enlighten us about the absolute facts and truths that lie behind the symbols and signs in the world of mathematics. As Dantzig (*Number, the Language of Science*) and Hogben (*Mathematics for the Millions*) have pointed out so clearly, numbers and signs constitute the arithmetical language and describe relationships and operations in a kind of shorthand. We must be able to visualize what the symbols describe; we must look behind them and through them to understand fully the *structure* of the arithmetical relationships with which we are concerned. To a true mathematician, the operations with figures are clearly visible and his figuring with symbols is concrete; he actually sees with his mental eyes how a series increases, what an equation indicates, and so on.

Structural Arithmetic is a new approach to the teaching of arithmetic with the goal of representing numbers by devices which have all the properties of abstract numbers. These devices allow pupils of all ages to carry out with their own hands and see with their own eyes what figures and signs tell them. Just as a teacher of French points to a chair when naming it *la chaise*, so the teacher of Structural Arithmetic may point to blocks or cubes or other devices and say: What you are seeing here is expressed by $5+4=9$, or, by $1+10+10^2+10^3$, or, by $1+2+2^2+2^3+2^4+2^5$, or, by carrying 1, etc.

In this article, I will attempt to show how the structure of the decimal system, the binary system, geometric progression, and the structural role of "carrying 1" can be shown with the proper devices of Structural Arithmetic. Let us see how a pupil may get first-hand experience with our number system, based on the powers of 10, and how he discovers the dramatic happenings which lie behind "carrying 1" if you will only unveil its secrets for him and go behind the strictly mechanical rules.

The abacus, which once again is enjoying a certain degree of popularity in schoolrooms, is sometimes used in an attempt to make the number system concrete. Demonstrations with an abacus show children that if certain mechanical rules of operation are followed, the correct answer comes out in the end, just as a piece of candy comes out of an automatic vending machine when a coin is put in the proper slot according to the directions. However, the general acceptance of the Hindu-Arabic system of notation four centuries ago has made it

possible to use figures in rows on paper in a more direct way than was ever possible with the abacus. For example, add 1 to 999:

$$\begin{array}{r} 999 \\ +\ \ 1 \\ \hline \end{array}$$

Here we see 9 in each row (we do not have to count beads as in the abacus), and if we apply the rule that ten 1's have to be exchanged by 1 ten, etc., it is much simpler to work with symbols than with beads. Unfortunately, children today are usually taught to use figures in adding just as mechanically as calculators move beads on the counting frame of the abacus. Listen to what a child says in solving the above example: "Nine and one are ten; put down zero, carry one. Nine and one are ten; put down zero, carry one. Nine and one are ten; put down zero, carry one." He writes the figures in the answer:

$$\begin{array}{r} 999 \\ +\ \ 1 \\ \hline 1000 \end{array}$$

He reads the result as "one thousand" just as the calculator records the total to be one thousand when he sees just one bead in the fourth row of the abacus.

What does the pupil *see?* Nothing but figures (or beads). What does he *think?* Only that he is not to forget to "carry 1." Instead of a fascinating number experience which provides pleasure and even thrill, the pupil stares blindly at his example (or at his beads in the counting frame), and he neither sees nor enjoys the experience that the operation has given him.

Let us study the same example with the materials of Structural Arithmetic. Our decimal system is built on the powers of ten. That means we have to deal with 10^0 or one, 10^1 or ten, 10^2 or one hundred, 10^3 or one thousand, and so on. In Structural Arithmetic, the pupil is given one unit, or cube, to stand for 10^0. Ten of these cubes equal a Ten-Block, which is ruled into 10 cubic units and thus seems constructed of ten cubes apparently "glued" together. This Ten-Block represents 10^1, or ten. There are 10 such Ten-Blocks that equal the next higher denomination, the Hundred-Square, which represents 10^2. It shows 10 tens fastened together, or 100 cubes. And 10 such Hundred-Squares put one on top of the other equal the Thousand-Cube that represents 10^3.

Any number between 1 and 1,999 may be constructed with these materials. The number 1,001 would show a Thousand-Cube and a Unit Cube. The number 1,350 would be constructed of 1 Thousand-Cube, 3 Hundred-Squares, 5 Ten-Blocks and no Unit Cubes. Let us construct the number 1,111. (See Figure 1.)

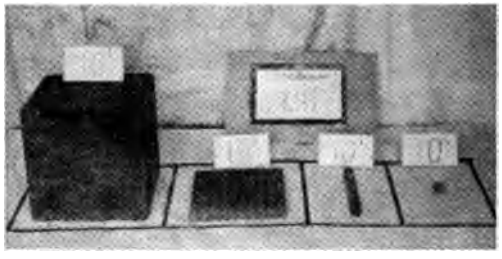

FIG. 1. The Number 1,111.

This is the number one thousand one hundred and eleven as built from 1 Thousand-Cube, 1 Hundred-Square, 1 Ten-Block and 1 Unit Cube. It is simplicity itself to show the pupil how each power of ten is expressed by a figure in a given *position*. One look at the materials shows the different value of each 1 in the number 1,111. It means a huge Thousand-Cube, a Hundred-Square that equals the bottom layer of the cube (being 1/10 of 1,000), a 10-row (or Ten-Block that is but 1/10 of the Hundred-Square), and, finally, a Unit Cube (that is 1/10 of the 10-row). And, inversely, we see that each figure is ten times as great as the figure to its right. The picture thus shows the first four terms of the geometric progression according to which the decimal number system is built:

$$1+10+10^2+10^3.$$

Now, let us construct the number 999.

(See Figure 2.) It is built up from 9 Hundred-Squares, 9 Ten-Blocks and 9 Unit Cubes.

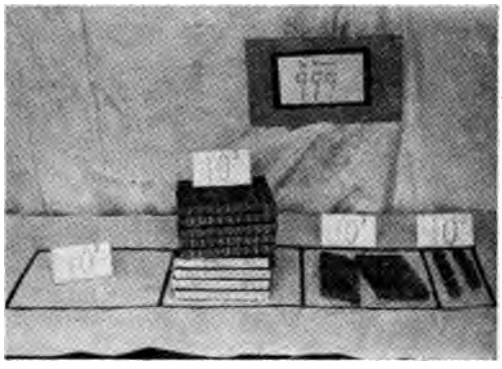

FIG. 2. The Number 999.

Add 1 cube, and the most dramatic happening ensues which only a movie can adequately show in operation. That 1 cube added to the 9 Unit Cubes makes 10 cubes. We may exchange 10 cubes for the equivalent Ten-Block which we actually carry to the next place. The 9 tens which are already there and this added Ten-Block form a Hundred-Square which is now substituted for the 10 Ten-Blocks. This Hundred-Square has to be carried to the next position, where it belongs. If we put this Hundred-Square on top of the pile of 9 Hundred-Squares, the figure of the Thousand-Cube emerges. Therefore, we carry again, this time the Thousand-Cube which we have substituted for the 10 Hundred-Squares. Thus, all the places up to the Thousand-Cube are emptied. (See Figure 3.)

When our pupils engage in this fascinating number experience, they often exclaim excitedly, "Carry! Carry! Carry! It gives the Thousand-Cube! It makes just 1,000!" The recording of this concrete operation is "999+1=1,000." If, later, the column form of recording is used, the carrying on paper is full of meaning. The children actually handle tens or hundreds or thousands just like units. However, they are fully conscious of how different the pieces are; form and size vary if a number is moved up the decimal scale, as 9, 90, 900. The numbers are not represented merely by 9 beads of the same size in different positions, but by 9 Unit Cubes, or 9 Ten-Blocks, or 9 Hundred-Squares. Do children understand the significance of these cubes and blocks of different sizes? Of course! Just try to give a child 1 candy in a different "position"—or wrapping—instead of 10 candies of the same size. Why should he exchange the 10 candies for only 1 just because of the different wrapping? Then, ask him whether he would rather take home 1 chocolate bar consisting of 10 squares or 10 separate squares of the same size. He sees clearly that each offer consists of the same amount of candy, but he will probably take the bar for convenient transportation. The exchange of 10 beads in Units place for 1 bead in Tens place is an adult convention which a child may learn to carry out obediently, but which does not make sense for him, since 1 bead and 10 beads of the same size will never look interchangeable.

FIG. 3. The Number 1,000.

With the materials of Structural Arithmetic, "borrowing" may be shown just as concretely as "carrying." The same vivid impression is gained if the Thousand-Cube is presented and the pupil is asked to take 1 from 1,000. A child often puts his finger on one unit of the Thousand-Cube, and says, "We can't break a cube off!" So, we exchange the Thousand-Cube for 10 Hundred-Squares, but, still we can't break one cube off. One Hundred-Square is replaced by 10 Ten-Blocks, but, again, no single cube can be taken away. Now, the Ten-Block is exchanged for 10

cubes. We can easily take 1 cube
he result, of course, is 9 Hundred-
es, 9 Ten-Blocks, and 9 Cubes, or
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us look at the geometric progression
represents the decimal number
m:

$$.0+10^2+10^3+10^4+10^5+10^6+\cdots.$$

this progression, we have shown the
our terms with 1 cube, 1 Ten-Block,
ndred-Square and 1 Thousand-Cube.
Fig. 1.) The sum of $1+10+10^2+10^3$
number 1,111 which we have shown
e. Must we stop here or could we
bly go beyond 10^3? At first sight, it
s impossible. How could we ever
ine a concrete representation of 10^4,
0^6—how could we render visible the
h, fifth, or sixth dimension!

order to solve this problem, we had
to visualize how we go from one
mination of 10 to the next higher one
mply multiplying by the base 10. If
ultiply 1 by 10, it means that we
allow the 1 to grow in length until the
full 10-row is reached. If we multiply 10
by 10, it means that we let the 10-row
grow in the second dimension until we
arrive at the 100-Square with the area
10×10. If we multiply once more by 10,
the 100-Square grows in the third dimen-
sion, i.e. into the Thousand-Cube with the
volume of $10\times10\times10$. We had but to go
on in this way: the next step is to handle
our Thousand-Cube as a "giant unit
cube" and take it ten times; the result is a
giant ten-row of Thousand-Cubes repre-
senting ten-thousand. Ten of these form a
giant Hundred-Square consisting of 100
Thousand-Cubes representing one hun-
dred-thousand; and, finally, ten layers of
these squares form another cube which the
children called the "Gigantic Cube"
because it contains a thousand Thousand-
Cubes. Its name is one million.

This concrete representation of 10^4 or
10,000, of 10^5 or 100,000, and of 10^6 or
1,000,000 is in accordance with the
nomenclature (ten-thousand for 10^4, one
hundred-thousand for 10^5) as well as with
the grouping of numbers within periods.
We talk about the "Unit Period" which
consists of 1's, 10's, and 100's of units.
We switch to the "Thousand Period"
which contains 1's, 10's, and 100's of
Thousands. We may go on to the "Million
Period" in which we find 1's, 10's, and
100's of millions. We remain in the first,
second, and third dimension, switching
simply to new units of measure. We have
the sequence of cube, row, square—cube,
row, square . . . ", and, consequently, we
are able to construct a visible image of
10^4, 10^5, 10^6, and so on.

Instead of a concrete representation of
10,000 or 100,000 or 1,000,000 in the deci-
mal system, we shall show these further
terms of the geometric progression con-
cretely by working with a number system
with a base smaller than ten. Any number
system which is constructed according to
the powers of its base (b) shows the struc-
ture of a geometric progression:

$$1+b+b^2+b^3+b^4+b^5+b^6+\cdots.$$

In place of ten, the *binary* system is built on the powers of two. Thus, b^6 would mean 2^6. Whereas b^6 in the decimal system must have the size of 1 million of our unit cubes, we need but 64 unit cubes to construct the number 1,000,000 in the binary system. Before showing the photograph of this small "gigantic cube," we have to study the notation in the binary number system.

The binary number system has a base of 2, and any number we may think of is recorded with just the two symbols 1 and 0. A 1 in units place means one unit, a 1 in the second place means 2, a 1 in the next place 2^2, and so on. If we look at the figures in the decimal system, each position characterizes a value ten times as much as the preceding one; in the binary system, it is twice as much. It is hard to talk about 100 in the binary system and to visualize that it means 2^2, or 4, if we still give "100" the name "hundred", which stands for ten tens and not for two twos. For clarity, we shall give the figures 10, 100, 1,000, etc., names which show that we are working with the binary system, or with 2, 2^2, 2^3, etc. Let us call "10" in the binary system "twin," instead of ten; 10^2, or "100," will be named "twindred," 10^3, or "1,000," will be one "twosand"; logically, then 10^4, or "10,000," must be called "twin twosand"; 10^5, or "100,000," "twindred twosand"; 10^6, or "1,000,000," one "twillion." We would go on with "twin twillion," "twindred twillion," etc.

In our decimal system, we find as many basic addition facts as there are possibilities to add the figures from 1 to 9 to one another and to each number itself. In the binary system, we have but to learn that "1+1=10," or, "one and one equal twin." The rest is done by "carrying." In both systems, one has to understand the "zero-facts," i.e., that any number to which zero is added equals itself. Here are a few addition examples in the binary system; first let us examine the series of "adding 1" which shows us how we count in the binary system:

Binary	Decimal
0+1= 1	1
1+1= 10	2
10+1= 11	3
11+1= 100	4
100+1= 101	5
101+1= 110	6
110+1= 111	7
111+1= 1,000	8
1,000+1= 1,001	9
1,001+1= 1,010	10
1,010+1= 1,011	11
1,011+1= 1,100	12
1,100+1= 1,101	13
1,101+1= 1,110	14
1,110+1= 1,111	15
1,111+1=10,000	16

Thus, the number series from 1 to 1[illegible] in the decimal system corresponds to th[illegible] numbers from 1 to 10,000 in the binar[illegible] system; counting 1, 2, 3, 4, 5, · · ·, 1[illegible] corresponds to: one, twin, twin-on[illegible] twindred, twindred and one, · · · twi[illegible] twosand. "Adding 2" in the binary syste[illegible] is adding 10 to any of the number[illegible] "adding 3" means adding 11, and so o[illegible]

The multiplication table in the binar[illegible] system contains the basic fact "1×1=1[illegible]" positional notation takes care of the w[illegible] if we have to multiply 11,110 by 111, [illegible] we must not forget how to carry if we a[illegible] the partial products. If we multiply b[illegible] 10, we multiply by the base. In the usua[illegible] terms, we "annex zeros"; in our terms w[illegible] change a number to the next higher de[illegible] nomination.

$$10\times 1 = 10 = 10^1$$
$$10\times 10 = 100 = 10^2$$
$$10\times 100 = 1000 = 10^3$$
$$10\times 1000 = 10000 = 10^4$$

Translated into the decimal system, "times 10" in the binary means *doubling*, since we are dealing with a base of 2. 10, or twin, is 2; 100, or twindred, is 4; 1,000 or twosand, is 8; 10,000, or twin twosand, is 16; 100,000 or twindred twosand, is 32; 1,000,000, or one twillion, is 64; and so on. The geometric progression which shows the structure of the binary system is

$10^0+10^1+10^2+10^3+10^4+10^5+\cdots$ or,

$$2^0+2^1+2^2+2^3+2^4+2^5+\cdots$$

in the decimal system.*

Let us construct this progression with the materials of Structural Arithmetic. We have the sequence of cube, row, square, cube, row, square, and so on. Look at the next photograph which shows this progression and represents the number 111,111 or twindred twin-one-twosand, twindred twin-one. We built it simply by putting 1 cube in first place; 2 cubes (in a row) in second place; 4 cubes in a square next; then a cube of 8; a 2-row of 8, or 16; a 2-square of 8, or 32. (See Fig. 4.)

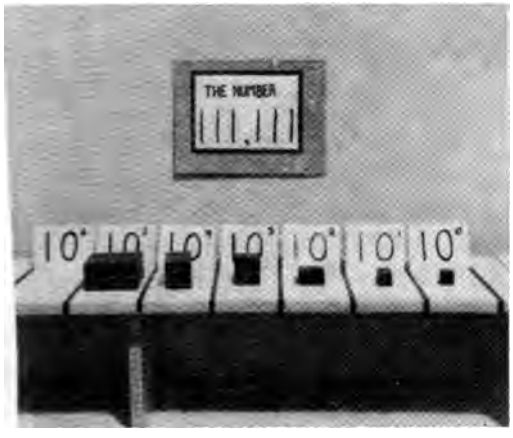

FIG. 4. The Number 111,111 in the Binary System.

Once more, we see how the "periods" repeat themselves, always using the cube as a new unit of measure. We have the "Ones Period," called here, ones, twins, twindreds. The "Twosand Period" follows with one twosand, twin twosand, twindred twosand. Then comes the "Twillion Period" with one twillion, twin twillion, twindred twillion, and so on.

We have seen how impressive an operation resulted when we "added 1" to the number 999 in the decimal system. As a parallel operation, let us "add 1" to the number 111,111 in the binary system.

* In order to show the parallel structure between the two systems, the indices are given in the familiar decimal notation even in the binary example.

How dramatic the scene is in action, yet so uninteresting when figured on paper without understanding.

$$\begin{array}{r} 111,111 \\ +\quad\quad 1 \\ \hline 1,000,000 \end{array}$$ Remember: $1+1=10$

Done mechanically, as our children are usually taught arithmetic, the figuring would follow this scheme: One and one are twin; write the zero, and carry 1. 1 plus 1 are 10; write 0, and carry 1, etc. The result is the number 1 twillion.

In Structural Arithmetic, we add 1 cube to the 1 cube already at that place. It makes 2, or twin, and has to be carried. That gives two twins—to be carried again to the 100's place. A Twindred-Square added to the Twindred already there gives two of the Twindred-Squares—which have to be carried as a second cube to the 1,000 place. We can't have *two* Twosand Cubes in the 1,000's place, so we have to carry again, and so on. Carry, carry, carry, until we have emptied all the places and have left only a 1,000 cube of 1,000 cubes, or 1,000,000, or 1 twillion! (See. Fig. 5.)

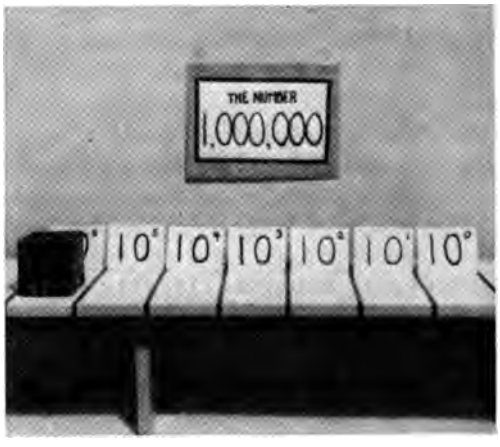

FIG. 5. The Number 1,000,000 in the Binary System.

These two parallel cases of "adding 1" to the number 999 in the decimal system or to the number 111,111 in the binary system, are instances of the same formula In any geometric progression with a base

(b), and whose first term is 1, we find the sum (S) of the first n terms as:

$$(1) \qquad S_n = \frac{b^n - 1}{b - 1}.$$

Let us multiply both sides of the equation by $(b-1)$:

$$(2) \qquad (b-1)S_n = b^n - 1.$$

Now add 1 to both sides of the equation:

$$(3) \qquad (b-1)S_n + 1 = b^n.$$

This last formula (3) states the following rule for all possible number systems of any base that are built as geometric progressions: If you sum up n members of the progression and multiply this sum (S_n) by $(b-1)$, the adding of "1 more" results in a total of b^n.

In the decimal system, the base is ten and the "base less one" $(b-1)$ equals 9. If we sum up the first three terms of the progression in the decimal system, we get

$$S_3 = 1 + 10 + 10^2$$
$$= 1 + 10 + 100$$
$$S_3 = 111.$$

Formula (3) for this case states:

$$9S_3 + 1 = 10^3$$

or,

$$(9 \times 111) + 1 = 1{,}000$$
$$999 + 1 = 1{,}000 \quad (\textit{Decimal System})$$

(See Fig. 2 and Fig. 3.)

If we substitute $b=2$ in the same formula, the "base less one" $(b-1)$ is 1. If we sum up the first six terms of the progression for the binary system, we arrive at

$$S_6 = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000$$
$$S_6 = 111{,}111$$

Formula (3) states for $(b-1) = 1$

$$S_6 + 1 = 10^6$$
$$111{,}111 + 1 = 1{,}000{,}000 \quad (\textit{Binary System})$$

(See Fig. 4 and Fig. 5.)

We have seen in the pictures an indication of the operation in which "adding 1" to 999 in the decimal system or to 111,111 in the binary system results in "carry, carry, carry . . . " until all the originally occupied places are swept clean. The total is always a number in the denomination which is just 1 *beyond* that of the last term of the progression: a thousand in the first example and a twillion in the second example.

We could work just as easily with a *quinary* number system and add 1 to a number like 4,444. The total must be 5^4, or 10,000 in the 5-system.

The "visible" representation of any number system makes arithmetical operations thrilling and unforgettable which, when figured out on paper, are often dull and hard to understand. Structural Arithmetic gives pupils an actual grasp of the meaning of operations which symbols and signs may accurately describe but also tend to conceal the true arithmetical characteristics. First-hand experiences unlock the secrets of mechanical rules, and such work as "carrying" on paper is done quickly and efficiently. For our children, it is the recording of a fascinating, understandable operation which they have carried out with their own hands.

Man has made his ten fingers the measure of "manyness" or plurality. He did not know then that such simple grouping of objects into tens and "tens of tens" was the first ingenius step to the invention of a number system based on the powers of ten. An unknown Hindu genius invented "positional notation" according to which the place of a number indicates its value, telling us whether we are dealing with 1's, 10's, 100's, 1,000's, and so on. This made a pure arithmetical step into a geometrical relationship: a number system built on the powers of its base forms a geometric progression. Man did not fully grasp what tremendous achievement is represented in his number system; and, unfortunately, it's unique beauty is not brought to life in today's classrooms.

Square Root

By FRED GRUENBERGER
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THE subject of square root cuts across the entire range of mathematics courses, from arithmetic to the calculus, and seems to cause trouble in every course.

Traditionally it is taught by a method stemming from the binomial theorem which involves remembering several rules, all quite mysterious to most students. While much has been written about simplifying this method and improved methods of teaching it, little or nothing has been done toward replacing it by a method fully as time-honored and much more intelligent.

In the usual algorithm, for each successive digit in the root, it is necessary to multiply by 20 the digits of the root found up to that point. This rule can be justified to an algebra class, and explained to a bright arithmetic student, but neither will remember it. Every teacher has had the experience of being asked to explain square root to someone who learned it at one time and has since forgotten it.

Ask the student who wants to learn, or relearn, square root what it is that he is looking for. Eventually he will furnish the definition: a number which can be multiplied by itself to produce the given number. This definition furnishes a better method.

Consider an example: the square root of 103,041 is desired. If we knew the answer exactly, then simple division would show divisor and quotient identical:

$$\begin{array}{r} 321 \\ \hline 321 \;\big|\; 103041 \end{array}$$

Therefore, to find the root, suppose we guess at it, using any information we have, and divide. We are free to borrow a bit from the traditional method; that is, starting from the decimal point we may separate this number into periods of two digits to ascertain that the root must have three digits (in this example) before the decimal point, and that it is clearly greater than 300, whose square is 90,000. Suppose that our first guess is 310. Simple division, to as many digits as our guess, gives:

$$\begin{array}{r} 332 \\ \hline 310 \;\big|\; 103041 \end{array}$$

and it should be clear that 310 is too small, since it "goes into" the dividend 332 times; similarly 332 is too large. The correct root must, then, lie between these two numbers and, lacking any better information, the second approximation should be half way between the two. Interpolation between 310 and 332 (half of their difference added to the smaller number) gives 321, which would be the second "guess" toward the root. Another such division will establish its accuracy.

This method was developed by Hero of Alexandria over 2000 years ago and is known as the Heroian algorithm. Hero showed, in fact, that if pq equals N, where p is not equal to q (if p equals q, then either is the desired root, of course), then any number d such that $p<d<q$ will lead directly to the square root of N, and in particular $d=\frac{1}{2}(p+q)$ is the simplest number with which to work. Hero worked entirely in fractions; our decimal notation is not only faster and more convenient, but the principle involved is much clearer.

How much must be remembered with this method? Just two things: the definition of what it is we seek, and the concept of interpolation. Obviously, the better the initial guess, the quicker the solution.

As another example; let us extract the square root of 10 by this method. As a first guess we can use 3; the first quotient

is then $3\frac{1}{3}$, and interpolation between 3 and $3\frac{1}{3}$ gives $3\frac{1}{6}$ (the first steps here can be done well in fractions, mentally). 10 divided by $3\frac{1}{6}$ equals 60/19, or 3.158. The root now lies between 3.167 and 3.158. The interpolation is

$$\begin{array}{r} 3.167 \\ 3.158 \\ \hline 2 \;|\; .009 \\ \hline .0045 \end{array}$$

and 3.158 plus .0045 equals 3.1625, which is correct to four significant digits.

The algorithm is especially adapted to machine computation (it is, in fact, the method which the makers of desk calculators publish for the extraction of any square root.) Consider its ease and speed for the computation of the square root of 5, using a desk machine. The first steps may be mental. We try 2 as a first approximation; 5/2 equals 2.5, and interpolation between 2 and 2.5 gives 2.25. Now, using the machine, 5/2.25 equals 2.222 and interpolation gives us 2.236, which is correct to four significant digits in one machine operation. If, now, we divide again, using 2.236 as our divisor, the quotient is 2.23613595, and interpolation gives 2.2360680. This result is correct to *eight* significant digits. One more such division with a standard desk calculator would yield, after interpolation, a result correct beyond the capacity of the machine. The entire operation described takes, with practice, only a few seconds.

Provided the initial guess is not too wild, this algorithm is fully as fast as the method usually taught in our schools. Indeed, we can show that the relative error, after interpolation, is generally less than the square of the relative error in the larger of p and q. This effectively doubles the number of significant digits which repeat in divisor and quotient. For example, if 1.414 be used as a first approximation to the square root of 2, the first quotient will be 1.41442715. Since 4 digits repeat in divisor and quotient, it is proper to interpolate 4 digits more, giving 1.4142135, which is correct to eight significant digits.

The method is accurate, being an iterative process; in fact, it automatically corrects errors, since an error in division means only that the next divisor is not as accurate as it might be. It is intelligible; there is nothing mysterious to be explained or justified. It is intelligent; it can be recalled after long periods of time. It is of particular value when it is desired to extend a known root to more decimal places. For example, if the square root of 5 is needed to more places than are given in a table (2.236), then this tabular value is the first approximation and the computation immediately extends the root without these first digits being re-computed; in general, the number of significant digits will be doubled.

Note that the algorithm offers no difficulty to the student in the extraction of roots of pure decimals. The fact that the root of a fraction is greater than the fraction itself usually causes difficulty. If the first approximation, using Hero's method, should overlook this fact, the method will quickly reveal the error.

It should be pointed out to students that there are many methods of extracting square root. Besides slide rule and logarithms, Horner's method of successive approximation can be applied to the equation $x^2 = N$. Or, it is possible to interpolate directly in a table of roots. For example, using a table of roots of numbers from 1 to 1000, the square root of 234.6 can be found by interpolating between the values given for 234 and 235 and adjusting the decimal point. For numbers which are close to unity, the square root may be found by halving the decimal part of the number; e.g., the square root of 1.00064 is 1.00032 and the square root of .999964 is .99982. The binomial series will give us square roots (particularly those of numbers lying near to a perfect square, such as 26). The method of subtraction of odd numbers is

also widely used on desk calculators.

The method outlined in this paper is usually credited to Newton, since it is, in fact, a special case of Newton's approximation method for the solution of equations. The method readily extends to higher roots, the n^{th} approximation to any root being given by the formula:

$$A_n = \frac{1}{i}\left[N/A_{n-1}^{i-1} + A_{n-1}(i-1)\right]$$

where i is the desired root and N is the number whose root is sought.

The advent of high speed automatic computing machinery makes this method of especial interest, since it is the method which these machines use. The first guess is usually taken as 3 (or 30, or 300, etc., depending on the position of the decimal point) arbitrarily, and the machine iterates until two successive trials agree to the number of digits desired.

The "Linear" Equation

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IN THE equation,

(1) $3x+y=6,$

if

$x=-1,\ 1,\ 4,$

$y=9,\ 3,\ -6.$

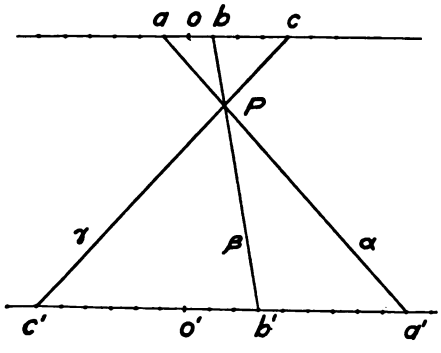

FIG. 1.

In Fig. 1 the line segment from the origin to a, which is identified with a, represents the value $x=-1$. a' marks the corresponding value $y=9$. In like manner b, b' and c, c' picture, respectively, the paired values $x=1, y=3$ and $x=4, y=-6$.

If the origins of the two scales are superimposed and the y-axis is rotated 90° we obtain Fig. 2 below.

In Fig. 2, $a(x=-1)$ and $a'(y=9)$ determine a point A* as the intersection of a horizontal through a' and a vertical through a. Thus,

$$A = a,\ a',$$

in the sense that the simultaneous values $x=-1$, $y=9$, pictured by a and a' are

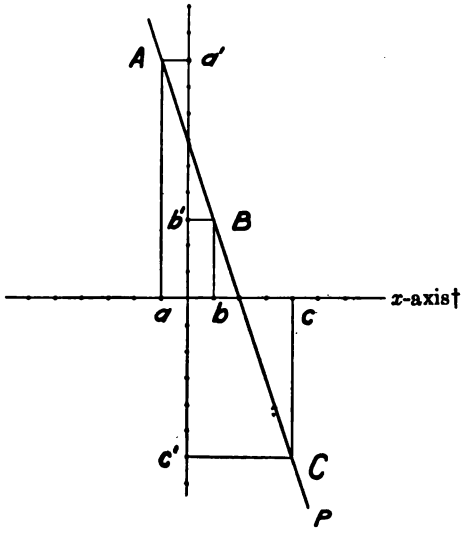

FIG. 2.

* $x=-1$, is called the abscissa (ab = away, scissa = cut) of A because it is the segment "cut off" the x-axis by the vertical line through A. Thinking of $y=9$, as the segment aA we refer to it as the ordinate (*order*, rank, elevation) of A.

† In the perpendicular arrangement the number scales are referred to as axes. $+y$ values which are to the right of O' in Fig. 1 are *above* the common origin in Fig. 2.

concentrated in the single point A. B and C serve similarly in place of the pairs b, b' and c, c'. In a dual manner we can think of the point pairs a, a', b, b', c, c', in Fig. 1, as equivalent to the respective connecting lines α, β, γ.

In Fig. 2 the points A, B, C, lie upon a line p and in Fig. 1 the lines α, β, γ, intersect in a point P. Moreover any x and y values which satisfy equation (1) simultaneously are coordinates of a point which is on the line p. Conversely, any point on p has such coordinates.‡ The line p is thus the geometric image or graph of equation (1). Since similar statements hold for the point P we can with equal right refer to it as the "graph" of equation (1).

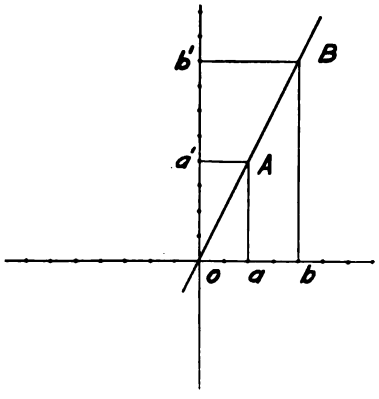

FIG. 3.

That an equation of the form,

$$ax+by=c,$$

has a straight line (or point) graph is a surprising and possibly intriguing§ discovery but at the expense of a clear understanding. Suppose, however, that we begin with a very simple equation such as,

$$y=2x.$$

In Fig. 3 the 2 to 1 ratio of the ordinates aA, bB, to the abscissas Oa, Ob, makes it obvious that the points A, B, C, lie on a straight line through the origin whose slope is 2.

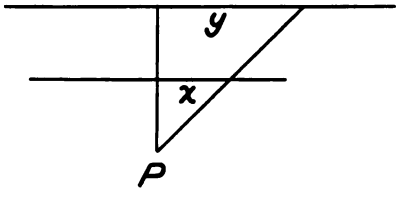

FIG. 4.

In a dual manner lines drawn through a point P (Fig. 4) whose distances from the y and x scales are in a 2 to 1 ratio, evidently cut off corresponding y and x segments which are in the same ratio. In either Fig. 3 or Fig. 4 the transition to $y=mx$, offers little difficulty after working with numerical instances of the slope constant m.

In the general case,

$$y=mx+b, \tag{3}$$

it is easy to see in Fig. 3 that the effect of the b is simply to move the graph b units measured along the y-axis. Writing equation (3) in the form,

$$y=m(x+b/m), \tag{4}$$

we see that in Fig. 4 the effect of the b term is to shift the x origin a distance b/m.

If the y term is present ($b\neq 0$) equation (2) can always be solved for y as in equation (3). If the x term is also present ($a\neq 0$, $b\neq 0$) we can solve for y as in equation (4).

‡ The formal proof of this comes much later, if at all.

§ Recalls the response of a Junior High miss, who was asked how she liked algebra. "Fine, they do such funny things with funny numbers."

Contributions of the Activity Movement to the Teaching of Arithmetic

By LOIS KNOWLES
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HAS THE activity movement which had its beginning some fifty years ago and which has been discussed, tried out, and evaluated under many different conditions made any impression on the practices of teaching arithmetic in our public schools? If progress has been made in techniques, materials, and the selection of content in arithmetic, is it due in any way to the philosophy underlying the activity movement? Finally, what are some of these changes?

A brief review of the program for the half-century may help in understanding the activity movement and evaluating its contributions. Dr. Dewey's work, begun at the University of Chicago in 1896, emphasized the "reconstruction or reorganization of meaningful experience by means of creative intelligence applied in purposeful activity." The Laboratory School at the University of Missouri in which I work was organized by Dr. Meriam in 1904 on this philosophy. The four types of activities which were developed in this school were observation, play, stories, and handwork. I believe that arithmetic was purely incidental in the early program.

Much experimental work has been done since 1900, and we owe a great deal to the progressive men and women such as Dr. Dewey, Dr. Meriam, Dr. Kilpatrick, Dr. Jersild and others who have helped us to focus attention on children and the ways in which they may be expected to grow, behave, and learn in every experience, including arithmetic.

A study of practice in the activity schools of the past leads one to believe that perhaps arithmetic was subordinated to other fields of learning. The phrase "incidental theory of arithmetic teaching" crept into the terminology of the activity movement, and received a great deal of condemnation through the years. During the early period I believe that arithmetic was not talked about as much as the content of some other areas, particularly the language arts and the fine arts, in which the emphasis was placed not on conventional subject matter, but on freedom, spontaneity, and creative expression. Without a doubt these factors had an effect on arithmetic, but perhaps in rather an indirect way.

The so-called activity school has received much adverse criticism, some of it quite justly. On the other hand the emphasis which the activity movement has given to the ways in which the child reacts to his environment, the way he behaves when confronted with new situations, and the teacher's part in helping him to decide on a course of action for which he is willing to be responsible—these emphases have contributed a great deal to the practices which are used in the elementary school of today. The teacher with this philosophy uses discussion, visual aids, excursions, etc., and selects from the variety of teaching materials those which help the child to make generalizations, to set up a method of thinking, to define for himself the problems which confront him and to find satisfactory ways of solving these problems. In a school organized on this philosophy the pupil sees meaning in his activity, he has a sense of its value for him, and he has to this extent enlarged and gained control over his immediate world.

This description of a modified activity school—the child-centered school of today—makes one distinctly conscious of the fact that it is quite different from the first purely activity school. From a beginning in which the center was on child-interest

alone, there has been a re-focusing to include the results of research to improve learning: an integration of the child's interests, the best materials available, and the very best techniques of presentation and evaluation.

The activity movement and the development of the pupil-center-of-interest type of school, has been paralleled and identified with the study of children, their interests, their growth patterns, the ability of children at different stages of growth, variations of ability in different individuals, etc. This effort to approach the goals of education from the inside, rather than from the outside, of the child has been one of the major accomplishments in elementary education. Child-study, the findings of psychologists, and the experimental programs for improving instruction have resulted in the principles and practices which most elementary teachers know and respect as readiness, maturation, individual differences, development of personality, integration, democratic techniques, group consciousness and the like. For the past ten years the literature in elementary education has been full of statements to the effect that we are teaching children not subject matter.

If the schools of the present and of the future are to make the best use of these results, the future of arithmetic should be commensurate with the future of any learning activity at the elementary level. I shall mention some of the aspects of arithmetic teaching which have been clarified by practices in recent years.

We may think of arithmetic as one of the areas in which we help children gain control over their environment. When a child has learned to use the clock to tell time, he can adjust his activities so that he can make the bus, join his friends at a certain time, get to school, and otherwise regulate his immediate life. This quantitative area extends and enlarges as the child grows and develops, but his reason for learning, his way of learning it, and his attitude toward learning is basically the same in arithmetic as in anything else. The content of arithmetic is not thought of so much as subject-matter-to-be-learned as a field of learning in which the child is helped to enlarge his world. Mrs. Mossman, in her book *The Activity Concept* says, "If we ask what makes subject matter essential or fundamental we shall eventually agree that it is value in living for the one who learns it." A felt need on the part of the child results in a desire and satisfaction in learning.

It is also believed that children can assume a great deal of responsibility for making intelligent use of their abilities. This is just as true in arithmetic as in anything else. Those who have used the self-instruction methods which Dr. Wheat and others have advocated believe that the independence and self-direction which result are examples of much-to-be-desired democratic techniques.

The literature of arithmetic is full of suggestions on how to make arithmetic meaningful. The use of concrete materials and visual aids and the deliberate emphasis on directed functional learning are examples of these techniques. Giving a child the techniques of how to find out what he wants to know is just as important in arithmetic as in any other field. The child who learns by manipulative discovery and by forming generalizations, has at hand a technique of finding out what he wants to know. This is in contrast to the pupil who learned by rote memory and relied on memorization.

The activity movement has also given emphasis to the interrelationship of ideas in different areas of learning. The understanding of new ideas often requires the interpretation of a quantitative description. Relational thinking is demanded. A child looks up from his reading and asks, "It says here, 'a party of'—how many is a 'party'?" A quantitative interpretation is demanded. Ideas such as these are presented many times a day in any group of children.

An examination of arithmetic textbooks, manuals, and teaching aids shows that changes have taken place in commercial materials. Teachers are given much help in developing a meaningful approach to arithmetic. They are supplied with materials and with suggestions for making arithmetic functional.

In conclusion, it should be pointed out that the activity school has contributed a great deal to our present philosophy and practice in the elementary school. The traditional school has taken on many of the attributes of the activity movement, and the activity school has been modified by research and experimentation. This change, or evolution, in both types of schools is good to see. For as a result of these changes our children are deriving greater benefits from their school experiences.

Teaching Plane Geometry to Mentally, Physically, and Emotionally Handicapped Pupils*

By ALLENE ARCHER
Thomas Jefferson High School, Richmond, Virginia

THOMAS JEFFERSON HIGH SCHOOL in Richmond, Virginia, is located in a "better class" neighborhood. For the most part its pupils come from homes above average in culture, education, and income; and between 50% and 60% of its graduates enter college.

For some time the teachers of plane geometry have been concerned about the number of failures in their classes and have tried several plans in an effort to improve the percentage of promotions. The failures fall into three groups—the mentally, physically, and emotionally handicapped pupils. About 84% of these pupils have I.Q.'s below 110, about 48% below 100, and about 13% below 90. Some have been victims of polio, rheumatic fever, scarlet fever, and other diseases which leave an after effect. Some of them are crippled and find writing difficult and many lack muscular coordination. Many are from homes disrupted by divorce, death, or severe emotional problems. These pupils are extremely sensitive and their feelings are easily hurt. They need to be given time to work without strain and encouragement to keep on working when the going is hard. Above all, they need consideration, sympathy, and love.

Whenever a counselor, after consultation with the first year algebra teacher and the homeroom teacher, believes that a pupil should not take plane geometry, she tries to guide him into another course which she feels would be of more value to him. In spite of the advice of the guidance department, there are always some pupils, who are ambitious or have ambitious parents, who insist that they want to study plane geometry. For some years the principal and the mathematics teachers have wondered what to do about these pupils, for they sat in class learning little or no geometry and just bothering the other pupils and the teacher.

Some people say that these pupils should not take geometry; however, the Fifteenth Yearbook of the National Council of Teachers of Mathematics says,[1]

"Backward pupils should not be taught only arithmetic, through the mistaken belief that arithmetic is the simplest mathematics since it is the most elemen-

* Presented at the 28th Annual Meeting of the National Council of Teachers of Mathematics, Chicago, Illinois, April 15, 1950.

[1] *The Place of Mathematics in Secondary Education* (Fifteenth Yearbook of the National Council of Teachers of Mathematics [New York, 1940]), p. 142.

tary. Parts of algebra and geometry are simpler than parts of arithmetic; furthermore, they may be more useful and more broadening. Slow pupils should be taught the simple parts of different divisions of mathematics. In all the branches of mathematics that are taught there must be a preponderance of concrete material and experience, abstract parts being introduced slowly and with moderation. A variety of subject matter can be successfully taught if it is presented in concrete form, if it is properly motivated, and if time is given for its mastery."

In February, 1943, a plan was tried at Thomas Jefferson High School by which the pupils who did not "see the light" in plane geometry at the end of six weeks were put in a special class that started back at the beginning of the course and took three semesters to cover one year's work. It was designated Geometry A, B, and C to distinguish it from Geometry 1 and 2. This plan helped a few students, but was discontinued because it had the following disadvantages:

1. The pupils did not like it because it took an extra semester. They felt that they were receiving no credit for the last one—Geometry C.

2. The Geometry A and B classes had so many drop outs and failures that there were hardly enough pupils left for a class of Geometry C.

3. The pupils still did little or no homework because they did not know how to study.

4. The teachers, as well as the pupils, became discouraged and did not like to teach the class.

Since September, 1947, a new plan has been tried in place of Geometry A, B, and C. Instead of waiting for a pupil to fail geometry, the ones who are likely to have trouble with it are picked at the end of their course in first year algebra. The Iowa Geometry Aptitude Test and the New California Short-Form Test of Mental Maturity are given. The results of these tests and the pupil's past record in mathematics are considered in determining whether he should go into the special geometry class—Geometry K. (Incidentally K does not stand for anything in particular.)

In order to overcome some of the disadvantages of Geometry A, B, and C, the class runs two consecutive fifty-minute periods, with one period for teaching and class discussion and the other for supervised study as a usual procedure. This gives time for a more leisurely approach to the subject. Since the pupils have more time in class, less work has to be done at home; and by working under the guidance of the teacher, correct methods of attack and good study habits are established. Pupils are encouraged to work in a relaxed manner without emotional stress. Tests of normal length for one period are given, but the strain of taking tests is lessened because pupils may use both periods if they wish. The first two periods in the morning when the pupils are not tired has been found to be the best time for the class. It is very important for this type of class to run smoothly, for when the pupils are upset, learning is blocked. There is more rustle and noise in such a class than in a regular one; and the teacher must decide whether the disturbance is a normal one or whether there is trouble. These pupils simply cannot sit still for a long time, and they must be allowed some freedom to move around.

Instead of the traditional way of teaching plane geometry, the class is taught by the laboratory method. Each pupil is encouraged to have an experimental approach to the subject and to discover geometric principles for himself rather than having them imposed upon him by the textbook or the teacher. As far as possible each pupil has his own model and is led by a series of questions to discover what happens if certain conditions are true. The "if-then" relationship is established, for the pupil knows that what is fixed on his model tells him what is "given" and that what results as he moves some

pieces of the model tells him what he must prove. The pupil states his own definitions and propositions, crudely at first, but these statements are later polished by the class; however, the pupil understands the proposition even though he can not state it clearly, because he has discovered it for himself. Afterwards the pupil proves the proposition. In this way he learns the difference between the experimental approach—inductive reasoning, and the geometric proof—deductive reasoning.

The laboratory method was found to be more effective in teaching these pupils than the traditional method which had been used in Geometry A, B, and C. Again the Fifteenth Year book points out.[2]

"Slow pupils, like very young children, learn best through experiences with concrete things. They must handle, measure, count, draw, make models, construct graphs, go on expeditions, hear talks, see pictures of things, etc., until the quantitative or spatial characteristics or relationships to be taught are really understood. After that the teacher may lead very gradually to the abstract, returning frequently to the concrete to fix and illustrate the generalizations."

A pupil who is physically handicapped or one who lacks muscular coordination can manipulate a model more quickly than he can draw the figures. Such pupils go through a terrific struggle to draw figures which often are so distorted that they can not see the relationships that exist. Freed from their physical handicaps and allowed to work with correct figures on models, they find geometry is fun, and they do some unique work. A boy who was a polio cripple did a marvelous piece of work on the trisection of an angle.

Emotional instability can be as great a handicap as mental immaturity. Perhaps you will be surprised that a boy with an I.Q. of 140 was placed in this class. After studying geometry six weeks in a regular class, he asked if he could come in Geometry K. In a conference with him the teacher asked him what she could do to help him.

He replied, "Teach me to be fast. Teach me to be fast."

His counselor said that he was on the verge of a nervous breakdown because he had brooded so much over geometry. His grades were low because he could not finish a test before the bell rang. He would not do anything outside of school except sit with his geometry book open before him from the time he reached home until he went to bed. He would not play or eat and neglected his other lessons. It was the only subject that kept him off the honor roll the first month. He was really pitiful to see. Once he started manipulating the models with his hands, all the tenseness left him and he learned to relax. Geometry became a fascinating subject which before had been dark and mysterious. He was still slow about finishing a test, but the strain of taking tests was relieved by the double period. He became a well adjusted normal boy and not the mental case his brother had been five years earlier.

Let's see how the pupils liked the laboratory approach to plane geometry.

BILLY—"Figures on paper are confusing. One model 'is worth a 1000 words.' Statements are easier to remember if you have an experiment and a model."

ROY—"On tests if you think of the model, you can work the problem."

JERRY—"You can see by the model what can be true and what is not true."

JOHN—"The experiments help one over the tough spots without doing the work for him."

DON—"With a model you are able to see what the figure looks like; the different ways it can be."

CHARLES—"You can tear down the figure and put it together again."

BOB—"Experiments help you learn and remember statements."

[2] *Ibid.*, p. 138.

KNOW YOUR NATIONAL COUNCIL REPRESENTATIVES

By KENNETH E. BROWN
Chairman of State Representatives, Department of Mathematics, University of Tennessee, Knoxville, Tennessee

WEST VIRGINIA REPRESENTATIVE

MISS MAY L. WILT has just completed her 25th year in the Laboratory High School at West Virginia University. The years have not dimmed her enthusiasm for mathematics or her interest in boys and girls in the high school. Miss Wilt has contributed pertinent articles in mathematics education to THE MATHEMATICS TEACHER, *Fifth Yearbook of the National Council of Teachers of Mathematics*, and the *West Virginia State Journal of Education*. She is the chairman of a committee which is developing a teacher training program for West Virginia. During the last seven years the National Council membership in West Virginia has doubled and Miss Wilt expects it to more than double again in the next seven years.

MAY L. WILT

PENNSYLVANIA REPRESENTA

MISS JOY E. MAHACHEK, He Department of Mathematics, Sta ers College, Indiana, Pennsylv been the National Council Repre in Pennsylvania since 1947. Du period the membership has gr 400 to 550 members. She recent "Pennsylvania has the third larg bership of the states. We hop distance New York and Illino

Miss Mahachek is a native where she received the B.A. deg leaving Iowa, she has been awa the master's and doctor's deg has shown a great interest in ma education both as a supervisin and as a contributor to the Se Yearbook.

JOY E. MAHACHEK

THE PRESIDENT'S PAGE

WHAT IS the National Council doing about cooperating with other national organizations?

In our new status as a Department of the National Education Association, we are in an ideal position to cooperate with any of the thirty other departments, seventeen of which maintain headquarters offices in the NEA building. Also, we have available the services of the fifteen NEA Divisions, among which are the Division of Accounts, Audio-Visual Instructional Service, Business, Legislative and Federal Relations, Publications, Records, and Research. While these opportunities of cooperation are open to us and much is to be realized from the development of our relationships with these organizations, we do not wish to limit our cooperative efforts to these alone. There are other national organizations, not within the NEA family, with whom we have worked in the past and with whom we expect to continue to cooperate.

For years the National Council and the Mathematical Association of America have worked together on major projects. One of the most important contributions to the field of mathematics in recent times is the final report of the Joint Commission of the Association and the Council which appeared as the Fifteenth Yearbook of the National Council of Teachers of Mathematics under the title of "The Place of Mathematics in Secondary Education." Since the cooperative work carried on in the past by the Association and the Council has been most agreeable and productive, it should be continued.

Because of our desire and increased efforts to give all help possible to the elementary teacher of mathematics, for the past three years we have been sending representatives to the Conference of Leaders in Elementary Education, an annual conference arranged and conducted by the Office of Education of the Federal Security Agency. Three years ago, Martha Hildebrandt, Proviso Township High School, Maywood, Illinois, represented the Council at the conference. Two years ago, Agnes Herbert, Clifton Park Junior High School, Baltimore, Maryland, was our representative. This past year we sent two representatives, Miss Herbert again and Mary C. Rogers, Roosevelt Junior High School, Westfield, New Jersey. More than forty national organizations participated in the fourth annual conference held in Washington last May.

Our representatives at these conferences have advised continuance of our cooperation with The Leaders in Elementary Education. They feel that the National Council is not only making valuable contacts with these other participating national organizations, but, at the same time, we are placing ourselves in a position to be of service to the U. S. Office of Education in its efforts to help the teaching situation in the elementary schools. Our contacts and relationship with the Office of Education promise to be mutually helpful. Particularly, and at the moment, this office can be of help to us in publishing leaflets and pamphlets on teaching aids and distributing these to the teachers of mathematics.

Incidentally, our participation in these conferences has emphasized again the need for a specialist in mathematics in the Office of Education. There are specialists in foreign languages, science, health, etc., but none in mathematics. This has been a matter of concern to the Council and we are taking advantage of every opportunity to urge that a specialist in mathematics be placed in the Office of Education. We are advised that this will be done as soon as a sufficient appropriation is granted by Congress to make it possible, and we are told that interested organizations will be given an opportunity to nominate candidates. We are keeping in close touch with developments along this line.

H. W. CHARLESWORTH, *President*

AFFILIATED GROUPS AND THE FUTURE OF THE NATIONAL COUNCIL

JOHN R. MAYOR, *Chairman*
Committee on Affiliated Groups

THE National Council of Teachers of Mathematics with more than 7000 members is already one of the largest professional organizations of teachers in any subject matter area. If the Council is to carry out its function to improve mathematics teaching at all levels of instruction everywhere, it not only should have a much larger membership but it will also have to constantly improve its services and make sure that its members and mathematics teachers everywhere make use of these services.

The Affiliated Groups can play an exceedingly important role in promoting the growth of the Council and in making available to teachers the services of the Council. In so doing, they will, of course, also be serving their own best interests. As an organization becomes larger it becomes much more necessary for the organization to work through smaller Affiliated Groups both to maintain the strength of the organization and to insure that the organization keeps in close touch with the needs of its individual members and others whom its services should benefit.

A major purpose of the Delegate Assembly is to determine how the National Council can most effectively work through the Affiliated Groups. The Affiliated Groups can greatly assist in building up the membership of the Council. The Affiliated Groups can make certain that the services of the Council reach their members and other teachers in their areas. The Affiliated Groups can have a real responsibility in maintaining the services of the Council and in determining the kinds of services that the Council should offer. The Second Delegate Assembly this month in Pittsburgh should find means for the Affiliated Groups to meet these opportunities.

The Delegate Assembly is a forum which makes recommendations to the Board of Directors concerning activities and policies of the Council. It may be that as the Council grows in effectiveness and membership that the Delegate Assembly will be called upon to assume greater responsibilities in the constitutional organization of the Council. At present the Delegate Assembly is not representative of the membership of the Council in the true democratic sense, since many members of the Council are not also members of an Affiliated Group, and, of course, the members of the Affiliated Groups are by no means all members of the National Council. An optimistic estimate would be that 40% of the members of Affiliated Groups are also members of the National Council. But even though the Delegate Assembly has no "legislative" responsibilities in the National Council, it can be of major importance and influence in the total program of the National Council through its recommendations to the Board of Directors and the Business Meetings and through its planning for the Affiliated Groups and National Council relationships.

Too, it must be kept in mind that the Delegate Assembly represents only a small part of the relationships between the Affiliated Groups and the National Council. It is given special emphasis in this Report because the Delegate Assembly is the point of emphasis for this month in the total Affiliated Groups program.

Goal for 1951

While there are now Affiliated Groups with which teachers in 34 states and the

province of Ontario may be actively associated, there are still areas with no state or local mathematics teachers group affiliated with the National Council. This means that mathematics teachers in these areas do not have as great an opportunity for participation in the total program of the National Council as teachers in other cities and states and it means that the National Council cannot as effectively carry its services to these areas.

Examination of the list of officers of the Affiliated Groups in the December number of THE MATHEMATICS TEACHER will show which areas are not yet affiliated with the Council. A letter from you to a friend in one of these states may get the movement started. Let's all help!

The Louisiana-Mississippi Branch of the National Council and the Association of Teachers of Mathematics in New England are two of the most active Affiliated Groups. Teachers in these areas have found it advantageous to cross state lines for area organization. This may also be the best plan in other parts of the country.

Pennsylvania Organizes

A copy of a Constitution for the Pennsylvania Council of Teachers of Mathematics was mailed to nine Convention Districts in time for their fall meetings. Each District thus had the opportunity to discuss it and to elect a delegate to the state-wide convention at Harrisburg after Christmas. At this time the state organization was formally approved and plans were made for affiliation with the National Council. Pennsylvania teachers deserve special praise for the energy with which they have carried on state organization plans while also planning for the big 1951 Annual Meeting of the Council in Pittsburgh.

A First for California, Colorado, and Oklahoma City

The California Mathematics Council, the Colorado Council of Teachers of Mathematics, and the Oklahoma City Mathematics Council were the first three Affiliated Groups to complete their annual renewal of affiliation for 1950–51.

Coming soon after these renewals were the official forms and fees from the Indiana Council of Teachers of Mathematics, Iowa Association of Mathematics Teachers, Association of Mathematics Teachers of New Jersey, Mathematics Section of the Texas State Teachers Association.

Association of Mathematics Teachers of New York State, Spring Meeting, May 12, 1951, Syracuse University, Syracuse, N.Y.

Just a year ago an interested group of mathematics teachers in New York State began plans for an organization of mathematics teachers within the state. Their purpose in forming such an organization is to unify the study of problems in the teaching of mathematics. A preliminary meeting, December 2, 1950, in Syracuse, N. Y., was attended by representatives of 35 New York counties. There, plans were made for an all-day meeting in the spring of 1951.

The temporary chairman, Miss Alice Reeve, 37 Clinton Avenue, Rockville Centre, N. Y., announces that plans are complete for a meeting on Saturday, May 12, at Syracuse University, Syracuse, N. Y. There will be general speakers as well as sectional meetings for groups ranging from college teachers of mathematics to elementary school teachers. Every one interested is invited to attend. For details concerning the program and the organization write to Miss Alice Reeve at the above address.

At this spring meeting the suggested name of ASSOCIATION OF MATHEMATICS TEACHERS OF NEW YORK STATE will be voted upon. A constitution will be presented for consideration and a permanent slate of officers will be elected.

MEETINGS, INSTITUTES, CONFERENCES

Annual Summer Meeting, National Council of Teachers of Mathematics, St. Olaf College, Northfield, Minnesota, August 20–23, 1951. Outstanding teachers and nationally known leaders in the field of mathematics will conduct meetings and present demonstrations at all levels of mathematics. The meetings will include general sessions, sectional meetings, laboratory sessions, discussion periods, and study groups. These study groups will be under the direction of prominent teachers and leaders for a continuing three-day session and will have limited enrollments. The latest in audio-visual aids will be exhibited: films, filmstrips, and models. Anyone having a model that has proved useful in the teaching of mathematics is asked to exhibit the model at this conference. Write to Mr. Emil Berger, Monroe High School, St. Paul, Minnesota, for information if you wish to exhibit material. It is hoped that models from all sections of the United States and Canada will enhance the exhibit of teaching aids.

The meetings will take place on the St. Olaf College Campus, forty miles south of Minneapolis and St. Paul on Federal Highway 65 (State Highway 218). The Jefferson Bus Lines and the Rock Island and Milwaukee Railroads provide excellent transportation facilities for north and southbound travel and make good connections with east and westbound travel. College dormitories will provide adequate accommodations with charges of $1.50 per day per person in double rooms and $2.00 per day per person in single rooms. All linen and towels will be furnished by the dormitory. All rooms will be available from Sunday, August 19 through Wednesday night, August 22.

Meals may be obtained at the college cafeteria with an average cost of $1.30 for breakfast and lunch. Special plans have been made for each evening as follows: Monday a picnic ($1.00), Tuesday a Norwegian Dinner ($2.00), and Wednesday a banquet ($2.50).

Reservations for rooms and registration for study groups should be made as soon as possible. Send reservations or requests for information and registration forms to Professor C. S. Carlson, St. Olaf College, Northfield, Minnesota. A registration and reservation form will appear in the May issue of THE MATHEMATICS TEACHER.

Facilities for recreation on the campus include a gold course, swimming pool, and tennis courts. There will be an evening trip to the observatory at Carleton College, folk dancing, and excursions. Northfield is the "southern gateway" to the magnificent northern Minnesota resort area with its numerous lakes and the famed North Shore Drive along beautiful Lake Superior. Plan a vacation trip to the "Land of 10,000 Lakes," and invite the mathematics teachers in your school or in other schools to meet you and your friends in Northfield at the end of August.

University of California at Los Angeles, Institute for Teachers of Mathematics, July 9–20, 1951. This Institute is sponsored by the Department of Mathematics and will be directed by Professor W. W. Rankin of Duke University. Two lectures daily will be given by outstanding men from business, industry, and teaching. The following study groups will be conducted daily by leaders of recognized ability chosen from teachers in the various fields of mathematical instruction.

1. Aids in the teaching of geometry and algebra
2. The development and coordination of basic ideas in mathematics
3. Teaching for understanding in arithmetic
4. Teaching applications of mathematics as used in industry and other fields
5. Use of mathematical instruments
6. Modern trends in the organization of secondary curricula in mathematics

Certificates of attendance will be issued to those registered in the Institute. It will also be possible for registrants to earn two units of college credit by attending lectures and participating in a sufficient number of the study groups.

Application blanks for the Institute and further details may be obtained by writing to Clifford Bell, Mathematics Department, University of California, Los Angeles 24, California.

Second Annual Meeting, Michigan Conference of Mathematics Teachers, St. Mary's Camp (Battle Creek), Friday to Sunday, May 11–13, 1951. The program of the conference is being planned to provide stimulating professional activities for mathematics teachers at all grade levels from the elementary school to the junior college. Enough time will be set aside to allow participants to take advantage of the excellent recreational opportunities which the camp affords, and to become well acquainted with their colleagues from other schools. The following program has been announced by the program committee: Friday—1:00 to 6:00, Registration; 4:30–5:30, Coffee Hour; 6:45–7:45, Dinner; 8:00–8:30, Short general meeting, address of welcome and formal opening of the conference; 9:30–(?), "Get-Acquainted" Hour, games and conversation, with coffee and "snacks" at 10:00. Saturday—8:00–8:30, Breakfast; 9:00–9:30, General Meeting; 9:30–10:30, Meetings of discussion groups; 10:30–11:30, Meetings of discussion groups; 12:00–1:00, Luncheon; 2:30–3:30, Meetings of discussion groups; 3:30–4:30, Meetings of discussion groups; 4:30–6:00, Recreation period; 6:00–7:00, Dinner; 7:30–8:30, General meeting of the conference with Mr. Walter H. Carnahan of Purdue University speaking; 8:30–(?), Recreation period, with coffee to 10:00. Sunday—8:00–8:30, Breakfast; 9:00–10:00, Meetings of

discussion groups; 10:00–11:00, General meeting of the conference; 12:00–1:00, Dinner.

The program committee is planning a varied array of topics with competent leaders for the small discussion groups, and is arranging also to provide observation, instruction, and practice in constructing multi-sensory teaching aids.

The cost of attending the conference is very moderate. The total cost to each individual will probably be twelve dollars which will include dormitory sleeping accommodations for Friday and Saturday nights and all meals beginning with dinner Friday evening and ending with dinner Sunday noon. This charge also includes the nightly coffee and "snacks" and the registration fee of one dollar. The camp can accommodate only about 150 persons. Reservations should be made early and each must be accompanied by a two-dollar deposit, which will be deducted later from the total charge. Requests for reservations should be sent to one of the following persons: Donald Worth, Lincoln Junior High School, Kalamazoo, Michigan; or William H. Nault, W. K. Kellogg Junior High School, Battle Creek, Michigan.

The Association of Teachers of Mathematics in New England, Institute for Teachers of Mathematics, Connecticut College, New London, Connecticut, August 23–29, 1951. The Association is happy to announce early plans for its third seven-day summer Institute. The response to the 1949 Institute at Wellesley College and the 1950 Institute at Tufts College has been most enthusiastic.

The Institute affords teachers of mathematics opportunity to discuss their subject with other men and women in the field, renew contact with sources of inspiration and enthusiasm, strengthen their confidence in the methods they are using, and enjoy pleasant association in an atmosphere of congeniality and good fellowship. Teachers in elementary schools, secondary schools, and colleges will all find features of interest at their particular level. You are invited to attend and have a part in making the Institute a great success.

The program will include outstanding speakers on topics in pure and applied mathematics, discussion groups on teaching methods and curriculum trends under the leadership of recognized teachers, a mathematics laboratory and workshop where models and other visual aids are constructed, and exhibits of latest texts and a variety of teaching materials. There will be abundant opportunity for "shop-talk" and a program of recreational features is being arranged for your enjoyment.

Connecticut College is ideally situated for the Institute, both in location and in facilities. The historic whaling port of New London is almost exactly half way between New York and Boston, on U. S. Route 1 and served by the Shore Line of the New York, New Haven and Hartford Railway. The spacious campus is located on a hilltop, with views of Long Island Sound and the Thames River. The College Arboretum with its bridle paths and small lake, the nearby Ocean Beach Park, the miles of shore line and various state parks, all within easy driving distance, offer a wealth of recreational facilities. Teachers from a distance may combine a delightful New England vacation with a stimulating and enriching professional experience.

A tentative list of lectures and discussion groups follows. A pamphlet containing the complete program will be available in April.

Lectures

1. "The Quantitative Study of Interaction in Small Groups" Fred L. Strodtbeck, Assistant Professor of Sociology, Yale University, New Haven, Connecticut
2. "Cartesian Instead of Euclidean Geometry" Julia Wells Bower, Chairman, Department of Mathematics, Connecticut College, New London, Connecticut
3. "Models and Methods" Robert C. Yates, Professor of Mathematics, United States Military Academy, West Point, New York
4. "The Foundations of Arithmetic" H. W. Brinkmann, Professor of Mathematics, Swarthmore College, Swarthmore, Pennsylvania
5. "Map Projections" A. Day Bradley, Assistant Professor of Mathematics, Hunter College of the City of New York
6. "Graphics: The Neglected Mathematical World" John T. Rule, Head, Section of Graphics, Massachusetts Institute of Technology, Cambridge, Massachusetts
7. "Galaxies" (Illustrated) Harlow Shapley, Professor of Astronomy, Harvard University, Cambridge, Massachusetts
8. "Slides and Demonstrations in Connection with Geometry," H. von Baravalle, Professor of Mathematics, Adelphi College, Garden City, New York
9. Banquet Speaker: James R. Killian, President, Massachusetts Institute of Technology, Cambridge, Massachusetts

Discussion Groups

I. A. "The Junior-Senior High School Laboratory"
Leaders: Alice Archer, Thomas Jefferson High School, Richmond, Virginia, and Gladys Schaefer, Lane High School, Charlottesville, Virginia

B. How I Teach
1. Exposition
2. Proof
3. The Problem Method
4. The Exposition Method
Leaders to be announced

C. "Calculus for Teachers of Twelfth Grade Mathematics"
Leader: Edward M. Pease, Chairman, Department of Mathematics, Rhode Island State College, Kingston, Rhode Island

D. "Contributions to Enliven the Teaching of Mathematics in College"
Leader: H. von Baravalle, Professor of Mathematics, Adelphi College, Garden City, New York

II. A. "Arithmetic and Junior High School Mathematics"
Leader: Ben A. Sueltz, Professor of Mathematics, State Teachers College, Cortland, New York

B. "The Senior High School Laboratory"
Leader: George W. Kays, Instructor in Mathematics, State Teachers College, Montclair, New Jersey

C. "Modern Mathematics after the Tenth Grade"
Leader: Eugene P. Northrop, Chairman, College Mathematics Staff, and Associate Dean of the College, University of Chicago, Chicago, Illinois

III. A. "The Status of Mathematical Concepts and Skills at the End of the Sixth Grade"
Leader: Kenneth Fuller, Professor of Mathematics, Teachers College, New Britain, Connecticut

B. "Mathematics Clubs and Recreations"
Leader: E. H. C. Hildebrandt, Associate Professor of Mathematics, Northwestern University, Evanston, Illinois, and Editor of THE MATHEMATICS TEACHER

C. "The Correlation of Algebra and Geometry in Grades Ten and Eleven"
Leader: Virgil S. Mallory, Chairman, Department of Mathematics, State Teachers College, Montclair, New Jersey

D. "The Place of Postulates in College Freshman Courses"
Leader: Albert A. Bennett, Professor of Mathematics, Brown University, Providence, Rhode Island

The only fee for the Institute will be $10.00 for registration and a $1.50 laboratory charge for materials for those who join that group. Room and board in attractive modern college dormitories will be available at reasonable rates. To be sure of accommodations, reservations should be made before June 1. Commuters will also be welcome.

For any additional information, write to Charles H. Mergendahl, Newton High School, Newtonville, Massachusetts.

Ohio Council of Teachers of Mathematics, April 20–21, 1951. In November, 1950 a group of mathematics teachers met in Columbus, Ohio, to organize the Ohio Council of Teachers of Mathematics. The first general meeting will be held at the University High School, College of Education, Ohio State University, on April 20th and 21st. The topic is *Functional Mathematics.* Teachers of mathematics are cordially invited to attend. Notices and complete programs were sent the first of March to all on our mailing list. The secretary will supply others on request.

Temporary officers are: President, Herschel E. Grime, Cleveland; Vice-President, Harold L. Lee, Athens; Secretary, Ona Kraft, Cleveland; Treasurer, Oscar F. Schaaf, Columbus. H. C. Christofferson is chairman of the Program Committee, and Harold P. Fawcett of Arrangements.

The meetings begin with a dinner, April 20th at 6:30, with greetings by Dean Donald P. Cottrell, College of Education, and *Audio-Visual Aids in Teaching Mathematics*, by Edgar Dale, College of Education. At 10:00 A.M. on the 21st, *Mathematics as It Functions in Agriculture*, will be discussed by Chester S. Hutchinson, Assistant Dean, College of Agriculture. At 12:15, at a lunch and business meeting, a constitution will be adopted and officers elected. At 2:00, *Mathematics as It Functions in Industry*, will be presented by Charles E. MacQuigg, Dean, College of Engineering, and Salvatore M. Marco, Professor of Mechanical Engineering. Discussion groups are arranged for each session.

For further particulars address the secretary, Miss Ona Kraft, 11328 Euclid Avenue, Cleveland 6, Ohio.

Illinois State Normal University, Conference on the Teaching of Elementary and Secondary School Mathematics, Saturday, April 14, 1951, 9:00 A.M.–3:00 P.M. Speakers: Elementary Section, H. G. Wheat, West Virginia University; Secondary Section, E. H. C. Hildebrandt, Northwestern University. Sectional meetings will be held for both elementary and secondary teachers. For further information write T. E. Rine, Illinois State Normal University, Normal, Illinois.

Illinois Council of Teachers of Mathematics, Sectional Meeting, Saturday, April 28, 1951. This Conference is being held at Western Illinois State College and is sponsored jointly by the college and the Public Schools of Macomb. Principal speakers will be Miss Mary Potter, Supervisor of Mathematics, Racine, Wisconsin and Dr. H. Vernon Price of the University of Iowa. For further information write H. G. Ayre, Western Illinois State College, Macomb, Illinois.

North Western Wisconsin Mathematics Conference, April 7, 1951. This meeting will be held at the Eau Claire State Teachers College, with Dr. H. P. Fawcett of Ohio State University speaking on "Significant Problems in the Teaching of Geometry" and "The Interplay of Induction and Deduction in the Teaching of Geometry." Registration from 9:00–9:30 A.M. Registration fee: $1.00 (without luncheon), $1.50 (with luncheon). For further information write Lawrence Wahlstrom, State Teachers College, Eau Claire, Wisconsin.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

19. Solutions of the Quadratic Equation.

Our *Miscellanea 5. A Geometric Solution of the Quadratic Equation*[1] brought us several interesting pieces of mail.

Professor Carl B. Boyer of Brooklyn College wrote to call our attention to Euclid's geometrical solutions of the quadratic. These solutions are to be found in Book II, Propositions 5, 6, 11 and Book VI, 27, 28, 29 of Euclid's *Elements*. The best known of these is II, 11 which solves $x^2+ax-a^2=0$ (a regarded as positive). This is the algebraic form of the division of a line in "extreme and mean ratio" and hence is the "golden section" which yields the "golden ratio" named the "divine proportion" by Pacioli (1509) and utilized in recent discussions of "dynamic symmetry." This problem is discussed in many places. Perhaps the most recent is an article by Carnahan[2] which is titled *Geometric Solutions of Quadratic Equations*.

Problems 85, 86 of Euclid's *Data* are also geometric equivalents of the solution of quadratics.

At this point, it should be noted that John Wallis' third solution which we gave in *Miscellanea 5*, figure 5, page 280 is, essentially identical with that given by René Descartes in *La Géométrie. Livre Premier* to be found on page 302 of his *Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences* (Leyden: 1637). Also, it would take only a slight modification of Wallis' first method (our figure 3, page 279, *loc. cit.*) to make it identical with the solution given by Descartes on page 303 of his work.

A letter from Willian J. Hazard of the Department of Engineering Mathematics of the University of Colorado includes the following list of 18 ways to solve $ax^2+bx+c=0$ taken from an article which he published in January 1924 in the *Colorado Engineer:*

1. By factoring by inspection.
2. By factoring after a substitution, $z=ax$, which leads to $z^2+bz+ac=0$.
3. By factoring in pairs by splitting bx into two terms.
4. By completing the square when a is 1 and b is even.
5. By completing the square as usual after dividing through by a.
6. By completing the square by the Hindu method ("the pulverizer"), i.e. by multiplying through by $4a$ and adding b^2 to both sides.
7. By completing square as given, adding $b^2/4a$.
8. By the formula.
9. By trigonometric methods (see Wentworth-Smith, *Plane Trigonometry*).
10. By slide rule (see Joseph Lipka, *Graphical and Mechanical Computation*. John Wiley and Sons, Inc. [1918], p. 11 ff.)
11. By graphing for real roots. (All modern textbooks.)
12. By graph, extended for complex roots. (See: Howard F. Fehr, "Graphical Representation of Complex Roots," *Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of the National Council of Teachers of Mathematics* [1945] pp. 130–138. George A. Yanosik, "Graphical Solutions for Complex Roots of Quadratics, Cubics, and Quartics," *National Mathematics Magazine*, 17 [Jan. 1943], pp. 147–150.)

[1] THE MATHEMATICS TEACHER, vol. 43 (Oct. 1950), pp. 279–280.

[2] Walter H. Carnahan, "Geometric Solutions of Quadratic Equations," *School Science and Mathematics*, 47 (Nov. 1947), pp. 687–692.

13. Real roots by Lill circle. (d'Ocagne, *Calcul graphique et nomographie*, from which L. E. Dickson got his reference to it in his *Elementary Theory of Equations*.) (Also see J. W. A. Young's *Monographs on, Topics in Modern Mathematics* "Constructions with Ruler and Compasses.")
14. By extension of the Lill circle to include complex roots.
15. Using the graph of $y=x^2$ and $y=-bx-c$ to find real roots. (Lipka, *op. cit.* p. 26, modifies and extends this solution; Schultze, *Graphic Algebra;* Hamilton and Kettle, *Graphs and Imaginaries*.)
16. By extending (15) to include complex roots (Hamilton and Kettle, Schultze).
17. By use of a table of quarter squares. This is a practical method of handling an equation having large constants, as we already have the table in print (Jones' *Mathematical Tables*).
18. By use of "Form Factors."

Professor Hazard adds that methods 12, 14, 17, 18 are original with himself, and that 13, 14 and 17 will be discussed in his book *Algebra Notes* to be published soon. He also suggests a 19th method:

"Make a template of transparent plastic, very slightly smaller and parallel to the unit parabola $y=x^2$, so that the pencil or ruling pen will draw the true curve. Scratch co-ordinates on it for accurate placing, and for any equation move the template to a position parallel to the Y axis and with its vertex at the point

$$\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right).$$

While this can't be called a different 'method' of solution, I think it should be included as a very handy and practical 'variation of approach' to the graphic solution of the quadratic in one unknown."

We hope that Professor Hazard and others will extend our list and include expositions of some of the less obvious or less easily available procedures.

Before leaving this topic, however, we should mention the reprint sent us by Professor H. D. Larsen of "Solutions of the Quadratic Equation" which appeared in the Spring 1950 issue of *The Pentagon*. Professor Larsen, editor of the journal, writes that he has some copies of the article which he will sell for thirty-five cents each. The article was written by Raymond H. Gillespie as an Honors Thesis in Mathematics while he was an undergraduate at Albion College, Albion, Michigan where Professor Larsen is also Chairman of the Mathematics Department. This thesis includes an historical survey, early solutions (including Viete's solution via the substitution of $x=u+z$ in $x^2+ax+b=0$ and the equating of the coefficient of u to 0), derivations of the formula (including that via determinants by Euler and Bezout), methods of factoring, graphical solutions, determination of imaginary roots, and solutions by trigonometric methods. Although the tabulation of procedures is not, of course, absolutely complete and the historical survey ignores entirely recent discoveries in regard to Babylonian achievements, nevertheless it is well done and contains much interesting material which would enrich both classroom teaching and club programs.

20. Notes on Werner Todd's Trisection

Professor Norman Anning handed us some notes on *Miscellanea 4. Trisecting Any Angle* (THE MATHEMATICS TEACHER, vol. 43 (Oct. 1950), pp. 278–279), with the admonition to destroy his message and then to keep away from trisections as we promised earlier. We can no more destroy them without some mention than he could resist the temptation to give them to us.

The curve which Mr. Todd constructed is one half of the Trisecant of Delanges[3]

[3] Described by P. Delanges in *La trisegnante nuova curva* (Verona: 1783) according to Gino Loria writing in *Enciclopedia delle Mathematiche*

of which a cartesian equa-
$^2)(x^2+y^2)+4m^2=0$.
ıning further notes that
:appeared in 1943 in the
liam Joseph Frederick of
Aichigan, who constructed
a clever combination of
a T-square. The year
same curve was used for
r Harvey P. Sleeper (Har-
Eliot House). Sleeper's
to be found on the end-
ly printing of R. C. Yates,
Problem (1942) an excellent
his topic which is still
the author at the United
r Academy, West Point,

, *the Quadratic, and the*
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0, 1, or 2 solutions occur
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ase. This is desirable visu-

in the acute angled case
ıd b and angle A are given,
$=b \sin A$ is computed and
ıe solutions determined by
ith a. This is convenient since $b \sin A$ is simultaneously the natural first step in determining B via the Sine Law. If A is obtuse, $a>b$ is necessary and sufficient to guarantee a unique solution.

Another approach which may be used as an exercise or extra credit project for the pedagogical purpose of making an interesting display of the interconnections between this problem, the quadratic discriminant, and conditional inequalities is as follows.

As above consider a, b, and A to be given. Then $a^2=b^2+c^2-2bc \cos A$ may be considered as a quadratic equation in c. From this

$$c=b \cos A \pm \sqrt{b^2 \cos^2 A-b^2+a^2}.$$

Since negative or zero values for a, b, c have no geometric significance, we see that (1) if $\cos A$ is positive (i.e. if A is acute) there may be 0, 1, or 2 solutions while (2) if $\cos A$ is negative (i.e. if A is obtuse) there will be one and only one solution if and only if $b^2 \cos^2 A-b^2+a^2 > b^2 \cos^2 A$ which condition simplifies to merely the requirement that $a>b$.

In situation (1) there will be no solution if $b^2 \cos^2 A-b^2+a^2<0$. But $b^2 \cos^2 A-b^2 = -(b \sin A)^2$. Hence the original condition reduces to the requirement that $b \sin A>a$ or that $h_c>a$. From this it readily follows that there will be a single unique solution if $b \sin A=h_c=a$, and if $h_c<a$ there will be two solutions as long as $b \cos A>\sqrt{b^2 \cos^2 A-b^2+a^2}$. There will be only a single solution however if $h_c<a$ and $b \cos A \leqq \sqrt{b^2 \cos^2 A-b^2+a^2}$. This latter condition is easily seen to be equivalent to $b \leqq a$.

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plementi, volume II, part II, p. 394 ff. See also Heinrich *elle Ebene Kurven* (Leipzig:

Annual Mathematics Institute, Louisiana State University, June 24-30, 1951

study groups for elementary, junior high and senior high school teachers on igh school mathematics. In addition there will be study groups on enrichment slow learner. Leaders for these groups are outstanding specialists from various try. Also there are to be general sessions. Some of the best talent in the field of athematics and other fields of interest will speak at these sessions. A new feature ry where models will be demonstrated and made. Rooms and meals will be avail- us at reasonable rates. Everything possible is being done to insure that everyone time together and a profitable experience. Plan now to attend this second L.S.U. itute. A printed program will be available after March 1st. For further informa- on T. Karnes, Department of Mathematics, Louisiana State University, Baton ıa.

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, New York

THE PROBLEM OF THE LION IN THE WELL

IT IS proposed to treat under this head the problems dealing with the time needed to go a certain distance at a constant rate when this progress is hampered by a constant retrograde motion.

The problem seems to be of Hindu origin, and as will appear from the following examples, the situation was elaborated in various ways. In the work of Mahāvīra (*c.* 850), there are at least two examples of this problem:

"In the course of 3/7 of a day, a ship goes over 1/5 of a krosá in the ocean; being opposed by the wind she goes back during the same time 1/9 of a krosá. Give out, O you who have powerful arms in crossing over the ocean of numbers well, in what time the ship will have gone over 99 2/5 krosá?"

"A powerful unvanquished excellent black snake which is 32 hastas in length, enters into a hole at the rate of 7 1/2 angulas in 5/14 of a day; and in the course of 1/4 of a day, its tail grows by 2 3/4 of an angula. O ornament of arithmeticians, tell me by what time this serpent enters fully into the hole."

About three hundred and fifty years later, Fibonacci gave the problem this way:

"There is a lion in a well whose depth is 50 feet. He climbs 1/7 of a foot daily and slips back 1/9 of a foot. In how many days will he get out of the well?"

Fibonacci (1202) has a more complicated case in which two serpents are approaching each other from the top and bottom of a tower respectively. Each advances a certain distance each day and retreats a certain distance each day. It is asked how long it will be before they meet. *Borghi* (1484) applies the same idea to two brothers who are trying to pay off a debt. Each earns a certain amount a day and each has certain daily fixed expenses. When will the debt be paid?

A distinctly new form of the problem which appeared in the Middle Ages showed that the periods of progress and regression might alternate. It should be noted that this introduced a further complication for the progress of the last day is all gain. Fibonacci for example gives the case of a lion in a well who climbs up in the daytime and slips back at night. Similar problems were given by Italian and by German authors. An additional modification was to have two animals such as a hawk and a dove approaching each other from the top and the bottom of a tower. Calandri (1491) thought the problem worthy of a woodcut showing the serpent emerging from the well.

One of the most absurd variations occurs in Pacioli's *Summa* (1494):

"A mouse is at the top of a poplar tree 60 feet high and a cat is on the ground at the foot of the tree. The mouse descends 1/2 of a foot each day, and at night it turns back 1/6 of a foot. The cat climbs one foot a day and goes back 1/4 of a foot at night. The tree grows 1/4 of a foot each day between the cat and the mouse and shrinks 1/8 of a foot at night. In how many days will the cat reach the mouse, how many feet does the tree grow in this time, and how far does the cat climb?"

Buteo (1559) makes use of a situation which was characteristic of navigation in the Mediterranean:

"Two ships 20,000 stadia apart weighed anchor to set sail straight toward each other. It happened that the first one set

EVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

HIS section is being published as an avenue gh which teachers of mathematics can favorite learning aids. Readers are invited id in descriptions and drawings of devices they have found particularly helpful in teaching experience. Send all communica- concerning Devices for a Mathematics atory to Emil J. Berger, Monroe High l, St. Paul, Minnesota.

THE SPHEROMETER*

E radius of a sphere and the radius urvature of a spherical mirror or al lens can be found rather con- ntly by using a spherometer. De- tions of this instrument can be found tbooks on physics.

is the object of this article to suggest s for construction of a homemade rometer which high school students athematics will find both interesting challenging. The principle of opera- involved is about the same as that mmercial instruments. Properly com- d and calibrated this inexpensive in- nent will provide students with a nice ication of two important mathe- cal principles; these are: (1) that an e inscribed in a semi-circle is a right e; and (2) that the altitude on the otenuse of a right triangle is the mean portional between the segments of the otenuse.

onstruction materials needed include tandard one-pound coffee can, an harpened pencil, and a block of wood 3"×3". The device is assembled in following manner (See Figure 1.):

1) Nail the block of wood to the inside of the cover of the can.
2) Drill a hole at E through the center of the cover and through the block of wood. This hole should be just large enough to accommodate the pencil snugly.
3) Remove the bottom of the can with a can opener. An opener with a rotary blade such as the "swing-away" type will leave the circular edge nice and smooth.
4) Replace the cover on the can and push the pencil through the hole at E. The instrument is now complete except for calibration of the pencil.

An alternative method of making the device is to fasten the block of wood to the inside of the bottom of the can, drill a hole through the center of the bottom, invert the can, and insert the pencil from the outside through the bottom. With this method of construction the bottom of the can is not removed, and the cover is not

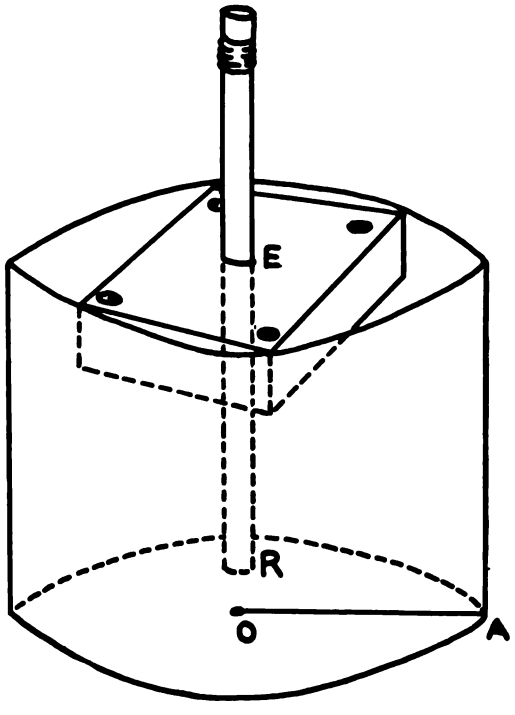


FIG. 1

* Plans for the construction of The Spherom- were developed in *The Mathematics* Labo- ry, *Monroe High School, St. Paul,* Minne-

needed at all. Advantages of the first method are that it is easier to fasten the block of wood to the cover than to the inside of the bottom, and also the rim retained on the bottom after removing the tin is sturdier than the thin edge at the top of the can.

In order to calibrate the pencil—that is, to put marks on it so that various radii of curvature may be read directly—it is necessary to take into account several facts about the instrument which are features of its construction. The open end, or base, is a circle whose radius is OA. See Figure 1. For the usual coffee can the length of this radius is approximately 2.5″. If the circular base is placed on a plane surface and the pencil pushed completely down, the end point R will coincide with O. This position of the pencil may be recorded by making a notch in it at the point E where it enters the top of the can. Subsequently this first notch can be used as a reference point for recording other positions of R. Attention is called to the fact that when the notch just made coincides with the top of the can, or what is the same thing, when R coincides with O, the radius of curvature is infinite.

With the foregoing facts in mind let us proceed to determine the radius of an actual sphere; and, for the sake of concreteness, let this sphere be a basketball. Place the circular base of the instrument on the ball, and apply enough downward pressure to the top of the pencil to insure that the point R will rest on the surface of the ball. Observation will reveal that the original notch on the pencil has now moved up to the position E'. See Figure 2. Thus OR, which is the distance of R above the plane of the circular base, can be found by measuring EE'. Having determined the two measurements OA ($=2.5''$) and OR ($=EE'$), the radius of the ball may readily be computed. Let RS be the diameter of the ball; then triangle RAS is a right triangle, and OA is the altitude upon the hypotenuse. Thus OA is the mean proportional between the ments OR and OS, and the radius of ball ($RS/2$) can be obtained from following relations:

$$\frac{OS}{OA}=\frac{OA}{OR}$$

$$OS=\frac{\overline{OA}^2}{OR}$$

$$RS=OS+OR$$

$$\frac{RS}{2}=\text{radius of the ball.}$$

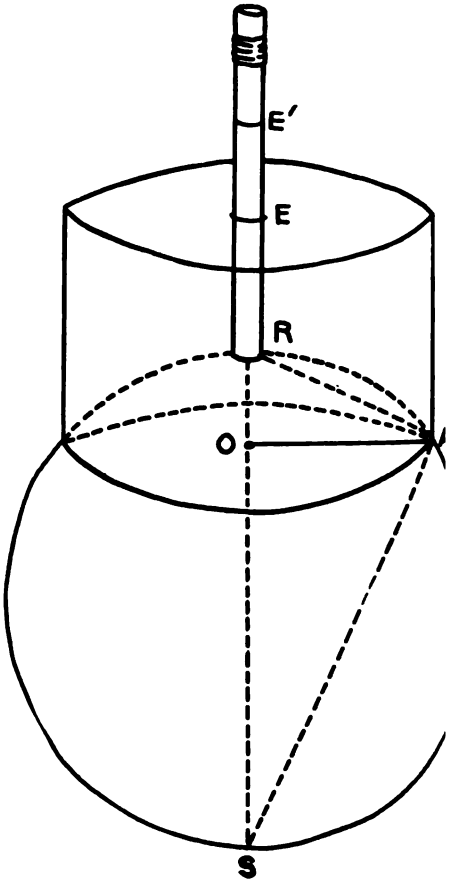


FIG. 2

By notching the pencil again at level of E, and keeping a record of length of EE' for the radius just compu one operation in the process of calibra will have been completed. By repeating above process with different sized sph

various radii of curvature may be computed and recorded. Actually, however, the calibration for a desirable range of radii may be completed by measuring EE' for different positions of R, and simply doing the computation. Once the device has been calibrated it may be used to find the radius of curvature of some particular spherical surface simply by noting which notch appears at E and reading the computed radius corresponding to this notch from the recorded data.

Two items of interest claim attention at this point. The first is that it is not possible with this device to determine the lengths of radii which are less than 2.5". The second is that while the discussion thus far has been concerned only with convex surfaces, the device can also be used to find radii of curvature of concave surfaces. To do so, however, the pencil must be calibrated in the direction above the original notch.

It hardly seems necessary to caution that measurements obtained with this device will be rather crude approximations; the materials and methods of construction employed are certainly not of the kind that should encourage much reliance on results if a high degree of accuracy is needed. The all important thing to remember is that the entire object of having students build a device of this kind is that their understanding of the principles involved will be enhanced.

A Device for Introducing the Number π

The usual approach to the introduction of the number π at the junior high school level places emphasis on two points: (1) that π is defined as the ratio of the circumference of any circle to its diameter; and (2) that the approximate value of the number is somewhere in the neighborhood of 3.1416 or $3\frac{1}{7}$. In early approaches the value $3\frac{1}{7}$ usually finds more favor among teachers. Undoubtedly every teacher of mathematics has some technique of his own for giving meaning to the ratio definition of π and also some handy intuitive approaches for showing that a close approximation to the value of the number is $3\frac{1}{7}$. The device suggested here for developing these two points of emphasis was called to the attention of this department by Miss Ruth Knaus of Waseca High School, Waseca, Minnesota. It is a clever little teaching aid and much more tidy than the usual ones.

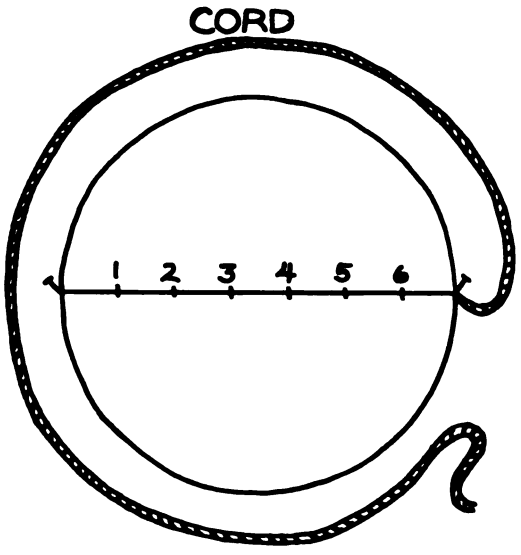


Fig. 3

Materials needed to construct the device include a piece of plywood 7" square, a piece of cord 22" long, and two right-angled curtain hanger hooks. See Figure 3. Cut a circle of diameter 7" from the plywood square, draw a diameter on its surface, and mark off seven one-inch divisions on it. Number the division marks along the diameter for easy reference. At the ends of the diameter screw the right-angled curtain hanger hooks into the sawed edge of the circle, and fasten the 22" cord to one of these hooks. If wound around the sawed edge of the circle, the cord should measure about one circumference. When wrapped around the two hooks at the ends of the diameter the cord will measure $3\frac{1}{7}$ times the diameter.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
School of Education, New York University, New York 3, N. Y.

THE APPEAL of this Department to the readers of THE MATHEMATICS TEACHER for materials and problems which would enrich, enliven, and motivate the classroom teaching did not go unheeded. The number of letters and replies from the readers has intrigued the secretary of the Mathematics Department of the New York University School of Education, MISS JOAN GEASA, so much that she has made a contribution of her own. Here it is.

Two parts of a cross.
A circle complete.
Two semi-circles, and
A line down to meet.
Angle, triangle, standing
On two feet.
Two semi-circles
And a circle complete.

Don't peek. The answer is given at the end of this column. This problem should be very intriguing in a second-term geometry class.

And now, here is a problem which was sent in by Miss PAULINE WILSON of James Hillman Junior High School, Youngstown, Ohio. She writes: "I wrote this problem about three years ago and have used it with my beginning algebra class. I have continued using it (although I am conscious of its flaws) because the response is always gratifying. The pupils (and even some of their other teachers) take an interest in the solution. The pupils' reactions to the little mystery are far different from those to the problems about when and where two cars will pass."

THE CASE OF THE MISSING LAUNDRY WHO STOLE MRS. MURPHY'S WASHING?

Mrs. Murphy was naturally very angry when she learned that her washing had been stolen from her clothes line while she was away. She asked Detective Curlock Foams to find out who the thief was. Upon investigating the movements of all who had been in the neighborhood about that time, Detective Foams discovered the following facts.

There was but one road between Waterloo and Daleville (24 miles due north). Mrs. Murphy lived on this road just 15 miles north of Waterloo. At 10:00 a.m. a bus left Waterloo and arrived at Daleville at 10:40 a.m. without making any other stops. The same bus made a return trip at the same speed, leaving Daleville at 10:50 a.m. Passengers on the northbound bus remembered seeing the laundry on the line as they passed. The driver and some passengers on the southbound bus were certain that there was no washing on the line when they passed Mrs. Murphy's home.

At 9:30 A.M. Claude Bennington was known to have left his home 13 miles north of Waterloo. He walked north, past Mrs. Murphy's home, arriving at a service station, 18 miles from Waterloo, at 11:10 A.M. (Curlock Foams counted on a man walking at an average of 3 miles per hour.)

Claude's brother, Ralph, left the same house thirty minutes later, and he also walked north. When he was 17 miles north of Waterloo he was picked up by George Pumpernickle and was driven by him into Daleville.

Mr. Pumpernickle had left his home (4 miles north of Waterloo) at 11:10 A.M. and had driven at an even speed of 40 miles per hour all the way, stopping only a moment to pick up Ralph. He was certain that Ralph was carrying no packages of any kind.

At 10:00 A.M. Alex Martin left his

home (11 miles north of Waterloo) and walked to a garage (16 miles north of Waterloo), arriving there at the same time as did George Monroe.

George Monroe had travelled on his bicycle at the rate of 6 miles per hour from his home (10 miles north of Waterloo), having left home at 10:40 A.M.

Curlock Foams sat down at his desk, drew some lines on a piece of graph paper, picked up his hat, and went out to arrest the guilty person.

WHOM DID HE ARREST?

The answer to this problem will be found in the May issue of THE MATHEMATICS TEACHER. Please do not write to Miss Wilson. However, this problem does not require any special abilities other than those which are acquired in the study of simple graphical methods employed in any ninth year algebra.

Professor JULIUS SUMNER MILLER of Dillard University, New Orleans, La. submitted several very intriguing problems. We shall publish two of them now.

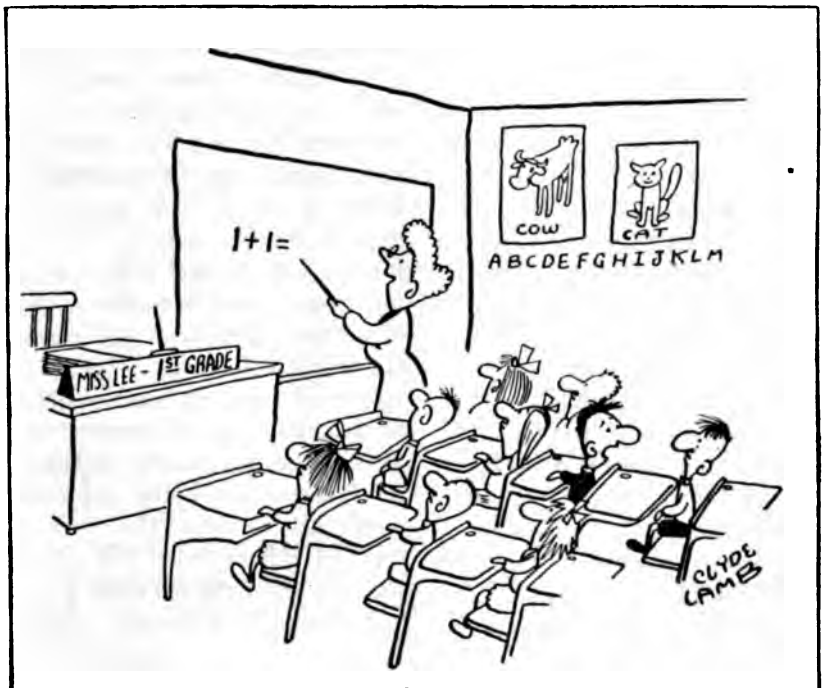
1. The product $2^5 \cdot 9^2$ was accidentally set up by the printer as 2592. Curiously enough there is no mistake. Is there another product like this?

2. On an examination a student copied.

$$\frac{5^2}{8}+\frac{3}{8} \quad \text{as} \quad \frac{3^2}{8}+\frac{5}{8}.$$

Curiously enough, their sums are the same. Is there another such pair?

The answer to the little ditty is TOBACCO. Every letter should be printed in capitals.



"*THIS* is what I've been dreading ever since school began . . . CALCULUS!"

THE SATURDAY EVENING POST

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TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

The Numerical Solution of Equations

TO OBTAIN the solution of quadratic, cubic and quartic equations, a variety of algebraic methods are available. But in engineering and technological work it is frequently desirable to obtain approximate numerical values of all the roots not only of such equations, but also of higher degree polynomial equations as well as transcendental equations. Perhaps the best-known of several approximation methods is the scheme introduced by William George Horner in 1819, a method which is very efficient and relatively simple, although it entails considerable computation, and is limited to finding the real roots of a rational, integral algebraic equation. Inasmuch as Horner's method is quite familiar, it will not be described here.

Another well-known scheme is Newton's method, which, according to Cajori, is not really Newton's method at all, but a modification of it developed by Joseph Raphson (c. 1690); hence it is frequently called the Newton-Raphson method. Actually, the basic idea was anticipated by Vieta (1540–1603). To solve an equation $y=f(x)=0$ by Newton's method, we first find an approximation x_1 to the root r by graphical or other methods. If x_1 is chosen sufficiently close to r, we may then obtain a second and closer approximation x_2 by finding the point $(x_2,0)$ at which the tangent to the curve at the point (x_1,y_1) intersects the x-axis. The slope of this tangent is $y_1'=f'(x_1)$; hence

$$\frac{0-y_1}{x_2-x_1}=y_1',$$

or

$$x_2=x_1+\frac{-y_1}{y_1'}.$$

This is equivalent to using the first two terms (dropping all higher-degree terms) in Taylor's expansion of $f(x)$ about the point whose abscissa is x_1 and assuming that $y_2=0$. If $y_2=f(x_2)$ is not sufficiently small, we apply the method once more, compute a second correction $-y_2/y_2'$, and obtain a third approximation $x_3=x_2-y_2/y_2'$. The Newton-Raphson method can also be used to solve certain kinds of transcendental equations.

The method of false position (*regula falsi*) is essentially a method of inverse linear interpolation. It consists of making a fairly close estimate of a root, say r, then substituting $(r+h)$ in the equation, dropping the terms in h of higher degree than the first (since they are relatively small), and solving the resulting linear equation for h. The process is then repeated, using the approximation just found $(r+h)$ in place of r. In effect, the desired root r is inclosed in a nest of intervals whose end points are approaching r on at least one side. A modification of the method of false position is known as the method of chords.

The above four methods are all alike in that they consist of repetitive processes that make it possible to secure successively better and better approximations of a real root. However, the Newton-Raphson method differs from the other three in that an approximate value for the root is substituted in a formula, namely,

$$r_n=r_{n-1}-\frac{f(r_{n-1})}{f'(r_{n-1})},$$

in order to find a better approximation. The latter approximation is substituted *in the same formula* to find a still closer approximation, etc. In this respect the

Newton-Raphson method is a special case of the so-called method of iteration. In general, a method of iteration refers to any procedure whereby an approximation to a root is substituted in some arbitrarily selected function in order to yield a better approximation; the latter approximation is then substituted *in the same function* to yield a still better approximation, and so on. Various iterative methods use different functions for this purpose.

Still another scheme is Graeffe's method, known for some time, but not often mentioned in algebra textbooks. It was known to Dandelin (1794–1847), but not widely used in his day. It had been anticipated by Edward Waring (c. 1770). Brought into prominence by Carl Heinrich Graeffe (1799–1873), it was further developed by the astronomer Johann Franz Encke (1791–1865). The basis of Graeffe's method is a transformation called the "root-squaring" process, the details of which cannot be given here. The method is described in such well known treatises as Whittaker and Robinson, *Calculus of Observations* (London, 1924); Runge and Kōnig, *Numerisches Rechnen* (Berlin, 1924); Doherty and Keller, *Mathematics of Modern Engineering*, Book I (New York, 1936); Scarborough's *Numerical Mathematical Analysis* (Baltimore, 1930); and Uspensky's *Theory of Equations* (New York, 1948). A very illuminating discussion is also to be found in the paper by C. A. Hutchinson, listed below. The chief advantage of Graeffe's method is that it yields the imaginary roots as well as the real roots.

1. General Approximation Methods

Bailey, V. A. "Prodigious Calculation." *Australian Journal of Science*, 1941, 3: 78–80.

A method for approximating rationally to a root α of the equation

$$\phi(x) \equiv x^m - c = 0, \qquad c = \alpha^m > 0$$

using the function

$$f(x) = x \cdot \frac{(m-1)x^m + (m+1)c}{(m+1)x^m + (m-1)c}$$

in conjunction with a recursion formula.

———. "Solutions of Algebraic Equations by Means of Tables of Logarithms." *Philosophical Magazine*, 1934, s7, 18: 529–539.

Bateman, Harry. "Halley's Method for Solving Equations." *American Mathematical Monthly*, 1938, 45: 11–17.

Edmund Halley (1723) deserves the credit of having found approximation formulas which usually make the convergence of the iteration process extremely rapid when the first approximation is fairly good. His method forms the basis of modern iteration methods.

Bell, Clifford. "The Solution of Numerical Equations." *National Mathematics Magazine*, 1940, 14: 435–438.

Describes a practical method for finding approximate values on opposite sides of the root when applied to a proper interval, thus determining the maximum error in the approximation of an irrational root; the method makes use of the osculatory parabola drawn to the curve.

Bolz, H. A. "Higher Degree Equations Solved by Trial and Error." *Product Engineering*, 1943, 14: 217; 384–385.

Boon, F. C. "The Solution of Equations by the Use of Proportional Differences." *Mathematical Gazette*, 1937, 21: 182–187.

Discussion of an iterative process for the approximate solution of algebraic and transcendental equations.

Brock, J. E. "Note on the Method of Iteration." *Journal of Engineering Education*, 1949, 40: 263–264.

Buchner, P. "Zur Berechnung der Wurzeln höherer Gleichungen." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1927, 58: 163 ff.

Cajori, Florian. "A History of the Arithmetical Methods of Approximation to the Roots of Numerical Equations of One Unknown Quantity." *Colorado College Publications, General Series*, Nos. 51, 52. 1910.

Coate, G. T. "On the Convergence of Newton's Method of Approximation." *American Mathematical Monthly*, 1937, 44: 464–466.

The author considers a function $f(x)$ which satisfies the following conditions: (1) the equation $f(x) = 0$ has a single real root within an interval $0 < a \leq x \leq b$, which root is irrational; (2) the function $f(x)$ is single-valued and continuous in this interval; (3) the derivatives $f'(x)$ and $f''(x)$ are continuous and have no real zero in this interval. For such a function, the following theorem holds true: If Newton's method is applied a sufficient number of times to such a function, the value of the root of $f(x) = 0$ may be found to any desired degree of accuracy.

Conkwright, N. B. "Horner's Method and the Algorithm for the Extraction of Square Roots and Cube Roots." The Mathematics Teacher, 1944, 37: 31–32.

Corey, S. A. "Method of Solving Numerical Equations." *American Mathematical Monthly*, 1933, 40: 163–164.

A variation of the customary iterative

methods, applicable to transcendental equations as well as polynomial equations.

Cronvich, L. L. "On the Graeffe Method of Solution of Equations." *American Mathematical Monthly*, 1939, 46: 185–190.

Recommends that Graeffe's method be used for the solution of equations containing irrational roots and imaginary roots which are desired only approximately; a further discussion and extension of the eleven rules given by Hutchinson (q.v.).

Didden, C. A. "Practical Solution for Equations of Higher Powers." *Civil Engineering*, 1943, 13: 327.

Douglas, W. A. "Higher Degree Equations Solved by Recurrent Methods." *Product Engineering*, 1943, 14: 451–452; 672–673.

Faber, G. "Über die Newtonsche Näherungsformel." *Journal für Reine und Angewandte Mathematik*, 1910, 138: 1–21.

Frame, J. S. "A Variation of Newton's Method." *American Mathematical Monthly*, 1944, 51: 36–38.

A method which uses the approximation formula

$$X_2 = x_1 + \frac{-y_1}{y_1' + (y_1''/2)(-y_1/y_1')},$$

instead of the usual approximation formula of Newton's method.

———. "Remarks on a Variation of Newton's Method." *American Mathematical Monthly*, 1945, 52: 212–214.

Another iterative method, more general than Bailey's method.

Gibbins, N. M. "Application of the Newton Approximation to the *m*th Root of *N*." *Mathematical Gazette*, 1944, vol. 28, Mathematical Notes, No. 1781.

Hatch, Hollis D. "Horner's Method—Shortened." *School Science and Mathematics*, 1936, 36: 1007–1008.

When using Horner's method, the depressed equation is of the type ending in . . . $bx^2 + cx + k = 0$; the next approximation is found by taking $cx + k = 0$, or $x = -k/c$. The author suggests that the following approximation be used instead:

$$\frac{1}{x} = \frac{b}{c} - \frac{c}{k}.$$

By so doing the most tiresome last two or three steps of the usual procedure are avoided without loss of accuracy when finding a root, for example, to six decimal places.

Hausmann, B. A. "Graeffe's Method and Complex Roots." *American Mathematical Monthly*, 1936, 43: 225–229.

Introduces a modification of Graeffe's method, extending it to the case where there are more than two pairs of complex roots in a given equation, and simplifying the procedure in such cases.

Heinrich, H. "Das Hornersche Schema als Hilfsmittel bei der Untersuchung ganzer rationaler Funktionen." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1933, 64: 331–336.

Hutchinson, C. A. 'On Graeffe's Method for the Numerical Solution of Algebraic Equations." *American Mathematical Monthly*, 1935, 42: 149–161.

An excellent description of Graeffe's method with illustrative examples worked out.

Hüttig, F. "Über Näherungsauflösung von Gleichungen." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1928, 59: 436 ff.

Jeffreys, H. "Numerical Solution of Algebraic Equations. *Nature*, April 1927, 119: 565.

Lowry, H. V. "Approximation to the Roots of an Equation." *Mathematical Gazette*, 1945, 29: 225 ff., Mathematical Notes, No. 1848.

Mitchell, Merle. "An Interesting Algorithm." *The Pentagon*, 1943, 2: 69–73.

A short-cut method for finding the values of a polynomial and the numerical solution of equations, using a modified method of synthetic division.

"On Root Approximation." *National Mathematics Magazine*, 1943, 17: 312–314.

A method of finding the approximate value of the root of a polynomial equation by successive substitutions, using synthetic division, rather than by diminishing the roots of successively derived equations, as in Horner's method.

Pawley, Myron G. "New Criteria for Accuracy in Approximating Real Roots by the Newton-Raphson Method." *National Mathematics Magazine*, 1940, 15: 111–120.

The expression commonly given in the literature for the inherent error involved in the Newton-Raphson method is shown to be incorrect. New criteria are derived which may be safely used, and which may likewise be applied to extensions of the Newton-Raphson method involving derivatives of higher order.

Shain, J. "Method of Cascades." *American Mathematical Monthly*, 1937, 44: 24–29.

A method invented by Michel Rolle (1690), which consists essentially of first finding two numbers between which the root of a polynomial equation lies, and then converging upon the root by the use of cascades. This means the process of taking successive derivatives of the given equation, which is first suitably transformed so that all its real roots are positive. A cascade of an equation is its derivative, and the principle involved is that the roots of an equation are separated by the roots of its cascade.

Steen, F. H. "A Method for the Solution of Polynomial Equations." *American Mathematical Monthly*, 1937, 44: 637–644.

A method for finding a fractional approximation to a real root of a rational integral equation with numerical coefficients. The method described usually requires less computation than is required for Horner's method or for the customary extraction of square roots or cube roots, and is better adapted to

of computing machines. It is especially
le where a high degree of accuracy in
proximation is desired.
h, D. L. "Numerical Solution of Equa-
Electrical Engineering, 1941, 60: 480–

. S. "A Modification of Newton's
d." *American Mathematical Monthly*,
5: 90–94.
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2. ON THE LOCATION OF ROOTS

). H. "On the Location of the Roots of
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ual." *American Mathematical Monthly*,
6: 209–212.
P. "Der Sturmsche Satz in graphischer
llung." *Zeitschrift für Mathematischen
aturwissenschaftlichen Unterricht*, 1927,
ff.
. J. "Upper Limits to the Real Roots of
Algebraic Equation." *American Mathe-
Monthly*, 1939, 46: 334–338.
, E. C. "Concerning Nearly-equal
" *American Mathematical Monthly*,
8: 42–43.
"Concerning Upper and Lower Bounds
Roots of a Real Algebraic Equation."
al Mathematics Magazine, 1939, 14: 76–

"Root Isolation through Curve Analy-
lational Mathematics Magazine, 1940,
–378.
, R. H. "A Lower Limit of the Roots of
nal Integral Equation." *Scripta Math-
*, 1945, 11: 379–380.
. J. "Upper Limit for the Roots of an
on." *American Mathematical Monthly*,
0: 35–36.
ld, E. C. "New Bounds for the Roots of
braic Equation." *American Mathemat-
onthly*, 1931, 38: 30–35.
N. R. "Separating the Roots of Poly-
Equations; a Modification of Sturm's
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935, Ser. 3, 29 sec. 3: 69–76.

SCELLANEOUS METHODS OF SOLVING
UBIC AND QUARTIC EQUATIONS

ach, W. "Zur Berechnung der Wurzeln
her Gleichungen in der Zahlenebene."
*rift für Mathematischen und Naturwis-
senschaftlichen Unterricht*, 1926, 57: 70 ff.
Davis, W. R. "Graph Solves Cubic Equation when Cardan's Formula Fails." *Civil Engineering*, 1948, 18: 100.
Fettis, H. E. "On Various Methods of Solving Cubic Equations." *National Mathemaics Magazine*, 1942, 17: 117–130.
Grant, J. D. "Graphical Solution of the Quartic." *American Mathematical Monthly*, 1933, 40: 31–32.
Hacke, J. E. "A Simple Solution of the General Quartic." *American Mathematical Monthly*, 1941, 48: 327–328.
Llewellyn, F. T. "Solving Cubic and Quartic Equations; Method for Use in Every-day Structural Design." *Engineering News Record*, 1930, 105: 254–256.
Lorey, W. "On Dieffenbach's Method for the Solution of Biquadratics." *National Mathematics Magazine*, 1937, 11: 217–220.
McMahon, F. J. "New Method for the Solution of Cubic Equations." THE MATHEMATICS TEACHER, 1947, 40: 33–35.
Miller, G. A. "Solution of the Cubic Equation." *Science*, 1944, 100: 333–334.
Nogrady, H. A. "New Method for the Solution of Cubic Equations." *American Mathematical Monthly*, 1937, 44: 36–38.
Pettit, John T. "A Speedy Solution of the Cubic." *Mathematics Magazine*, 1947, 21: 94–98.
Plummer, H. C. "Numerical Solution of a Type of Equation." *Philosophical Magazine*, 1941, s7, 32: 505–512.
Pöschl, Theodore. "Ein Verfahren zur angenäherten Auflösung der kubischen und biquadratischen Gleichungen." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1934, 65: 329–336.
Risselman, W. C. "On the Solution of Cubic Equations." *American Mathematical Monthly*, 1932, 39: 229–230.
Ritter, A. "Graphical Method for Cubic Equations." *School Science and Mathematics*, 1915, 15: 804–805.
Rocard, Y. "Méthode pratique pour la résolution de l'équation du quatrième degrè." *Revue Scientifique*, 1939, 77: 4–7.
Running, T. R. "Graphical Solutions of Cubic, Quartic and Quintic." *American Mathematical Monthly*, 1943, 50: 170–173.
Sah, A. Pen-Tung. "A Uniform Method of Solving Cubics and Quartics." *American Mathematical Monthly*, 1945, 52: 202–206.
Schucker, M. G. "Solution of Cubic Equations by Straight Line Graphs." *School Science and Mathematics*, 1920, 20: 818–820.
Widder, W. "Bestimmung reeller Wurzeln bei Gleichungen dritten Grades." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1930, 61: 116 ff.
Yates, Robert C. "A Mechanical Solution of the Cubic." THE MATHEMATICS TEACHER, 1939, 32: 215.

BOOK SECTION

Edited by J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

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BOOKS RECEIVED

HIGH SCHOOL

General Mathematics

General Mathematics by Virgil S. Mallory, State Teachers College, Montclair, New Jersey; and Kenneth C. Skeen, Union High School, Taft, California. Cloth, vii+478 pages, 1951. Benj. H. Sanborn and Co., 221 E. 20th Street, Chicago.

Algebra

First Algebra by Virgil S. Mallory, State Teachers College, Montclair, New Jersey. Cloth, vii+443 pages, 1950. Benj. H. Sanborn and Company, 221 E. 20th Street, Chicago.

COLLEGE

Analytical Geometry

Analytical Geometry by R. Walker, Mathematical Master, Stowe School, Sometime Scholar of King's College, Cambridge. Cloth, 248 pages, 1950. Edward Arnold and Company, London; Longmans, Green and Co., Inc., 55 Fifth Avenue, New York. $2.25.

Calculus

Operational Calculus by Balth. Van Der Pol and H. Bremmer. Cloth, xii+415 pages, 1950. Cambridge University Press, American Branch, 51 Madison Avenue, New York 10, N. Y. $10.00.

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TEACHING OF MATHEMATICS

Space and Geometry by Aaron Bakst, School of Education, New York University. Paper, iii+77 pages, 1950. New York University Bookstore, Washington Square, New York 3, N. Y. $0.90.

REVIEWS

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Everyday General Mathematics. William Betz, A. Brown Miller, F. Brooks Miller, Elizabeth B. Mitchell, and H. Carlisle Taylor. New York, Ginn and Co., 1949. xii+436 pp., $2.40.

This book incorporates the advances that have been made in general mathematics in recent years, but at the same time retains the best features of the old methods. The book has a wealth of problem material, drill material, diagnostic and remedial features, provisions for measuring achievement, and provisions for individual differences. It includes material in which emphasis is placed on understanding, interpretation, and use.

Sixty percent of the material utilizes the basic concepts of numbers as used in actual life situations. For example: 7.5 tons of coal at $18.00 per ton; shop drawings which show eight holes to be drilled; planning to make six aprons; planning the family budget and instalment buying. Less than twenty percent of the material presented is called "Geometry" and the authors have succeeded in presenting this section as practical problems in mensuration. Slightly more than twenty percent of the material is called "Alg-

the use of computing machines. It is especially valuable where a high degree of accuracy in the approximation is desired.

Waidelich, D. L. "Numerical Solution of Equations." *Electrical Engineering*, 1941, 60: 480–482.

Wall, H. S. "A Modification of Newton's Method." *American Mathematical Monthly*, 1948, 55: 90–94.

Instead of using the conventional formula of Newton's method, viz.:

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Buchner, P. "Der Sturmsche Satz in graphischer Darstellung." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1927, 58: 61 ff.

Corliss, J. J. "Upper Limits to the Real Roots of a Real Algebraic Equation." *American Mathematical Monthly*, 1939, 46: 334–338.

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Moorman, R. H. "A Lower Limit of the Roots of a Rational Integral Equation." *Scripta Mathematica*, 1945, 11: 379–380.

Myers, R. J. "Upper Limit for the Roots of an Equation." *American Mathematical Monthly*, 1933, 40: 35–36.

Westerfield, E. C. "New Bounds for the Roots of an Algebraic Equation." *American Mathematical Monthly*, 1931, 38: 30–35.

Wilson, N. R. "Separating the Roots of Polynomial Equations; a Modification of Sturm's Theorem." *Royal Society of Canada, Transactions*, 1935, Ser. 3, 29 sec. 3: 69–76.

3. Miscellaneous Methods of Solving Cubic and Quartic Equations

Breidenbach, W. "Zur Berechnung der Wurzeln kubischer Gleichungen in der Zahlenebene." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1926, 57: 70 ff.

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This compact text can easily be used as a source book by the student in later years as it contains many useful tables and a complete index. The authors suggest the use of supplementary material to enrich the course whenever there is a need or opportunity to introduce it. Illustrations and graphs add to the attractiveness of the book. The content of the chapters entitled "The Use of Measures on the Farm," "Farm Credit and the Wise Use of Money," "Crops, Soils and Fertilizers," "The Dairy Herd," "General Livestock and Poultry," "Farm Building and Engineering," and "Farm Economics and Management," should help the student apply basic arithmetic to farm problems.—HUMPHREY C. JACKSON, John D. Pierce Junior High School, Grosse Pointe, Michigan.

Everyday General Mathematics. William Betz, A. Brown Miller, F. Brooks Miller, Elizabeth B. Mitchell, and H. Carlisle Taylor. New York, Ginn and Co., 1949. xii+436 pp., $2.40.

This book incorporates the advances that have been made in general mathematics in recent years, but at the same time retains the best features of the old methods. The book has a wealth of problem material, drill material, diagnostic and remedial features, provisions for measuring achievement, and provisions for individual differences. It includes material in which emphasis is placed on understanding, interpretation, and use.

Sixty percent of the material utilizes the basic concepts of numbers as used in actual life situations. For example: 7.5 tons of coal at $18.00 per ton; shop drawings which show eight holes to be drilled; planning to make six aprons; planning the family budget and instalment buying. Less than twenty percent of the material presented is called "Geometry" and the authors have succeeded in presenting this section as practical problems in mensuration. Slightly more than twenty percent of the material is called "Alge-

bra" and is presented in a very simple and elementary manner.

The teacher will find the book suitable for classes of average ability and also easily adjusted for the retarded classes. The text should definitely be considered when selecting a text for general mathematics.—KENNETH ALLEE, Hartford City High School, Hartford City, Indiana.

A Second Course in Algebra (Second revision). N. J. Lennes and J. W. Maucker. New York, The Macmillan Co., 1950. xv+522 pp., $2.48.

The basic philosophy of Lennes and Maucker centers in the idea that the average student can learn more by doing a simple example through his own efforts than by struggling with a difficult example which requires continuous and detailed explanations by the book and the teacher. The emphasis is always on *why*, not *how*.

The text is amply supplied with sight work, exercises and problems, as well as Chapter Reviews, Cumulative Reviews and Special Practices. There is no lack of examples in this book. Exercises, problems, and supplementary material are labeled (A), (B), (C)—thus grading the work in three steps from easy through difficult, the (C) being included primarily for the neglected bright students we are talking so much about and for whom we are doing so little.

The type is excellent; the illustrations are excellent; the language is simple and direct. Sample examples are plentiful and effectively enclosed in rectangles. Chapters twenty through twenty-three take up permutations and combinations, probability, determinants, and a historical sketch; thus providing additional material for the abler students.

I enthusiastically commend this book to the attention of all second-year algebra teachers.—DOROTHY S. WHEELER, Bulkeley High School, Hartford, Conn.

Commercial Algebra (Third Edition). Thomas M. Simpson, Zareh M. Pirenian, and Bolling H. Crenshaw. New York, Prentice-Hall, Inc., 1950. xi+173 pp., $2.50.

This book is designed to meet the needs of those who wish (1) preparation for a course in compound interest and annuities, or (2) a course in college algebra of a more practical nature than the traditional courses.

It differs from the usual college algebra text in that a chapter on elementary statistics is included, and many of the applications involve interest, discount, partial payment, equation of payments, equation of value, equation of accounts, and other commercial problems. Among topics usually found in college algebra texts which are absent from this book are variation, synthetic division, remainder and factor theorems, permutations, combinations, probability, mathematical induction, and De Moivre's Theorem.

The teacher of a traditional course in college algebra would object to some of the omissions. For the purposes stated, however, the book seems to be adequate, and well organized. Features particularly appealing to the reviewer are: (1) the explanation of and exercises in "translation" from word language to algebraic language, and (2) the wealth of illustrative examples and review exercises.

Instructors of courses having the purposes mentioned above should examine this text carefully.—CLEON C. RICHTMEYER, Central Michigan College, Mount Pleasant, Michigan.

Primer of College Mathematics. John F. Randolph. New York, The Macmillan Company, 1950. xii+544 pp., $4.75.

This is another attempt to unify the material usually found in courses in college algebra, trigonometry, and analytic geometry. There is also an introductory chapter on the calculus. The unifying idea is the notion of functions and their representations.

The chapter titles gives an idea of the arrangement of material. They are: (1) "Arithmetic, Algebra, Probability," (2) "Functions and Rectangular Coordinate Systems," (3) "Equations, Inequalities, Identities, Determinants," (4) "The Binomial Theorem and Applications," (5) "Logarithms," (6) "Introduction to Calculus," (7) "Trigonometry," (8) "Solution of Triangles," (9) "Graphical Analysis," (10) "Theory of Equations and Identities," (11) "Solid Geometry."

The author points out that he has presented the work on solutions of triangles in Chapter 8 and the standard material on conic sections in Chapter 9 in such a way that they may be easily omitted. Why one should wish to do so is not quite clear.

There seems to be an adequate supply of exercises with answers provided for about half. A brief set of tables and an index are included.—FRANK C. GENTRY, Arizona State College, Tempe, Arizona.

Analytic Geometry and Calculus. Harold J. Gay and Raymond K. Morley. New York, McGraw-Hill Book Company, 1950. vii+524 pp., $5.00.

This is a text which is designed to integrate the teaching of analytic geometry and calculus. Most of the proof seemed to be presented in a form somewhat comparable to methods found in traditionally organized textbooks in the separate subjects.

The attempt to integrate the material, however, makes for a much more meaningful development of concepts, and is more likely to insure the establishment of relationships between the various fields of mathematics included.

The textbook is carefully written so as to make it possible for students to follow, in a very carefully organized step by step manner, the most appropriate procedures for the solution of problems. If carefully used by instructors, this book should eliminate a great deal of repetition in the development of new mathematical concepts in both analytic geometry, as well as in the differential and integral calculus.—HARRY C. JOHNSON, University of Minnesota, Duluth Branch, Duluth, Minnesota.

LETTERS

Questions on Per Cent

Consideration of two questions on per cent in the Sueltz paper in the January 1951 MATHEMATICS TEACHER leads to further questions about per cent. He asked college freshmen, considered at least average in intelligence: How is one fourth of one percent written as a decimal? A large orange contains 150% more juice than a small one. This is how many times as much juice?

Correct answers selected: 1st question, 27%; 2nd, 16%.

From this should we conclude that per cent ought to be taught better at some level; or that it ought not to be taught at all?

Is this awkward double standard—both hundredths and per cents for expressing the same parts—due merely to business men's lack of complete faith in the decimal system, and failure to apply it in this situation?

For those who believe that this is so:

Is education a force which should stand up against business when business is foolish and wasteful in such a practice? Or should education teach merely what business wants taught? Recalling how many generations it took business to accept the decimal system originally, should education now resist this clumsy practice, or consider resistance helpless and say nothing?

Suppose that we decided to fight, and to show every pupil, by grade IX, that per cent is a useless, burdensome, expensive, extra terminology. Could the resulting effect on the everyday use of percent, after say 20 years, give a sound test of the effectiveness of our education?

HARRY C. BARBER
Newton Center, Massachusetts

Answer to Harry C. Barber on Per cent

Questions:

First: Should *per cent* be taught better at some level?

Second: Should *per cent* be taught at all when decimal fractions serve the same purpose?

Answers:

The answer to the second question involves, among other things, the elements of human nature and a basic ethic. Can the schools in one generation or even in 100 years change a mode of expression and thinking which society has adopted? If this could be done, should it be done by the schools? Historically it seems easier for the population to adopt new ideas and procedures than it is to drop old patterns. This seems particularly true when the older patterns are identified with a functional use. It chagrins many mathematics teachers to note the use of fractions (common and decimal) currently used in market quotations: 151⅛ and 1 11/32 (stock); 109.16 (meaning 109 and 16/32, bond); .3229 (cotton); and .9016 and .73¼ (hides). Apparently business continues in use and invents or adopts *practices* that serve its needs but does so without due regard to best possible practice in terms of all of the elements involved. Can the schools change this? Stevin and later Erastus Root and others tried to drop common fractions but the public did not follow their lead. Assuming that the proposal to drop *per cent* has merit and that this could be achieved, should the schools set out deliberately to achieve this goal? That is a serious question. Should the schools lead society or merely interpret it? My own impression is that, in a democracy, our larger emphasis should be devoted to the role of interpretation and to provide ideas for bettering society but should not attempt to remake society by fiat. Without arguing the merits of *per cent* in comparison with *decimals*, I doubt that we should attempt to make this change in the public schools.

The first question. Assume that *per cent* will continue to be used in our society. Recognize many per cents used in business and industry are not the simple ones to which the elementary school restricts its teaching. Then, to provide an intelligent citizenry, *per cent* ought to be taught at an age level at which the more complex numbers and situations can be made meaningful and intelligible to pupils. This does not mean that percentage should be removed from the elementary school. It argues for the extension of basic learning at a more advanced level. *In the high school.* The extension of arithmetic topics into the high school is desirable. Where can we find the time? Would mathematics teachers and curriculum workers agree that, for a large part of our population, the extension of arithmetic is more valuable and important than certain parts of algebra and perhaps half of the typical one-year course in plane geometry? If *per cent* were dropped and *decimal* used exclusively we would still be faced with the problem of meeting the needs of society by instruction beyond grade eight.

To Mathematics Teachers:

Consider Mr. Barber's proposition very seriously. Assuming that it could be done, do you believe that the best interests of our country would be served by dropping the terminology and framework of *per cent* and basing all of this work on the idea and notation of *decimal*? Shall we argue the proposition even though you may consider this academic?

BEN A. SUELTZ
Cortland, New York

Solving Equations by the Factoring Method

In an article in the March 1948 issue of THE MATHEMATICS TEACHER, William R. Ransom recommends that the "factoring method" of solving equations be dropped in favor of a "division, two-case method." The fault he finds with the "factoring method" is that most students cannot be taught to understand or to use properly the theorem that "If a product is zero, at least one of the factors is zero."

rs and textbook writers who use this
ı explaining the "factoring method"
t expect good results. The reverse
the proper one to use: "A product is
r one of its factors is zero." Then, if
s aware of what he is seeking (and a
ıre of the "factoring method" is that
this understanding of the definition
it is easy for him to see the value of
equation in the form of a product of
ıal to zero. He understands why set-
actor to zero leads to an answer, and
rorried by such doubts as "If I know
ıt least one of these is zero, what right
ıet both of them to zero?"
ɔn, if it is desired to show that this
ves all the answers, the theorem can
ıted to include "and only if." Mr.
"two-case method" has merit (al-
annot easily be used for equations of
ree because of the need of sub-cases),
not discard the "factoring method"!

KENNETH W. WEGNER
Northfield, Minnesota

THE MATHEMATICS TEACHER

yone ever suggested the possibility of combining all the issues of a single volume into a single book? If this could be done feasibly, I for one would be very happy, as this would enable me to keep my file of THE MATHEMATICS TEACHER all together and in one place where it belongs.

Also has the idea of membership cards ever been broached? This would seem to me to bring a certain definiteness to the membership such as is brought about by other professional groups who use membership cards.

FRANK J. HARDMAN
Baltimore, Maryland

EDITOR'S NOTE: How many readers would purchase covers, if they could be made available? We will welcome your postcard reply.

A Problem Department

I wish to take this opportunity to suggest the addition of a Problem Department in THE MATHEMATICS TEACHER. Problems of varying difficulty could be offered in one issue and solutions (submitted by readers) in a subsequent issue.

H. SIDOFSKY
New York, N. Y.

MEETINGS OF AFFILIATED GROUPS

lo Council of Teachers of Mathematics: eting in late April or early May
natics Section, Eastern Division, Colo-ation Association: April 20–21, Colo- College of Education, Greeley

Council of Teachers of Mathematics: Tampa; August, Gainesville

Council of Teachers of Mathematics: -24, Atlanta; two days in May, Mil- eleven meetings by districts in Oc-

Mathematics Club (Chicago and ın Area): March 16, Chicago Bar ı; April 20 and May 18, YMCA

's Mathematics Club of Chicago and March 16, Chicago Bar Association; useum of Science and Industry

a Council of Teachers of Mathematics: eeting, April 28, Indiana University, on

ınd

ıtion of Teachers of Mathematics of New England: May 12; August 23–30, Summer Institute at Connecticut College, New London

New Jersey

Association of Mathematics Teachers of New Jersey: May, Rutgers Univ., New Brunswick

Ontario

Ontario Association of Teachers of Mathematics and Physics: March 27–28, Toronto

Tennessee

Mathematics Section, Eastern Tennessee Education Association: October 26, University of Tennessee

Texas

Mathematics Section, Texas State Teachers Association: Eleven district meetings in the spring

Virginia

Richmond Chapter, National Council of Teachers of Mathematics: April 16, W.R.N.L. Theatre

Wisconsin

Wisconsin Mathematics Council: Joint meeting with the Wisconsin Section, Mathematical Association of America, May 12, Carroll College, Waukesha

AIDS TO TEACHING

Edited by

HENRY W. SYER
School of Education
Boston University
Boston, Massachusetts

and

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.56—Personal Money Management

Local representative of American Bankers Assoc., 12 East 36th Street, New York 16, New York.

Booklet; 36 pp.; 7″×9¾″; Single copies—$.25.

Description: The booklet is composed of eleven brief essays on methods of controlling expenditures economically and on means of determining the suitability of one's present spending habits. Individual chapters are devoted to the factors which consume the largest portions of one's income—savings, shelter, household operating expenses, food, clothing, and advancement. The final chapter of the booklet is especially good; it is entitled, "Are You as Old Financially as in Years?" The author classifies all people in three somewhat artificial, but nevertheless useful, categories according to savings—minimum, middling, and master. A clever graph serves to illustrate the "financial life-line" of people within these classes so that one may plot his own economic condition and determine his monetary age. A second excellent feature of the booklet is an enclosed table, "Suggested Distribution of Income After Income Taxes." It is an up-to-date revision of a table which is in the booklet. Budgets are suggested for families of two, three, and four members whose incomes range between $150 and $1,000 per month.

The booklet is printed upon light brown paper in dark brown print. The format is excellent in every respect. Each chapter is followed by a helpful bibliography for those who care to pursue the subjects further; the chapters are well-arranged and presented in terms understandable to the layman.

Appraisal: The topics presented are certainly consistent with the general objectives of socialized mathematics courses. Increased ability in money management is a skill vital to every wage earner.

Each of the essays has been written by a person well qualified to expound upon theories in his particular field; the essays are therefore authoritative. They are probably most useful on the senior high school level for the vocabulary is difficult for younger students.

The teacher may well utilize the booklet in directing brief projects which require solutions to problems involving present and future financial status of the pupils. Such work may well contribute toward the students' formation of vocational plans.

It would indeed be advantageous to supply each student with an individual copy, but where the school budget does not permit, the teacher can reproduce the context in class and make the single copy available for student examination. The booklet is certainly worth twenty-five cents. (Reviewed by Bernard Singer, Hyannis, Mass.)

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MATHEMATICS TEACHER

Volume XLIV Number 4

A Five Year Program of Preparation for Mathematics Teachers*

By MYRON F. ROSSKOPF
Syracuse University, Syracuse, New York

the past ten years there has
than usual interest in the
teacher training. The shortage
brought on by total mobiliza-
ır participation in the Second
r, the flight from teaching of
omen into industry and govern-
ice in the post-war economic
and the increase in the number
ı entering the schools of the
ed to keep this problem con-
ore those responsible for teacher
The gap between supply and
qualified teachers of science
matics is particularly impor-
reports that have appeared
stress the problem of attracting
number of students into pre-
mselves for teaching science
matics. In addition these re-
evidence of much dissatisfac-
he present program of teacher
ı. There is need for a new
oth to courses in the sciences
ematics and to professional
education.

ident from a study of recent
l articles that there is a strong
toward a longer period of prep-
ır teachers. The President's
Research Board indicates its point of view in the following words:

"The graduate offerings in science and mathematics are usually too highly specialized and narrowly defined to serve the needs of any individual who does not intend to become a research scientist.

"Some teacher training institutions should provide for at least one year of graduate work in which ample courses in science and mathematics are offered to meet the needs of the teacher instead of the research scientist.

"Such offerings will make it possible for many teachers who now are forced to do their graduate work in education only, to return to their teaching fields for a majority of their graduate study."[1]

Commenting on a part of the foregoing report, Professor Dodds of Purdue University writes, "It might even be suggested that this area of work is a true inter-departmental responsibility and that there are people available for university staffs who might hold joint appointments in both education and the subject matter department."[2] There is a belief that, "There should be more seminars in our teacher-training programs for the purpose

t the Twenty-Eighth Annual Meet-
National Council of Teachers of
s at Chicago, Illinois, April 15, 1950.

[1] The President's Scientific Research Board, *Manpower for Research*, p. 107. Volume four of *Science and Public Policy*. Washington, D. C.: U. S. Government Printing Office, 1947.

[2] B. L. Dodds, "Cooperative Planning in Teacher Education," *School Science and Mathematics*, L (March, 1950), 226.

of integrating subject matter, and for the purpose of discussing the utilization of mathematics in other fields of knowledge."[3] The program that I am going to describe attempts to provide just those opportunities; that is, to provide an environment for student teachers that will facilitate integration of their knowledge of subject matter in their major area, psychology and professional education, and general education.

The all-university School of Education was established at Syracuse University in 1934.[4] From its inception a cardinal point of its philosophy has been furnishing future teachers with a unified and functional curriculum. An implication of this objective is that students have contact during their training period with a faculty that is sympathetic to this point of view. An immediate corollary is the appointment of personnel who will bridge the gap between a college and a school of education.

The personnel referred to in the foregoing paragraph were given the title "dual professor." The title emphasizes the sort of teacher training that a student obtains and underlines the philosophy of the School of Education. In each of the academic areas and in each of the special areas at least one dual professor has been added to the staff of the School of Education and the College of Liberal Arts or School of Fine Arts or any other college that is taking part in the training of teachers. The dual professors divide their time about equally between the two colleges. For example, the dual professor in mathematics teaches half his academic load in the mathematics department of the College of Liberal Arts and half his academic load in the School of Education. To talk about the academic load of a dual professor is to speak of the least important phase of his responsibilities. He is the cement that binds his department to the School of Education; thus, he establishes liaison between subject matter and teaching in a secondary school. In addition the dual professor supervises practice teaching and acts as adviser of juniors, seniors, and graduate students in his major area.

What of the students in the program? Does the School of Education welcome with open arms anyone who thinks he might like to teach in our public schools? No, the applicants are subjected to a well-organized selection program.[5] Applicants who are accepted become dual enrollees; that is, they retain enrollment in their first college choice and accept dual enrollment in the School of Education. During their freshman and sophomore years, the only contact that the faculty of the School of Education has with its probable future students is in the capacity of advisers or in a one-credit course, *Education Orientation*. Normally a student makes application for dual enrollment in the School of Education during the second semester of the sophomore year.

The Selection Committee is composed of three members of the School of Education faculty and a fourth member, the dual professor for the area in which the applicant intends to major. The folder of information that is considered when an applicant's admission is decided upon consists of two individual interview reports, one with the dual professor and one with the chairman of the Selection Committee, several independent interview reports on the applicant's group interview, and results on the following tests:—American Council on Education Test of Aca-

[3] C. V. Newsom, "Mathematics and Modern Educational Trends," THE MATHEMATICS TEACHER, XLII (November, 1949), 343.

[4] The Curriculum Committee of the School of Education, Syracuse University, *A Functional Program of Teacher Education as Developed at Syracuse University*. Washington, D. C.: American Council on Education, 1940. This monograph gives a complete report of the total education program, the several chapters treating in detail its phases.

[5] A detailed account of the selection program, together with an analysis of three cases, is given in a recent paper by Verna White, "Selection of Prospective Teachers at Syracuse University," *The Journal of Teacher Education*, I (March, 1950), 24–31.

demic Aptitude, Cooperative General Culture Tests, Cooperative Reading Test, Minnesota Multiphasic Personality Inventory, and the Strong Vocational Interest Inventory (for men) or the Kuder Preference Record (for women). An applicant is accepted; or he is rejected and guided through counselling into other outlets for his abilities and interests; or he is accepted on "release," pending the removal of weaknesses that his folder of information reveals. During the past two years, approximately 58 percent of those applying for admission to dual enrollment were accepted.

As was pointed out at the beginning of this paper, it has long been believed by forward-looking educators that a longer training period is necessary for teachers. Recently an important committee arrived at this statement, "Colleges and certification authorities should work toward a five-year program for the preparation of high school teachers. . . . There would then be more time for obtaining the broad liberal education which promotes growth of the student as a person and makes him a better teacher."[6] Under the urging of educational leaders the Board of Regents of the University of the State of New York adopted new requirements for certification in 1943. Basically these new requirements are summarized in the statement that five years of training now are necessary for certification to teach in the public schools of the state. Because of the shortage of qualified teachers during the next several years, enforcing the new requirements was postponed until 1951; then only students preparing to teach in the academic areas will come under the five-year requirement. Meanwhile the School of Education at Syracuse University moved ahead with plans for a training program that would not only satisfy the requirements of the law but would carry out the best thinking of educators. Committees were set up to study the present program, to search out its strengths and weaknesses, and to work out a philosophy for the training of the third, fourth, and fifth years of a prospective teacher.

The certification law states that thirty credit hours of advanced work beyond the bachelor's degree is required of all teachers of the academic subjects; the first application of the law will be made in 1951. As the faculty of the School of Education investigated this problem questions arose such as, "Could the present master's program be used to satisfy the new requirements? Can a three-year sequence of integrated experiences be developed that will better fit a teacher for his role in the classroom?" After examination of these questions and related problems, the faculty voted to re-plan the whole teacher preparation program in terms of a three-year sequence. Following is a short description of the courses in professional education, together with an account of the experiences in which all students participate.

Introduction to Human Development, Adjustment and Learning, 3 semester hours. This is the first course in the education sequence; it is taught during the first semester of the junior year. Through class discussion, work in Syracuse youth agencies, reading, and motion pictures, students obtain an understanding of the psychology of human adjustment and development.

Human Development, Adjustment and Learning, 4 semester hours. This course, taught during the second semester of the junior year is a continuation of the foregoing course. The major emphasis is on contemporary theories of the psychology of learning, In place of work in youth agencies students observe in Syracuse public schools. Each week a two-hour seminar is held by the dual professor for students majoring in his area. At this

[6] Report No. 4. The AAAS Cooperative Committee on Science Teaching, "The Preparation of High School Science and Mathematics Teachers," *School Science and Mathematics*, XLVI (February, 1946), 113.

time attention is directed to the application of the psychology of learning in the major area; observations are discussed and methodology of observed teachers is analyzed.

Creating the Learning Environment in the Classroom, 4 semester hours. During the first semester of his senior year, a student spends almost all of his time with his dual professor; the exceptions are seminars and laboratories in the two phases, audio-visual aids and equipment, and business machines and equipment. Emphasis is placed on methods of classroom teaching, evaluation in the major area, and careful planning for classroom work. Each student teaches one class per day for a period of six weeks in one of the Syracuse public schools.

The Exceptional Child, 3 semester hours. At this time, in the second semester of the senior year and after a practice teaching experience, it is believed the students are ready for an intensive study of the child as an individual. By means of class discussions, readings, class demonstrations, films, and field trips all the students have a chance to become familiar with characteristics of the normal as well as the exceptional child. Eyesight, hearing, and speech difficulties; rheumatic fever and cerebral palsy damages; heredity and environment factors; all these are considered in the course. The primary objective is not to make prospective teachers experts in these special fields but to make them sensitive to the problems that these children present in a classroom so that special considerations or special care can be recommended for them.

The Teacher as Educational Leader, 12 semester hours. Because extramural teaching requires that a student be away from the campus for six weeks, the first semester of the graduate year consists solely of courses in education. The faculty believes that it is easier for the School of Education to adapt its program to these special needs of its students than several liberal arts college departments. Although this block of work is considered an integrated whole, for purposes of instruction it is divided into five parts; these units are described in the following paragraphs.

The American School, Its Evaluation and Curriculum and *The American School: Its Organization and Administration* account for one-half the number of credits in the foregoing block. The titles indicate in an adequate fashion the content considered in these two units.

Interpretation of Educational Measurement, 2 semester hours. Emphasized in this unit is the practical knowledge of statistics that a teacher can use to analyze his classroom tests and to participate intelligently in a program of evaluation in a school.

Research in Teaching, 2 semester hours, and *Seminar in Teaching*, 2 semester hours, are conducted by the dual professor. Emphasized in the former are the research methods in the major field and reading of professional journals. Instruction as well as practice is given in organizing material for publication. During this semester the student, working closely with his dual professor, must prepare himself for living and teaching for a period of six weeks in a small community. Each student is responsible for teaching three classes per day, supervising study halls, attending faculty meetings, and helping the cooperating teacher in any way that might offer itself. After the students return to the campus from their extra-mural teaching there is planned for them a unique experience. In order that they may be familiar with the details incident to the taking of a field trip, they plan and take trips to such places as a bank, a television broadcasting station, a technical high school, the statistical department of a manufacturing plant, or a broker's office. It is believed that participation in these enrichment experiences will encourage them to use comparable community resources when they go out to positions as teachers.

Since most of our students come from urban communities and since most of

will begin their teaching in a small
unity, a three semester hour spe-
organized course in *Rural Sociology*
o taken during the first semester of
aduate year.

the second semester there is a two
ter hour course in *Philosophy and*
of the Teacher and a three semester
course with the self-explanatory
of *Curriculum Workshop*. However,
nts will work in this curriculum
hop in both heterogeneous and
geneous groups. Thus, they will
a chance to work with people in
subject matter fields, as on a school
y, in studying curriculum problems,
ll as working cooperatively with
lists in their own areas.

addition to meeting all course re-
nents for the two degrees, bachelors
aster's, a student must undertake
arch project. The problem chosen
nparable to one that a classroom
r might undertake. The research
t may be an individual problem or,
se of its nature, a cooperative at-
by several students may be made.
ples of research projects that pre-
udents in the program have already
to work on are a unit of work for
l mathematics organized like a
book, a test on critical thinking,
lection and organization of material
onsumer mathematics course in the
high school.

tudent majoring in mathematics is
ed to earn eighteen semester hours
d the calculus to satisfy the re-
nents for a bachelor's degree. Courses
s *Advanced College Algebra, Infinite Series, College Plane Geometry, College Solid Geometry, Projective Geometry, Statistics, Introduction to Modern Mathematics, History of Mathematics, Fundamentals of Analysis*, and the like are included. Programs vary from student to student depending upon his background, abilities, and needs. The department has set nine semester hours as satisfying the mathematics requirement for the master's degree in education. Hence a student earns a total of twenty-seven semester hours in mathematics beyond the calculus by the time he finishes the five year program. Although a minor teaching subject is not required, each student is strongly urged to establish a minor in the sciences. There is ample room in each student's program for elective courses, either of a general education nature or in related areas of education like audio-visual aids and guidance.

At the present time students in the five year program are completing their fifth year. Even though it is too early to assess the effectiveness of the curriculum that has been designed for them, the evidence that has been gathered to date indicate that the faculty made a wise decision when it voted to plan an integrated program. In class discussions, readings, and practice teaching students show steady growth toward a mature understanding of boys and girls. Their work does not seem to them a series of disconnected courses. The whole program is an experience that forces them to study the problems of a teacher and to think about the implications for teaching that exist in a learning situation.

May Expirations

he May issue of THE MATHEMATICS TEACHER is the last you are to receive until your mem-
is renewed, you have received notice of this. *Please renew your membership now.* If this is
efore May 15 your name will *not* be dropped from the mailing list and you will be sure to
the October issue. Thank you for your cooperation. HWC

Making "Pi" Meaningful

By HILLARD C. TRIPP
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ONE OF the most difficult experiences that junior high school mathematics students encounter is the understanding of *pi*. The approximate value of 3.14 can be memorized in a short time, but what meaning does it have for the student after he has memorized it? As Professor Breslich points out:[1]

> Memorizing is not learning and drill alone is not conducive to learning.

The typical mathematics student has the idea that *pi* is a number, 3.14, which is used in working with circles. His concept of *pi* is quite vague. His experience with *pi* has been limited to using it in formulas for finding the circumference of a circle, the area of a circle, or the volume of a sphere. Unless he is an exceptional student, he does not readily comprehend the idea that *pi* is always the relationship between the diameter and the circumference of a circle.

Why is this memorization of *pi* not sufficient for the student's understanding if he can substitute 3.14 in a formula and get the correct answer? Again, as Professor Breslich states:[2]

> The pupil who in arithmetic has acquired the habit of relying upon rules of procedure that are presented in the textbooks or by the teacher, without training himself to gain real understanding of rules and processes, usually continues to memorize the rules in algebra and the demonstrations in geometry. He will fail because the processes of algebra are far more intricate than those of arithmetic. Unless teaching is such as to change these harmful methods cultivated by years of study of arithmetic he will not progress very far in high school mathematics.

Dr. Harold P. Fawcett raises the question concerning the guiding of the student in the exploration of this very important ratio.[3] Will the student be given time and opportunity to acquaint himself with this important ratio? Time should be given for the development of important concepts for nothing is gained when progress is too rapid for understanding.[4] The question then arises as to just how this very important concept can be made clear to the average junior high school mathematics student.

Human beings are endowed with five senses as well as with minds and the most effective approach to minds is by way of the senses.[5] It would seem, then, that if a situation could be set up by which the student could experience *pi* in more ways than just memorizing it, he would understand this ratio more clearly, and this concept would have a definite and lasting meaning to him. If he, himself, could actually measure the diameter and circumference of various sized circles, and then be guided in the arithmetical combination of these sets of numbers, his concept of *pi* would evolve as a natural part of his experiences.

The following guide sheet for the investigation of the relationship between the diameter (d) and the circumference (c) of any circle has proved to be quite beneficial for junior high school mathematics students in establishing the *pi* concept.

To have the maximum amount of understanding evolve from this experience, the student must have an understanding of approximate numbers and the meaning of

[1] Breslich, E. R., "Teaching Mathematics as a System of Understandings," THE MATHEMATICS TEACHER, Vol. XLII, No. 1, January, 1949, p. 61.

[2] *Ibid.* Breslich, p. 65.

[3] Fawcett, Harold P., "Mathematics in the Core Curriculum," THE MATHEMATICS TEACHER, Vol. XLII, No. 1, January, 1949, p. 10.

[4] Breslich, *op. cit.*, p. 63.

[5] Carnahan, Walter H., "Enrichment of Mathematics Teaching," THE MATHEMATICS TEACHER, Vol. LXII, No. 1, January, 1949, p. 14.

Guide Sheet For the Investigation of the Relationship Between the Diameter (d) and the Circumference (c) of Any Circle.

1. Measure the diameters and circumferences of ten or more circular objects to the nearest one quarter of an inch and tabulate these measurements in columns 3 and 4 below.

1	2	3	4	5	6	7	8
Circle No. or object	Instrument used to measure	C	D	$C+D$	$C-D$	$C\times D$	$C+D$

2. Is there any one of the columns above that seems to result in approximately the same number? _____.
   If so which column? _____.
3. Take the average of the numbers in this column. Average _____.
4. What is the name of this constant (number)? _____.
5. Make a general statement concerning the relationship of the circumference and the diameter of any circle. State this also in a mathematical formula.

precision and accuracy in measurement.[6] It will also be necessary for the student to be familiar with the addition, subtraction, multiplication and division of mixed and decimal numbers, for his measurements will be taken in either one or the other of these numbers.

The day before the class begins their experience with *pi*, each student is asked to bring to class a tape measure or about three feet of string and a ruler. It is desirable to have at least one half as many circular objects of various sizes as there are students in the class. The class may be divided into groups of two and each member will have the opportunity of taking the measurements of the diameter and circumference of at least ten different sized circular objects.

Column No. 1 is filled out first in each case thus identifying each object measured. This is necessary in case a re-check is

[6] Butler and Wren, "The Teaching of Arithmetic," Chapter XI, *Teaching of Secondary Mathematics*, New York, McGraw-Hill, Series in Education, 1941, p. 249.

desired of any one of the measurements taken. In Column No. 2 the measuring instrument or instruments used may be the same for any one group of students throughout the investigation. However, this information is important, for each measuring device will have its inherent degree of error and this will be one of the reasons why the *pi* ratio will not be exact for each circular object. The measurements of the circumference and diameter are entered in Columns No. 3 and 4 respectively, preferably to the nearest quarter of an inch, if the measuring instruments allow for that degree of accuracy.

Some might question why other operations on the data are included in the guide sheet when only division produces a ratio and a result which is constant. If the division operation is the only one suggested on the guide sheet, the student may wonder what the result would be if these two special numbers were added, subtracted, or multiplied. If a nearly constant number results every time the circumference is divided by the diameter, the intellectual curiosity of the student may be aroused as to the possibility of the existence of other constants with other arithmetical combinations of these two related numbers. Let us not forget it is the student who is experiencing. When we can anticipate a question on the part of the student, let us subject him to experiences by which he can answer his own questions through a certain amount of guidance. Let him perform the addition, subtraction, and multiplication operations along with the division. This will give a contrast and make the circumference-diameter ratio more impressive in his experience.

Question No. 2 is inserted to direct the student's attention to the C/D ratio. In the third statement, the student gets the average of the ratios in Column No. 8 and arrives at a number that will be quite close to the value of *pi*, 3.14. The student may know what this constant is called from his past experience or the teacher may name it for him, giving some of the historical background of this ratio.[7]

The fifth and last question on the guide sheet gives the student the experience of summarizing and making a generalized statement concerning his experiences with the relationship of the circumference and diameter of a circle. He should be encouraged to make the statement in his own words and then transpose his generalized statement into a mathematical formula, $C/D=\pi$, or $C=\pi D$ by cross-multiplication.

The letter π should now have meaning to the mathematics student. When he sees it, his experiences have been such that he will recall the very important ratio of C/D. The concept of *pi* becomes a cornerstone in the structural foundation of the student's mathematical literacy.

[7] Bell, E. T., "The Age of Empiricism," Chapter 2, *The Development of Mathematics*, New York, McGraw-Hill Book Company, Inc., 1945, pp. 40, 46.

LETTER

A Suggestion

"I would like to offer one suggestion which would increase the value of the magazine—to me, at least.

"In order to enable those members who desire to file the various articles under subject matter to do so without destroying any articles, it would seem to be a good idea to arrange the format of the magazine so this could be done. Thus, if an article ends on an even-numbered page, no difficulty arises; the article can be cut out of the magazine and retained intact. If an article ends on an odd-numbered page, however, this cannot be done without losing something of either one or the other articles. This might be overcome by using the even-numbered page immediately following an article which concludes on an odd-numbered page for advertisements exclusively. In this fashion every article could be retained intact."

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Linear Equations Without Determinants*

By C. C. MacDuffee
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Determinants have become such a tradition in algebra that it is perhaps well to remind teachers of mathematics that the method of determinants is not in general the easiest way to solve systems of linear equations. In the special case of n consistent and independent equations in n unknowns, the method of determinants is very effective, but it is not well adapted to the treatment of special types of systems.

The primitive method of using elementary transformations, refined and systematized, is capable of producing the general solution of a system of any number of equations in any number of unknowns without preliminary separation into cases. All systems are treated by the same method. In the case mentioned at the beginning, this method is about as easy as the method of determinants, in most other cases it is easier. Moreover, it is simpler in theory and easier to teach.

Let us consider a typical problem, to find the general solution of

$$\begin{aligned} 2x+\ 3y-\ 5/2z+\ 5w&=2,\\ 3x+\ 5y-\ \ z+\ 2w&=3,\\ 7x+11y-\ \ 6z+12w&=7,\\ 3x+\ 4y-13/2z+13w&=3. \end{aligned}$$

There are three types of elementary transformation:

I. The interchange of two equations of the system.

II. The multiplication of each term of one equation by the same number $k\neq 0$.

III. The addition to each term of the ith equation of the system of l times the corresponding term of the jth equation of the system, $i\neq j$.

It is easily shown that an elementary transformation applied to a system of equations changes it into an equivalent system, two systems being equivalent if every solution of each is a solution of the other.

We shall define the *diagonal* of a system of equations as that sequence of coefficients which starts with the last coefficient in the last equation (not counting the constant term) and extends upward and to the left. Thus the diagonal in the system illustrated is

$$13,\ -6,\ 5,\ 2.$$

It will be shown that every consistent system in n unknowns can by means of elementary transformations be reduced to an equivalent system consisting of exactly n equations having the following properties, at which point the system is essentially solved:

1. Every coefficient above the diagonal is 0.
2. Every diagonal coefficient is 0 or 1.
3. Where the diagonal coefficient is 0, every other coefficient in that equation is 0.
4. Where the diagonal coefficient is 1, every coefficient below it is 0.

Let us return to our example. Since the second equation has the smallest coefficient of w, we may divide through by 2 and place this equation last:

$$\begin{aligned} 2x+\ \ 3y-\ 5/2z+\ 5w&=2,\\ 7x+\ 11y-\ \ 6z+12w&=7,\\ 3x+\ \ 4y-13/2z+13w&=3,\\ 3/2x+5/2y-\ 1/2z+\ \ w&=3/2. \end{aligned}$$

Clearly all the coefficients of w above the diagonal can be made 0 by adding -5 times the last equation to the first, -12 times the last equation to the second, etc. We obtain the equivalent system

* From a paper read before the Tenth Summer Meeting of the National Council of Mathematics Teachers, Madison, Wisconsin, August 22, 1950.

$$\begin{aligned}
-11/2x-19/2y+\ 0z &= -11/2,\\
-\ 11x-\ 19y+\ 0z &= -11,\\
-33/2x-57/2y+\ 0z &= -33/2,\\
3/2x+\ 5/2y-1/2z+w &= 3/2.
\end{aligned}$$

The unusual feature of the above system is that, in making all coefficients of w above the diagonal equal to 0, we also made all coefficients of z in and above the diagonal equal to 0. The proper move now is to increase the number of equations to 5 by the introduction of a vanishing equation:

$$\begin{aligned}
-11/2x-19/2y &= -11/2,\\
-\ 11x-\ 19y &= -11,\\
-33/2x-57/2y &= -33/2,\\
0x+\ 0y+\ 0z &= 0,\\
3/2x+\ 5/2y-1/2z+w &= 3/2.
\end{aligned}$$

Now the term $-57/2y$ appears on the diagonal. To change this coefficient to 1, we multiply the third equation by $-2/57$, obtaining as the new third equation

$$11/19x+y=11/19.$$

We multiply this new third equation by 19/2 and add to the first, by 19 and add to the second. The system now takes the form

$$\begin{aligned}
0 &= 0,\\
0x &= 0,\\
11/19x+\ y &= 11/19,\\
0x+\ 0y+\ 0z &= 0,\\
3/2x+5/2y-1/2z+w &= 3/2.
\end{aligned}$$

At this point the consistency or inconsistency of the system of equations will be evident. The constant terms (to the right of the =) have not been under our control. There are three equations whose left members vanish. If in one or more of these the right member did not vanish, the system would clearly be inconsistent. If these right members do vanish, as in our example, the system is consistent for a solution can be found.

We now drop or add vanishing equations until the system consists of exactly n equations. We have reached a system

$$\begin{aligned}
0x &= 0,\\
11/19x+\ y &= 11/19,\\
0x+\ 0y+\ 0z &= 0,\\
3/2x+5/2y-1/2z+w &= 3/2
\end{aligned}$$

possessing the first three of our desired four properties. The fourth property may now be imposed. The first 1 on the diagonal above the last equation is the coefficient of y in the second equation. We add $-5/2$ times the second equation to the last (and in general continue this process from right to left) until every coefficient below a 1 in the diagonal is 0. Our system

$$\begin{aligned}
0x &= 0,\\
11/19x+\ y &= 11/19,\\
0x+0y+\ 0z &= 0,\\
1/19x\ -1/2z+w &= 1/19
\end{aligned}$$

now has all four of the desired properties. It can be shown that this form is unique regardless of the steps taken to reach it.

Each unknown whose diagonal coefficient is 0 is a parameter, and each unknown whose diagonal coefficient is 1 is expressible in terms of these parameters. Thus in our example

$$\begin{aligned}
x &= p,\\
y &= 11/19-11/19p,\\
z &= q,\\
w &= 1/19-1/19p+1/2q.
\end{aligned}$$

This is a solution for every pair of values of p and q, and there are no others. This general solution may conveniently be written

$$\begin{aligned}
(x, y, z, w) = (1,\ &-11/19,\ 0,\ -1/19)p\\
&+(0,\ 0,\ 1,\ 1/2)q\\
&+(0,\ 11/19,\ 0,\ 1/19).
\end{aligned}$$

It should be clear from this example how any system of linear equations can be solved.

Projects for Plane Geometry

By MARIE L. BAUER
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PROJECTS help the student to appreciate more fully the application of geometry to life situations, by the recognition of geometric form, the direct application of geometric principles and the use of geometric logic or methods in everyday reasoning.

Projects may take the form of posters, bulletin board displays, models, demonstrations, notebooks or booklets. In order to be interesting they should be well organized and illustrated. Illustrations may be cut from magazines or newspapers, or if he desires, the student may make his own sketches. These projects may be written up as themes, giving the illustration subordinate place, or the illustration may be made the center, using brief statements to point out the geometrical relationships.

The following topics will serve as a guide to the students. In some cases suggestions are given that may be followed, in others the ideas are left entirely to the student. A few of the topics overlap, however, they would probably be developed in an entirely different manner by different students. Two or three will be carried out to show what can be done. An attempt has been made to include a wide variety of subjects so that every student may choose a topic in his particular field of interest.

Since the purpose of the project is to give the student an opportunity to develop his own ideas on the topic he chooses, he is encouraged to find his own references, especially in modern magazines and newspapers. However, in order to aid those who find it difficult, a number of references are given, and with the exception of those where the title is self explanatory, a brief exception of those where the title is self explanatory, a brief explanation of what can be found in each is added.

1. Geometry on the Highways
   Use of curves and lines and the proper combination of the two for safety
   Cloverleaf highway intersections
   Geometry in highway signs, bridges, etc.
2. Geometry of Faces
   Basic types: a. Oval, b. Round, c. Square, d. Pear-shaped, f. Diamond-shaped, g. Oblong
   Optical illusions provided by the type of hair, dress and make-up
3. Geometry in the Home
   Geometric designs in rugs, linoleums, laces, quilts, furniture, pottery, jewelry, tablecloths, pillow tops, towels, clothes, textiles, blankets, wallpaper, mosaic floors and arrangement of furniture.
4. Geometry in Furniture
   Lampshades, desk tops, pictures, windows, chests, curtains, mirror, fireplace, chandelier, and chairs
   Arrangement of furniture for balance, symmetry or efficiency
5. Geometry in Drawing
   Blueprints, maps, and scale drawings
   This might also include cartoons using geometric figures
6. Geometry in Music
   Staff and symbols used in writing music
   Forms and designs of a number of musical instruments
7. Geometry in Fabrics
   Actual samples of materials, or designs drawn and painted are very effective
8. Geometry in Buttons
   Samples of actual buttons, pictures cut from magazines, or hand-drawn designs may be used
9. Geometry in Weather
   Clouds, rainbow, lightning, moon, sun, tornado, snowflakes, weather maps, weather flags, weather vane, thermometer, barometer, thermograph
10. Geometry in Transportation
    Trains, trucks, automobiles, bicycles, and airplanes
11. Geometry in Insignia
    The use of circles, triangles and parallel lines
12. Geometry in Advertising
    Example of ads using lines, rectangles, circles, semi-circles, parallelograms, curves, and a contrast of several. Also examples of geometric trade marks, copies or originals
13. Geometry in Art
    Non-objective painting, cartoons drawn by the student or clipped from magazines
    Lettering, action figures, Indian art

Pictures showing geometric shapes and principles

14. Symmetry, Congruency, Equality, and Similarity in Everyday Life
    Congruency in manufacturing, e.g. parts of automobiles, ten cent store items, factory made clothes. (Different sizes exemplify similarity, the suit as a whole represents symmetry.)
    Spider web, dandelion, star fish, Easter lily, and snow crystal
    Rug designs, linoleum patterns
15. Geometry in Abstract Design
    Circles, squares, rectangles, triangles, squares, and diamonds present interesting designs if they are mixed together to produce different patterns. In looking at abstract designs you see geometric figures which are usually disguised by additional lines that set the pattern, such as flowers. (Use pictures to show the basic geometric structures of each design.)
16. Geometry in Sailing
    Triangular shaped sails, etc.
17. Geometry in Dressmaking
    Measurements
    How stout people look thin with geometry.
    Effects of pockets pleats, drapes, stripes, double-breasted coats, length of jackets in suits, etc.
18. Geometry in City Planning
    Scale drawing
    Street intersections forming angles
    Plantings in geometric designs
19. Geometry in Church Windows in Our Community
    Colored drawings of various church windows showing use of circles, rectangles, squares and triangles. (One student who was particularly interested and good in art went to a number of churches in town and sketched one window from each church. The sketches were made in pencil and then the various parts colored with crayon. The likeness was so real that it was possible for one acquainted with the church to identify it.)
20. Geometry in Farming
    Machinery
    Contour plowing
    Shapes of fields
    Planting in rows
    Planning or layout of the farm
21. Geometry in Sports
    Hurdles, cross bars, football fields, basketball floors, tennis courts, games played with geometrical objects
    Part played by geometry in training champions in track, pole vaulting or discus throwing
22. Geometry in Nature
    Spider web, snow crystals, honeycombs, cross section of seed pods, leaves, flowers, crystals, butterfly, morning glory, one-celled animals, one-celled plants, natural beauty spots of the world
    Triangular grouping in Easter lily, tulip and dog toothed violet
    Quadrangular arrangement in primrose
    Pentagonal arrangement in hyacinth and tiger lily. (An interesting booklet entitled "You've Been to Geometry-land" shows how Mother Nature used geometry in planning the beauty spots of the world. Full page colored pictures of the following scenic spots were used: a South American landscape taken from a hill, old wheels in a California landscape, a mountain range with a rail fence in the foreground, a scene from the Garden of the Gods, South American power lines, Parliament Building on the Thames, the top of an active volcano, the atomic bomb cloud taken off Bikini in the Pacific, and a famous waterfall. Following each picture there was a diagram emphasizing the geometric arrangements in nature; for example, in the South American scene the trees formed triangles and parallel lines; the mountains in the background, triangles; the peninsula, a circle and a triangle; and the hill in the foreground, a circle.)
23. Geometry in Bridge Construction
    Use of triangle for strength and rigidity
    Circular arc and circular segmental arches
24. Designs in Geometry
    Indian belts
    Sketches of designs based on parallel lines, equilateral triangles, isosceles triangles, squares within squares, stars, crosses, circles, trefoil
25. Geometry in Architecture
    Windows, gables, churches, ancient buildings, modern buildings
26. Geometry in Landscaping
    Parks, roadsides and intersections, homes, public buildings
27. Geometry in Carpentry
    Picture frames, tops for dressing tables, dormer windows, pigpen, roof for barn
    Use of carpenter's square and plane
28. Geometry in Machinery
    Railroads, farm machinery, belts, pulleys, etc.
29. Geometry in Railroads
    Construction of tracks, switches and signals
30. Geometry in Medicine and Surgery
    Pen and ink drawings of instruments and machines
31. Geometry in Churches
    Windows, designs in pulpits, arches, Gothic construction
32. Geometry in Modern Buildings
    Use of parallel lines and rectangles
33. Geometry in Everyday Life
    Curves in eye glasses
    Curved trolley tracks at street intersections
    Examples of planes and other geometric figures
34. Geometry in Millinery
    Drawings or pictures of hats showing effect of lines, symmetry, etc.

35. Geometry in Map Making
    Use of scale drawings
36. Optical Illusions
    Psychology in pictures
37. Making Geometric Figures Using only Straight Lines or Curves in Stitching
    Stitching designs on cards in colors from two basic designs, and the angle and the circle
38. Constructions in Geometry
    Ranging from simple constructions to more difficult ones.
39. Photographs of Uses of Geometry in the Community
    Show use of geometric designs, curves, symmetry and proportion
40. Harmony, Line, Rhythm, and Unity
    Table setting
    Arrangement of furniture
    Selecting drapes
    Designing and choosing clothes to better advantage
41. Geometry in Aviation
42. Geometry in Navigation
43. Geometry in Games
    Chinese checker board, cards, football, basketball, and shuffleboard
44. Geometry in a Camera
45. Geometry in Neckties
46. The Triangle in Everyday Life
    Buildings, bridges
47. The Circle in Everyday Life
    Nature, industry, and home
48. A Day with Geometry
    Follow through a typical day showing contacts with geometry
49. Areas of Plane Geometric Figures
50. Making Models from Erector and Tinker-Toy Sets
    Large models suitable for class demonstration
    Small models suitable for seat work
    Triangles (Isosceles, right, 30–60 right, 45–45)
    Quadrilateral, angle bisector, parallel rulers, protractor with sliding bar, cross staff, center finder, etc.
51. Vocations Requiring a Knowledge of Geometry
52. Reasons for Studying Geometry
53. History of Geometry
54. Great Men in Mathematics
    Pictures of Pythagoras, Archimedes and others with a short biography and their contributions
    History and travels of Pythagoras
55. Application of Triangles to Problems of Rigidity and Circles to Problems of Locomotion
56. Construction of Floor Plans of a House
    Construct plans of cardboard or other material according to scale
57. Compose an Assembly Program to Popularize Geometry
58. Pythagorean Theorem
    Show the mathematics necessary to prove the theorem
    Make an illustrated story including a list of all the theorems, constructions, postulates, axioms and definitions required to prove the theorem
    Construct a model of wood or plastic illustrating proof of theorem
59. Locus in Geometry
    Illustrate the story of an original problem by building a scene of a hidden treasure, etc.
60. Geometry of Flower Arrangement
    Show the use of the triangle, line, symmetry, and balance by diagrams
61. Examples of Ratio and Proportion
    Uses in everyday life
    Newspapers
62. Making a Pantograph
    Explain principle upon which it operates
    Explain its use by series of diagrams
63. Making a Transit or other Surveying Instruments
64. Linkages
    Construction and use
65. Construction of Nine-Point Circle
    Use of paper of at least 12" by 18"
    Begin on a 60–40 degree triangle
    Colored pencils increase readability of the drawing (The midpoints of the sides, the feet of the altitudes and the midpoints of the segments joining the vertices to the orthocenter of any triangle lie on a circle.)
66. Topology or Rubber Sheet Geometry
    Moebius strip
    Map coloring—only four colors are necessary to distinguish one country from another on a flat map
67. Geometry in the Kitchen
    Food, utensils, measurements, etc.
68. Conflicting Assumptions in Everyday Life
    Analyze reasons and their logical conclusions in common situations of school conduct
    Conflicting assumptions in government, advertising, etc.
69. Types of Reasoning in Geometry Applied to Everyday Life
    Give non-geometric examples of converses, negative analogy, irrelevant facts, special cases, generalizations, jumping to conclusions, fallacious reasoning, and critical thinking
70. Window Transparencies
    Cut original design out of black paper, and use thin paper of various colors to replace cut-out paper. The paper may be oiled or dipped in melted paraffin
71. Paper Folding in Geometry
    Explain a number of geometric principles by folding paper
72. Geometry in the Alphabet
    Show the use of lines and curves in capital letters
    (A very interesting notebook was produced showing the use of the triangle, circle, square, half-triangle, semi-circle, half square and double semi-circle in the

capital letters of the alphabet. First the square was set up as the unit of measure, and its relationship to the other figures mentioned above was shown by means of a diagram; for example, the area of a circle with diameter equal to the side of the square would be .7854 or approximately $\frac{3}{4}$. Then the letter "O" was constructed within a square. Finally the twenty-six capital letters were constructed in simple design with ruler and compass, one on a page with an explanation of each given below; for example; "The letter A is an isosceles triangle with crossbar parallel to the base and dividing the sides proportionally." A similar explanation pointing out the geometric processes used in each was given for each letter.)

Bibliography

Ahern, Lorella, "Art in Geometry," The Mathematics Teacher, 1939. 32: 156–162.

Biegeleisen, J. I. *The ABC of Lettering*. New York, Harper and Brothers, 1940, pp. 53–65.

Bradley, Carolyn and Detz, Meredith, *Costume and You*. Scranton, Pa., International Textbook Co., 1949.

(Good for geometry in faces, clothes and fabrics.)

Carson, Byrta, *How to Look and Dress*. New York, McGraw-Hill Book Co., 1949.

(Chapter three shows the use of proportion, harmony, lines and balance in clothes.)

Downer, Marion, *Discovering Design*. New York, Lothrop Lee and Shepard Co., 1947.

(Excellent for showing the use of symmetry, rhythm, line and the abstract in design.)

Encyclopaedia Britannica, Chicago, University of Chicago, 1947. Vol. 10, pp. 178–182.

(History of geometry)

Estrin, Micheal, *2000 Designs, Forms and Ornaments*. New York, Knickerbocker Publishing Co., 1947.

(An idea and reference book in the development of design)

Gaer, Joseph, *Everybody's Weather*. New York, J. B. Lippincott Co., 1944.

(Excellent illustrations to show the use of geometry in weather)

Hogben, Lancelot, *Mathematics for the Million*. New York, W. W. Norton and Co., Inc., 1937.

(Contains a chapter on what you can do with geometry.)

Huntington, Harriet, *Tune Up*. New York, Doubleday, Doran and Co., Inc., 1942.

(Illustrations of musical instruments)

Kasner, Edward and Newman, James, *Mathematics and the Imagination*. New York, Simon and Schuster, 1940.

Keniston, Rachel P. and Tully, Jean, *Plane Geometry*. Boston, Ginn and Company. 1946.

Morgan, J. McDowell, *Military Medals and Insignia of the United States*. Glendale, Calif., Griffin-Patterson Publishing Co. 1941.

(Excellent illustrations to show the use of geometrical design in metals and insignia.)

Morros, Nelson, "Topology Party," *Science Illustrated*, 1948. March: 20–23.

National Council of Teachers of Mathematics, 18th Yearbook, *Multi-Sensory Aids in the Teaching of Mathematics*. Bureau of Publications, Teachers College, Columbia University, New York. 1945.

(Locus, linkages, how to construct a transit, pantograph, etc.)

National Council of Teachers of Mathematics, Commission on Post War Plans, *Guidance Pamphlet in Mathematics for High School Students*. The Mathematics Teacher, 1947. 40: 315–338.

(Gives vocations requiring a knowledge of all branches of mathematics)

Norling, Ernest R., *Perspective Made Easy*. New York. The Macmillan Company. 1949.

(Emphasizes the use of line, cylinders and

parallels in perspective. Also shows some arrangement of furniture.)

Pingry, Robert E. "A Lesson in Appreciation; The Nine-Point Circle," THE MATHEMATICS TEACHER, 1948. 41: 314–136.

"Psychology in Pictures, "*Science Illustrated*, 1948. March: 57–59.

(Gives optical illusions)

Rockwell, F. F. and Grayson, Esther C. *The Complete Book of Flower Arrangement*. New York, The American Garden Guild, Inc.

(Use of triangles, lines etc. in flower arrangements)

Rosskopf, Myron I., Aten, Harold D., Reeve, William D. *Mathematics, A First Course*. New York, McGraw-Hill Book Co., 1951.

(Application of mathematics to aviation)

Row, Sundara, *Geometric Exercises in Paper Folding*. LaSalle, Illinois, Open Court Publishing Company, 1941.

(87 illustrations of geometric processes by paper folding)

Sanden, Arthur G., "How to Draw the Human Head, Head Proportions and Facial Expressions," *School Arts Magazine*, No. 108, Worcester, Mass.

Seymour, Eugene S., and Smith, James P., *Plane Geometry*. New York, The Macmillan Company, 1949.

(Good for types of reasoning)

Shuster, Carl N. and Bedford, Fred L., *Field Work in Mathematics*. New York, American Book Company, 1935.

(Shows uses of sextant, pantograph and transit)

Simon, Howard, *500 Years of Art in Illustration*. Cleveland, Ohio. The World Publishing Company. 1945.

(Shows the use of geometry in all types of art)

Sykes, Mabel, *Source Book of Problems for Geometry*. Boston, Allyn and Bacon, 1912.

Tucker, Albert W. and Bailey, Herbert S., Jr., "Topology," *Scientific American*, 1950. Vol. 182, No. 1, pp. 18–24.

Verrill, Alpheus Hyatt, *Minerals, Metals, and Gems*. Boston, L. C. Page and Company, 1939.

(Geometry in Nature)

ANNUAL SUMMER CONFERENCE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
San Francisco, California, July 2, 1951

Theme: *Improving the Teaching of Mathematics to Meet the Emergency*
Program: Morning Symposium: *Mathematics and the Present Emergency*
Luncheon: *Get-Acquainted Opportunity*
Afternoon Symposium: *The Training of Mathematics Teachers*
Hotel Reservations:
Hotel reservations will be handled by the NEA Housing Bureau, Room 200, 61 Grove St., San Francisco. Complete information as to available hotels and rates, and an application form appear in the *NEA Journal*. Application can also be made by letter to the NEA Housing Bureau at the above address. In writing please be sure to include:
Preferences of hotels and price range desired
Type of accommodation (single, double bedroom, twin bedroom)
Names and address of all occupants
Dates of arrival and departure
A deposit check, payable to the NEA Housing Bureau, for $10 per room requested
Local Chairman:
Dr. Arthur Hall
San Francisco State College
San Francisco, California

PRESIDENT'S PAGE

YOU WILL be interested in knowing something about the *Cooperative Committee on the Teaching of Science and Mathematics* of the American Association for the Advancement of Science, a committee of some importance and on which the National Council is represented. This committee now numbers fifteen members. Thirteen different national science and mathematics organizations and the United States Office of Education are participating. Among these organizations are: The National Council of Teachers of Mathematics, the Mathematical Association of America, the Central Association of Science and Mathematics Teachers, the National Science Teachers Association, and others from the fields of Astronomy, Physics, Geology, Zoology, Chemistry, and Biology. The National Council of Teachers of Mathematics has been represented on this committee for the past six years. The last meeting of the Cooperative Committee was held in Washington, D. C. at the Federal Security Building on November 17 and 18, 1950, as a part of the Science Education Conference arranged by the Office of Education. This was the second meeting of the Cooperative Committee with the Office of Education and it is planned to hold these meetings annually in the fall. At the first meeting of the Committee with certain members of the staff of the Office of Education in December 1949, the two groups agreed to strengthen the relationships between the Committee and the Office of Education through mutual exchange of information and advice and through joint meetings. The Committee also meets annually in Chicago in the spring.

The AAAS Cooperative Committee concerns itself with the improvement of science and mathematics teaching, as well as with the advancement of the sciences themselves. The Committee has been active for only a few years but their work has been effective. Some of the matters to which they have given attention are: certification of teachers, teacher training programs, the science curricula for college preparation and for general education, laboratory equipment and procedures. The Committee has come out with rather definite recommendations on these and other problems.

Their recent attachment to the United States Office of Education should make more effective the work of the AAAS Cooperative Committee. There will be a material exchange of information and advice on many vital problems such as, mathematics and science in general education at all levels, the early identification of talented youth and the proper educational opportunities for them, the training of mathematics and science teachers, and the problem of adult education in mathematics and science. The fact that the Cooperative Committee plans to hold at least one meeting a year with the Office of Education for the review and evaluation of the activities of both groups, for the discussion of problems of mutual concern, and for cooperative planning is bound to result in making more effective the work of both groups.

For the past two years John R. Mayor has represented the National Council on the AAAS Cooperative Committee. As it now stands, there are three mathematics representatives on the committee: George E. Hawkins, representing the National Council of Teachers of Mathematics; John R. Mayor, representing the Mathematical Association of America; and Donald W. Lentz, Ridge Road School, Parma, Ohio, representing the Central Association of Science and Mathematics Teachers. The former chairman, K. Lark-Horowitz, Purdue University, has found it necessary to resign because of pressure of other duties. The new chairman is Morris Meister, Bronx High School of Science, New York.

H. W. CHARLESWORTH, *President*

ATTENDANCE RECORD OF THE ELEVENTH CHRISTMAS MEETING

University of Florida, Gainesville, Florida
December 27-29, 1950

* Member of the National Council of Teachers of Mathematics.
(E) = Elementary School, (J) = Junior High School, (H) = High School, (JC) = Junior College, (C) = College, (V) = Visitor, (Ex) = Exhibitor, (Su) = Supervisor, (St) = Student.

ALABAMA
- Birmingham
  - *Garrett, Jeannette (H)
- Livingston
  - *Killebrew, H. A. (C)
- Tuscaloosa
  - *Lewis, F. A., Mrs. (C)

ARKANSAS
- Arkadelphia
  - *Young, Ralph W. (C)
- Hot Springs
  - Rapley, John H.

COLORADO
- Denver
  - *Charlesworth, H. W. (H)
  - Charlesworth, Mrs. (V)

DISTRICT OF COLUMBIA
- *Dickert, Eddie (H)
- *Douglas, Elinor V. (H)
- *Lane, Ruth (H)
- *Motyka, Agnes (E)
- *Schult, Veryl (C)
- *Wilson, Dorothy (J)

FLORIDA
- Avon Park
  - *Shearer, Mary C.
- Bradentown
  - *Fitzgerald, Louise (J)
- Cherry Lake
  - Dugan, Russ (Ex)
- Clearwater
  - *Hammock, Ivy (J)
  - *Johnson, Dorris (J)
- Cocoa
  - *Pomeroy, Jeanette (H)
  - Pomeroy, W. H. (V)
- Daytona Beach
  - Greer, C. W. (Ex)
- DeLand
  - Foard, Betty (H)
- Frostproof
  - *Webb, Celeste
- Gainesville
  - Awtrey, R. A. (St)
  - Boles, Ralph C. (C)
  - *Kidd, Kenneth P. (C)
  - *Kokomoor, F. W. (C)
  - *Phillips, Wayland (E)
  - *Phipps, Dorothy (H)
  - *Young, Laura (H)
- Haines City
  - *Barrett, Lerlin (H)
- Havana
  - *Evans, Jane (H)
- Jacksonville
  - *Haggard, Margaret
- Lakeland
  - *Brown, Ruth H.
  - *McIntyre, Maxine
  - *Mustoe, Anthony Q. (C)
  - *Reinsch, Bernard P. (C)
- Mayo
  - *Davis, Lera (H)
- Miami
  - *Annin, Elizabeth (H)
  - *Gibbs, Fletta (H)
  - *Kimler, Verna (H)
  - *Mahood, Mildred (J)
  - *Pierce, Gladys (H)
  - *Rollins, Annie L. (H)
- Miami Beach
  - *Carlton, Charlotte (H)
  - *Lyle, Lilla G. (H)
- Milton
  - *Hamilton, Geo. C. (H)
- Ocala
  - *Cromartie, Virginia (J)
  - *Green, Edna (J)
  - *Parrish, Josie (H)
- Orlando
  - *Echols, Beulah (J)
  - *McEntire, Evalyn (H)
  - *Phillips, Theresa (H)
- Palmetto
  - *Warren, Bertha
- Pensacola
  - *Habel, Elmer (JC)
- Plant City
  - *Futch, Cedora (H)
  - *Wilson, Margaret (H)
- St. Augustine
  - *Spencer, Mrs. J. J.
- St. Petersburg
  - *Barnes, Ruth
  - *Harper, Martha (H)
  - *Healey, Frances (J)
  - *Scott, Mrs. Wm. C. (JC)
- Sanford
  - *Lynch, Elizabeth (H)
  - *Riser, Ethel (H)
- Sarasota
  - *Ziegler, Mae (H)
- Seabring
  - *Titus, Julia (H)
- Tallahassee
  - *Badger, Mrs. W. V. (C)
  - Badger, W. V. (V)
  - Echols, Katie S.
  - *Hyman, H. H. (C)
  - *Outlaw, W. J. (C)
  - *Plant, Joe (C)
  - *Rehwinkel, Myrle (H)
  - *Trimble, Harold (C)
- Tampa
  - *Bolser, F. C. (C)
  - *Craver, Lucille (J)
  - *De Shong, Virginia
  - *Ellis, Irma (J)
  - *Gallant, Howard (H)
  - Hinton, W. C. (St)
  - Mitchell, Calvin (St)
- Winter Haven
  - *Norton, Elizabeth
  - *Smith, Phyllis (H)

GEORGIA
- Athens
  - Harrell, Bill (Ex)
- Atlanta
  - Crane, Thomas (Ex)
  - Durhman, W. C. (Ex)
  - Godwin, R. B. (H)
  - *Huie, H. Mark
  - Johnwick, E. F. (Ex)
  - *Lindsey, Edna E. (H)
  - *Patton, Bess (H)
  - Shipp, John S. (Ex)
- Collegeboro
  - *Moye, W. B. (C)
- College Park
  - *Dolvin, Kathleen (H)
  - *Wilson, Lottie C. (H)
- Macon
  - *Slocumb, Margaret (J)
  - *Waters, Gladys (J)
- Sandersville
  - *Thomason, Gladys (H)
- Savannah
  - *Blakeley, Laura (C)
- Valdosta
  - *Babcock, Leroy (C)

ILLINOIS
- Chicago
  - *John, Lenore S. (E)
  - *Johnson, John T. (C)
  - Mann, Orion A. (Ex)
  - Saastad, Arthur (C)
- Elmwood Park
  - Janicki, George (E)
- Evanston
  - *Bradfield, Geo. (H)
  - *Hildebrandt, E. H. C. (C)
- Macomb
  - *Ayre, H. Glenn (C)
- Oak Park
  - *Woodruff, R. S. (H)
- Pekin
  - *McCoy, M. Eleanor (H)
- Peoria
  - *Gault, A. E.
- Urbana
  - *Snader, Daniel W. (C)
  - Snader, Mrs. D. W. (V)
- Woodriver
  - *Hicks, Olive V. (H)

INDIANA
- Bloomington
  - *Ibson, Rebecca (H)
- Indianapolis
  - *Wilcox, Marie S. (H)
- Lafayette
  - *Carnahan, W. H. (C)
  - Carnahan, Mrs. W. H. (V)
- Muncie
  - *Edwards, P. D. (C)
- West Lafayette
  - *Cooper, Hazel (H)

IOWA
Cedar Falls
*Van Engen, H. (C)
Mason City
*Constable, W. L. (H)

KANSAS
Lawrence
*Ulmer, Gilbert (C)
Manhattan
*Sanger, R. G. (C)

KENTUCKY
Bowling Green
*Jones, Edwina (J)

LOUISIANA
Baton Rouge
*Karnes, H. T. (C)
Delhi
*Warden, Meta W. (H)
Lafayette
*Begnaud, Lurnice (H)
*Heard, Ida Mae (C)
*Hoag, Jessie May (C)
*Loflin, Z. L. (C)
McGuire, Norma (C)
Natchitoches
*Maddox, A. C. (C)

MARYLAND
Baltimore
*Herbert, Agnes (J)
Herbert, Mr. (V)

MASSACHUSETTS
Boston
*Betts, Barbara B. (Ex)

MICHIGAN
Ann Arbor
*Jones, Phillip S. (C)
East Lansing
*Blyth, Isobel (C)
Kalamazoo
*Bartoo, G. C. (C)

MINNESOTA
Northfield
*Wegner, J. W. (C)

MISSISSIPPI
Hattiesburg
*Thomas, Ed. J. (C)
Thomas, Mrs. Ed. J. (V)

MISSOURI
Columbia
*Erkiletian, D. (C)

NEW HAMPSHIRE
Exeter
*Adkins, Jackson B. (H)

NEW JERSEY
Montclair
*Mallory, Virgil S. (C)
Mallory, Mrs. Virgil S. (V)
Newark
*Farquhar, Hazel (H)
Trenton
*Shuster, Carl N. (C)
Shuster, Mrs. Carl N. (V)
Union
*Zofay, Agnes (H)
Westfield
*Rogers, Mary C. (J)

NEW YORK
Albany
*Newsom, C. V.
Buffalo
*Gehman, H. M. (C)
*Schneckenburger, Edith (C)
Canastota
*Daniels, Gertrude (H)
Canisteo
*Longley, Russell (E)
Jamestown
*LaVier, Maude A. (J)
New York City
*Alfred, Brother Bernard (C)
*Bakst, Aaron (C)
Bakst, Mrs. Aaron (V)
*Fehr, Howard F. (C)
Fehr, Mrs. Howard F. (V)
*Higgins, William B. (C)
*Jablonower, Joseph
Niagara Falls
*Horn, Ruth
Syracuse
*Farndacher, Karl (St)

NORTH CAROLINA
Durham
*Rankin, W. W. (C)
Rankin, Mrs. W. W. (V)
Greenville
*Phillips, Orval L. (C)
Marion
*Alley, Catherine (H)
Wilmington
Morris, Texys (H)

OHIO
Columbus
*Lazar, Nathan (C)
*Schacht, John F. (H)
Granville
*Rupp, E. C. (C)
Kent
*Brumfield, Ema Lou (C)
*Stapleford, Edward (C)
Medina
*Insprucker, J. H. (H)
Oxford
*Christofferson, H. C. (C)
Christofferson, Mrs. (V)

OKLAHOMA
Stillwater
*Zant, James H. (C)
Zant, Mrs. James H. (V)

OREGON
Portland
*Hoel, Lesta

PENNSYLVANIA
Bala-Cynwyd
*Lamb, Beatrice (J)
Elizabethtown
*Heilman, C. E. (C)
Heilman, Mrs. C. E. (V)
Kingston
*Jones, Freda
Philadelphia
*Wyatt, Ruth
Uniontown
*DiPrampero, Louise (H)

RHODE ISLAND
Providence
*Bennett, Albert (C)

SOUTH CAROLINA
Columbia
*Dicks, A. Lois (C)
Rast, Dorothy (St)
Greenville
*Werts, Lily Mae (H)
Rockhill
*Rogers, Hortense (C)
Silverstreet
*Werts, Eoline (E)

TENNESSEE
Knoxville
*Brown, Kenneth E. (C)
Brown, Mrs. K. E. (V)
Nashville
*Banks, J. W. (C)
*Miser, W. L. (C)
Union City
*Ray, Vera (H)

TEXAS
Dallas
*Fulton, Will P. (H)
Huntsville
*Lane, Ruth O. (C)
Smith, Lolie (C)
Nacogdoches
*Layton, W. I. (C)
San Antonio
*Rees, Dorothy (H)
San Marcos
*Cude, Don (C)

VIRGINIA
Arlington
*Burington, Artha (H)
Charlottesville
*Lankford, F. G., Jr., (C)
Lankford, Mrs. (V)
Lexington
*Knox, R. H. (H)
*Purdie, K. S. (H)

WASHINGTON
Seattle
*Haynes, Helen (J)

WEST VIRGINIA
Montgomery
*Nolan, Irene (C)
Morgantown
*Dorsey, Catherine (H)
St. Albans
Holmes, Margaret F. (H)
*Lynch, Kathryn (H)

WISCONSIN
Madison
*Mayor, John R. (C)
Racine
*Potter, Mary (Su)

CANADA
Toronto
*Petrie, P. A. (C)
Petrie, Mrs. P. A. (V)

CUBA
Havana
*Rivas, Silvia (H)
*Rodriques, Maragrita (H)

(*Continued on page 244*)

SIGNS OF PROGRESS OF THE AFFILIATED GROUPS PROGRAM

By JOHN R. MAYOR
Chairman Committee on Affiliated Groups

BY THE time these paragraphs are in print the Second Delegate Assembly will be past history. Early reports indicate that the Second Delegate Assembly, like the First in Chicago, should be an important event in the history of the National Council. In this Second Assembly nearly fifty delegates, representing members of Affiliated Groups, including some 5,000 members of the National Council and more than 10,000 teachers of mathematics, will have debated policies of the National Council and made decisions important for the future of the Council.

Members of the Affiliated Groups should start immediately to plan for the Third Delegate Assembly. These meetings can be most valubale if the local groups provide adequate opportunities for their members to express opinions on all issues debated. Your Group should give consideration throughout the year to ways and means of making the National Council, with the assistance and cooperation of its Affiliated Groups, a more effective agency for the improvement of mathematics instruction at all levels and in all parts of the country. There has never been a time in the history of American education when teachers of mathematics faced a greater challenge and a greater opportunity.

Following conferences at the Florida meeting of the National Council in December plans are being developed for a number of new activities to be sponsored through the Affiliated Groups. These plans include the organization of a Speakers Bureau on which a preliminary announcement was made in the December issue of THE MATHEMATICS TEACHER; a membership campaign, which should benefit both the National Council and the Affiliated Groups, and which is timed so that new members may take advantage of the special reduced price offer on the 19th and 20th Yearbooks; a cooperative investigation of some critical problems in mathematics education; and the organization of a traveling exhibit which may be shared by various Affiliated Groups.

Participation of the Affiliated Groups will, of course, be on a completely voluntary basis. Some of the larger Groups and those which have been longer established may wish to participate in all of these activities. Other groups may choose to take part in only one of these activities, or perhaps none for the first year. Naturally the Groups will be free to determine their own organizational set-up for carrying on this work. In many cases it may be desirable for the Affiliated Groups to organize coordinating committees to work in conjunction with the Committee on Affiliated Groups as proposed by the First Delegate Assembly. It is in the best interests of all concerned that as many different teachers as possible be brought into this work and that in no case a "chosen few" have all the privileges or all the burdens involved in active participation in Group work.

As plans materialize they will be reported in this section of THE MATHEMATICS TEACHER. In the development of plans the Newsletter of Affiliated Groups will be used as a means of communication. One of the purposes of early announcement of plans in this section is to give all readers of THE MATHEMATICS TEACHER an opportunity to make suggestions during the developmental stages and to offer assistance and cooperation to their own Affiliated Groups or to the national committee.

In the May number of THE MATHEMATICS TEACHER, in this section, an analysis will be made of the organization of the various Affiliated Groups with par-

ticular reference to the cooperation of city and county groups, within a state, with a state organization. This report is to be made upon special request.

New Groups in Philadelphia and in Pinellas County

The Association of Teachers of Mathematics of Philadelphia and vicinity is the first Group to apply for affiliation with the National Council in 1951. In the letter from M. Albert Linton, Jr., of the William Penn Charter School and president of the Association, which brought the good news that the Philadelphia mathematics teachers wished to become affiliated with the National Council was this sentence:

"It is my sincere hope that affiliation with the National Council will serve as an incentive to us to try to improve our services to the mathematics teachers of Philadelphia and suburbs." Since this so well expresses also the sincere hope of the Committee on Affiliated Groups for each and every Affiliated Group it is repeated with great appreciation to Mr. Linton.

At the Florida meeting, the attention of the Chairman of the Committee on Affiliated Groups was called to the omission of the Pinellas County (Florida) Council of Teachers of Mathematics from the list of Affiliated Groups in The Mathematics Teacher for December. The Chairman apologizes for this error and is happy to welcome Pinellas County to the family of Affiliated Groups especially after the fine contributions of teachers from Pinellas County to the Christmas meeting in Gainesville. All who attended the Florida Meeting are grateful for the hospitality shown by all of the Affiliated Groups in Florida which cooperated in making that meeting a success.

A Ten Year Record of 100 Per Cent Membership

The A.T.M.N.E. Newsletter for November, 1950 includes the following announcement:

"The Roxbury Latin School has had a 100 per cent membership in the A.T.M.N.E. for the past ten years. Can any other school equal or exceed this claim?"

This question, addressed to members of the Association of Teachers of Mathematics of New England, is repeated for all other Affiliated Groups. Can other Affiliated Groups point to examples of such long and loyal support?

Attendance Record, 11th Christmas Meeting

(*Continued from page 242*)

ATTENDANCE RECORD

State	Members	Guests	Total
Alabama	3	0	3
Arkansas	1	1	2
Colorado	1	1	2
District of Columbia	6	0	6
Florida	60	10	70
Georgia	11	6	17
Illinois	11	3	14
Indiana	5	1	6
Iowa	2	0	2
Kansas	2	0	2
Kentucky	1	0	1
Louisiana	7	1	8
Maryland	1	1	2
Massachusetts	1	0	1
Michigan	3	0	3
Minnesota	1	0	1
Mississippi	1	1	2
Missouri	1	0	1
New Hampshire	1	0	1
New Jersey	5	2	7
New York	13	2	15
North Carolina	3	2	5
Ohio	7	1	8
Oklahoma	1	1	2
Oregon	1	0	1
Pennsylvania	5	1	6
Rhode Island	1	0	1
South Carolina	4	1	5
Tennessee	4	1	5
Texas	5	1	6
Virginia	4	1	5
Washington	1	0	1
West Virginia	3	1	4
Wisconsin	2	0	2
Canada	1	1	2
Cuba	2	2	4
Totals	181	42	223

KNOW YOUR NATIONAL COUNCIL REPRESENTATIVE

By KENNETH E. BROWN
University of Tennessee, Knoxville, Tennessee

MISSOURI REPRESENTATIVE

MISS ELLA MARTH is a native of Missouri where she was granted three degrees, including the Ph. D. from St. Louis University. The special work in graduate mathematics and the 13 years as a classroom teacher in the elementary schools of St. Louis have provided her with a rich background of experience that makes her particularly fitted for her present position of the training of future mathematics teachers. In addition to her duties as a faculty member of Harris Teachers College and consultant in arithmetic for St. Louis Public Schools, she spends many hours in giving publicity to the work of the National Council. Her efforts have been rewarded by a substantial gain in membership in Missouri since she became State Representative in 1947.

MISS ELLA MARTH

MISS EUNICE LEWIS

OKLAHOMA REPRESENTATIVE

MISS EUNICE LEWIS was born in Tennessee, however her high school and college education was received in Oklahoma. Her experience in teaching in the high schools of Covington, Sapulpa, and Tulsa has given her a knowledge of high school problems that is invaluable in her present position of training prospective mathematics teachers at the University of Oklahoma. As Supervisor of Mathematics at the University High School she keeps in close touch with high school students and their problems. The mathematics teachers indicated their high regard for the professional zeal of Miss Lewis by electing her President of the Oklahoma Council of Teachers of Mathematics. The membership of the National Council in Oklahoma has increased more than fifty per cent during the last four years under her guidance.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

MODELS MADE FROM MAKIT TOY

THE following devices are easily constructed of MAKIT TOY, a colorful, sturdy, building set manufactured by the W. R. Benjamin Company of Granite City, Illinois. The set retails in three sizes, PRIMARY, SUPER, and JUMBO MAKIT. The JUMBO size costs about $2.50, contains 275 pieces, and is sufficiently complete to permit the simultaneous construction of two or three working models. Directions for construction in this article make direct reference to the names of the parts included in a set.

Angle Demonstration Device

Two colored wheels, a metal bushing, a collar, a short red rod, and three plain rods are needed to construct this device. (See Fig. 1.) Red rods fit loosely in wheel holes; plain rods fit tightly. Red rods capped with metal bushings also fit tightly.

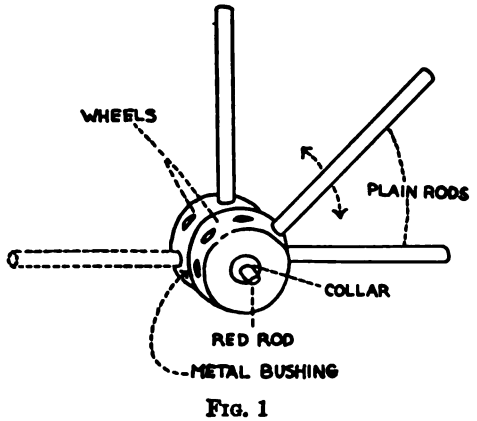


FIG. 1

Insert the red rod capped with the metal bushing into the center of one of the wheels. Slide the red rod through the center of the other wheel, and push the collar down over the exposed end of the red rod. The result is a potential vertex of the angles to be constructed. It may be designated as the "red vertex," the "yellow vertex," or the "red-blue vertex" depending on the choice of colored wheels.

Insert the two plain rods in the holes of one wheel at right angles to each other. Fix the third plain rod in the other wheel. The finished product is a right angle with a movable third line which may be used to show how angles are generated, to illustrate the concept that a degree is a unit of revolution independent of the length of the sides of an angle, to demonstrate the inverse variation of complementary angles, and to illustrate the definition of complementary angles.

The device can be adapted to supplementary angles by fixing the plain rods in the holes so as to form a straight angle. (Note the position of the rod drawn with dotted lines.)

Triangle Demonstration Device

To build this device, construct three vertices, each of a different color, by the method described above. Choose three plain rods and insert them in the wheel holes, making sure that no two rods are inserted in the same wheel. (See Fig. 2.) Keep one end of the third rod free for the purpose of demonstrating to the class that each angle varies freely. By fixing the third rod in the remaining wheel, the rigidity property of a triangle may be demonstrated, and also the corresponding

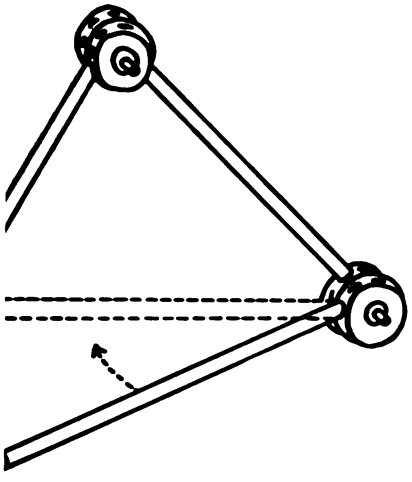


FIG. 2

t triangles are determined by the
of their sides.
ods in MAKIT TOY are so pro-
d that two of one length and one
ext longer will form an isosceles
angle. Also, combinations of short
th joining wheels are equal to
rods. Thus, models illustrating
triangles may be built quickly
lass time by students themselves.

eneral Quadrilateral Device

nore "wheel vertex," two equal
s, two equal shorter rods, and two
ds of sizes different from the two
ntioned are required to transform
ngle demonstration device into a
quadrilateral device. (See Fig. 3.)
ning with four plain rods of dif-
engths, it is possible to illustrate

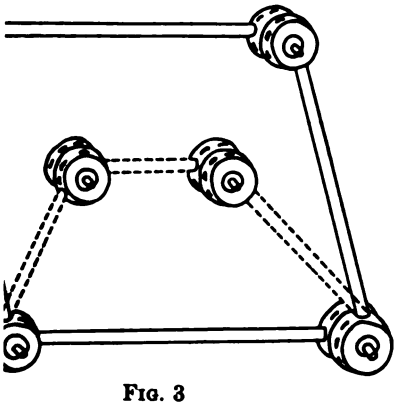


FIG. 3

the evolution of the various types of quadrilaterals by substituting rods in accordance with the special conditions of the definition of each figure. The parallelogram can be continuously deformed to show that no matter how the angles are permitted to vary, the sides remain parallel and equal.

By inserting the rods at one vertex into a single wheel at right angles to one another, the principle that a rectangle is a parallelogram with one right angle may be demonstrated.

The possibilities for constructing working models of MAKIT TOY are limited only by time and imagination, and the models are adaptable to classes in both the junior and senior high school.

Nona Mary Allard
Cromwell Public Schools
Cromwell, Minnesota

HEXAHEXAFLEXAGRAMS

Editor's Note. This article is being published in response to several special requests. After reading it you'll be as excited about it as the people who asked to have it printed. Upon receiving the manuscript the editor of this department couldn't wait to get his hands on a suitable strip of paper and produce one of "those things." The directions are remarkably clear and easy to follow—a tribute to the ingenuity of Miss Margaret Joseph who submitted the article. High school students will enjoy making the hexahexaflexagram and "flexing" it.

To make the hexahexaflexagram, obtain a piece of adding machine paper about 28 inches long. The exact length needed will depend on the width of the strip used.

Fold one end forming a crease about 1½" long. See Figure 4. Then fold the paper so that point *B* falls along crease *CD*

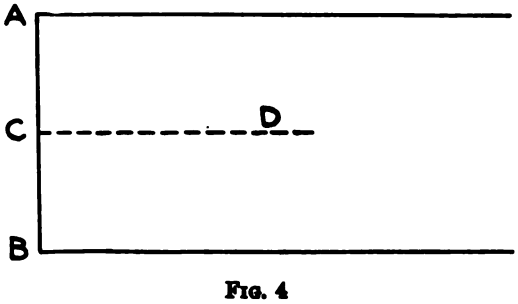


FIG. 4

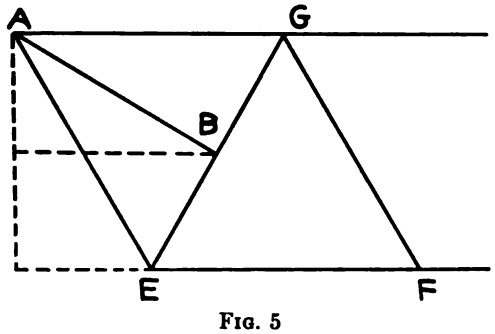


FIG. 5

forming triangle *ABE* as shown in Figure 5. This triangle will be a 30–60 degree right triangle with the 60-degree angle at *E*. Now fold *AE* along lower edge *EF* to form an equilateral triangle *EFG*. This and all subsequent foldings must be done carefully so that good triangles are formed. If carelessly made, there will be a flat space instead of a vertex for the top of the triangle.

Since the first triangle was formed by folding the paper forward, the second should be made by folding the paper under or away from you so that the triangles are made by repeatedly folding back and forth until 19 equilateral triangles have been made. Cut off the excess that remains as well as the first little right triangle that was made. Then lay the strip flat on a desk and number as shown in Figure 6. The last triangle will be a blank. Turn the strip over so that the end marked *A* will remain on your left. Number this side as shown in Figure 7. If a different colored pencil is used for each number the entire figure will be more attractive.

Now the strip must be folded. There is a rhythm to the folding which becomes apparent after you have made a few folds. Begin by holding the strip in the position *CD* shown in Figure 7. Fold the second triangle numbered 4 over the first one numbered 4, then fold triangle 5 on triangle 5 in the same manner, and triangle 6 on triangle 6. (You may have to look under the strip at times to see the numbers desired.)

Stop now and check your progress. In reading from left to right you should have the following arrangement: a blank triangle, triangle 3, triangle 1, and a second triangle 1 on top. If you do not have this arrangement recheck the directions already given.

Next fold triangle 1 on triangle 1, and another triangle 3 will appear on top. The series of folds made to this point are repeated twice more and then the folding is complete. However, when you come to the last arrangement you will have a blank triangle, and a triangle 1 remaining at the right end of the strip or hexahexaflexagram. Put the blank triangle from the extreme left over the triangle numbered 1. (You will note that there is a 1 on the underside of the blank triangle.) Finally paste the two blank triangles together. When the paste is dry the hexahexaflexagram will easily open or flex so that the same number will always appear on all triangles. Hold the figure towards the bottom and do not force it. If it will not open where you are holding it, change the position of your fingers and try again. All six numbers can be made to appear.

MISS MARGARET JOSEPH
South High School
Milwaukee, Wisconsin

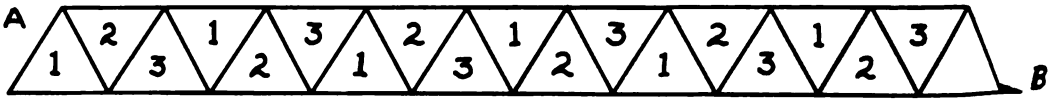


FIG. 6

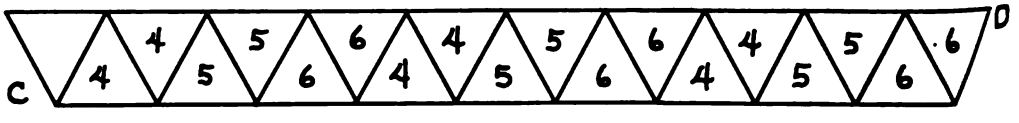


FIG. 7

ID GEOMETRY DEMONSTRATION*

'sical principle familiar to nearly igh school student is that the ' of illumination varies inversely iquare of the distance from the of light. The existence of this ı can readily be verified by using ate science laboratory apparatus. r, it is much more interesting and g to use essentially the same dem- ɔn materials and examine the from the standpoint of the mathe- principles involved.

pparatus to be used consists of a , yardstick, two wooden triangular supports, one opaque cardboard ı ft. square, and four smaller The central parts of the smaller should be cut out. Figures to be may include a circle, triangle, ınd pentagon. Either the cut-out or the figures cut out may be the demonstration. In the first base of the figure (to be described 'ill be an illuminated area; in the ase it will be a shadow.

pparatus is set up as illustrated . The large screen (Y) and one of l screens (X) may be fastened to gular wooden blocks with thumb 'he screens and the candle are ced on top of the yardstick as ı the diagram. The planes of the ıust be parallel to each other and cular to all long edges of the :. When the candle is lighted an lined as triangle A'B'C' will be ed on screen Y. For best results ɔnstration should be conducted in ıed room. Now let P represent ex of a pyramid with triangle s its base. LO' will be equal to the of this pyramid. Triangle ABC ı cut out of screen X) will be a f this pyramid, and LO will be the length of the perpendicular o the plane of triangle ABC.

ect application of a solid geome-

demonstration described in this article ɔped in The Mathematics Laboratory, igh School, St. Paul, Minnesota.

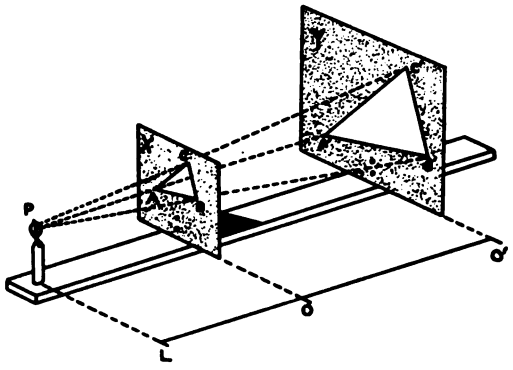


FIG. 8

try theorem and its corollary we obtain the following relation:

$$\frac{\text{Area of Triangle } ABC}{\text{Area of Triangle } A'B'C'} = \frac{(LO)^2}{(LO')^2}.$$

Theorem. If a pyramid is cut by a plane parallel to its base: (a) The lateral edges and the altitude are divided proportionally. (b) The section formed is a polygon similar to the base.[1]

Corollary. A section of a pyramid parallel to the base is to the base as the square of its distance from the vertex is to the square of the altitude of the pyramid.

Substitution of the small screens with the square, circular, and pentagonal cut-outs will help add a note of generality to the result demonstrated with the triangular pyramid. In the case of the circular cut-out it should be pointed out that a theorem and corollary similar to those stated above for the pyramid can be stated for a cone.

The connection between the mathematical principles just described and the principle concerning illumination stated earlier needs only a little clarification. From the theorem and its corollary it is apparent that the area illuminated increases as the square of the distance from the source of light. By the law of conservation of energy we know that the amount of light passing through the cut-out screen X is the same as that illuminating the large screen Y. Hence the intensity of illumination varies inversely as the square of the distance from the source of light.

[1] This statement of the theorem and the corollary which follows were taken from the textbook, *Modern-School Solid Geometry*, by Smith, Rolland R., and Clark, John R., World Book Company, Yonkers-on-Hudson, 1939.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York 3, New York

The Questions: How do students differ in what they do in attempting to solve problems? How can a teacher use his knowledge of these differences to improve general problem-solving ability?

The Study: Bloom, B. S. & Broder, L. J. *Problem-solving Processes of College Stuents.* University of Chicago Press. Chicago 37, Illinois. 1950.

The teacher of mathematics who sees evidence that his efforts are accompanied by an improvement in the general problem-solving ability of his students probably feels that he is making a major contribution to both general and special education. This exploratory study is a rich source of suggestions of ways of bringing about this much-desired result.

It is tempting to assume that the problem of problem-solving is the same for all students and, therefore, to infer that it is sufficient to standardize the teaching of problem-solving by patterning it after some such analysis as Dewey's famous five steps described in his book. "How We Think." To most of us it is not news that the fact of individual differences spoils this happy illusion. This study makes that reality even clear.

It is also tempting to believe that a study of the written work of students in doing problems gives all the clues needed for wise and adequate guidance in improving problem-solving ability. This isn't so, either. Hence, it is not surprising to learn that in this study there is used the *method of making a record of what the student says as he thinks aloud in attacking his problems.*

The purpose of the first part of the study was to discover the kinds of ways in which students differ in solving problems. One group of students consisted of six individuals with high scholastic aptitude test scores and with records of A's and B's on achievement examinations given at the College of the University of Chicago; the second group of six students had low aptitude test scores and achievement test grades of *D* and F. These two groups were designated the "successful" and "unsuccessful" problem-solvers respectively. Each of the twelve students worked with twenty problems selected from a variety of fields, such as science, social studies, English and mathematics. These problems were such that memory and puzzle aspects were reduced to a minimum; they were characterized by clear-cut goals and open to definite methods of attack. Each individual student attacked these problems in several sittings in a private conference with one interviewer. As he did so the *interviewer recorded what the student said as he thought aloud about the problems.*

The major differences between the two groups were found to fall into five general categories: understanding the nature of the problem, understanding the ideas in the problem, the general approach to the solution and the attitude toward solutions. The first of these involved difficulties in starting the attack upon the problem and in solving the problem as presented. The second revealed differences in the ability to bring relevant knowledge to bear upon the problem and in the capacity to realize the implications of ideas in the problem. The third showed differences in aggressiveness of attack, in the care and system in thinking about the problem and in the ability to follow through on a process of reasoning. The last uncovered differences in the faith placed in reasoning versus guessing, in the amount of confidence in attacking problems and in the tendency

to introduce personal reactions not implied by the problem.

These characteristics, which varied in these two extreme groups, were checked by employing a group of twenty-seven typical students. The same traits appeared. The major difficulties of this group were in trying to decide what to do, in maintaining an objective attitude about the problems, in reasoning logically and systematically, in completing a chain of reasoning and in bringing to bear relevant subject matter which apparently was known. Six to eight weeks later these students exhibited more carefulness and system, increased confidence, more thinking and, as might be expected, more knowledge of the subject matter involved. It seems apparent that these traits were related. It was noted that there was no change in objectivity toward the problems. Dislike, fear and personal and social values associated with the problems previously were still strong.

Having isolated the differences in the characteristics of the students in solving problems the experimenters next studied the kinds of differences in the problems themselves. Some of these of a trivial nature involved difficulties with the format of the problem, the method of indicating answers, the presence of multiple choice items having negative phraseology and the appearance of relative terms like "frequently" and "usually." The difficulty of the problems also varied with the ease of translating the ideas and terms of the problem into more familiar or more specific ones and with the possibility of reducing a problem to its essentials, bereft of excess verbiage.

Finally, two remedial programs were carried out. The first was an individualized one; the second involved procedures with groups of five to twenty students. The basic assumption was that each individual has his own peculiar cluster of problem-solving characteristics which had grown up with him over a considerable period of time. The criteria employed in selecting the problems for the programs were (1) that a small amount of specific information was needed, (2) that they were the kind usually met in the college examinations and (3) that they should differentiate between good and poor problem achievers. Next, the students were oriented and motivated for the remedial program. They were told, among other things, the principal aim was to produce an improvement in *general* problem-solving ability, that success would depend upon the effort exerted and the development of confidence, and that practice sessions would be essential. The remedial procedures consisted of a determination of the individual problem-solving characteristics by recording the "thinking aloud" comments of the students in two sessions of one and one-half hours. In the next session students were encouraged to analyze their methods and compare them with those of very successful problem-solvers. In the remaining sessions further analysis was made and practice in applying good problem-solving methods to new problems was provided. To facilitate these procedures a pamphlet of sample problems and a printed check list of problem-solving characteristics were made available for student use.

In the experiment with the groups of five to twenty the early sessions were the same as in the individualized approach. After the third session a sample problem would be flashed on a screen. A student volunteer would then give his solution for analysis and criticism by his fellow students. Eventually, the solution would be compared with a model solution for the general methods of problem-solving involved. During the practice sessions which occurred twice a week it was necessary and desirable to divide the large group into smaller groups of two or more students. In each of these the students would take turns presenting their solutions to the members of their small group for analysis and comparison with a model solution. At times the problems

attacked were centered about one kind of subject matter; at others the emphasis was on general methods of problem-solving.

These remedial programs were evaluated by the students themselves, by certain faculty members, by an analysis of the results of a problem-solving test and by a study of the improvement in examination results over the experimental period. The first three methods revealed improvement on the part of a varying number of students in a variety of aspects of problem-solving. One of the most frequent results was an increased confidence in problem-solving. The examination results showed that the gains made by the experimental groups of students were statistically significant to a high degree while the gains of other control groups of comparable ability and scholastic success were inconsequential.

CORRECTION: We regret that on page 153 of the February issue the heading "Annotated References to Research Studies" was inadvertently omitted above the references to Dr. Kramer's study. Readers may have inferred that the study was considered trivial because of the brief comment but we did not intend that such an implication be drawn.—J.J.K.

APPLICATIONS

Edited by SHELDON S. MYERS
University School, Ohio State University, Columbus, Ohio

CONCERNING Ar. 5 Gr. 7–9 "Famous Explosions on the Earth," Dr. John Kinsella writes as follows: "The January MATHEMATICS TEACHER arrived today. Your department pages came into view. There was something on 'famous explosions.' That reference brought back memories of University School where I tried hard to make 7th and 8th grade mathematics function in science materials. I'm not sure the credit for the six problems belongs to me; I do know that the content was provided by Dr. Charles C. Weidemann who was teaching there when I came in September 1942. I may or may not have provided the questions on the material." In giving credit your editor used the evidence of the initials "JJK" in the corner of the guidesheet. Therefore, it is likely that Dr. Kinsella made up the questions based on materials gathered by Dr. Weidemann.

Al. 6 Gr. 9–10 TRANSLATING STATEMENTS INTO ALGEBRAIC FORMULAS

Paul J. Eby, mathematics instructor in General College, Boston University, sent us the following problems about which he wrote: "I use them to give practice in symbolizing verbal relationships and solving simple equations. The following problems[1] may provide interesting sidelights for a class in elementary algebra. An excellent way to use them is to present them as they are and ask the pupils to write a formula to represent the statement. When this is done, they may collect their own data and solve the problem for the desired unknown."

1. How much wall paper do you need to redecorate a room?

This is one of the shortcuts of the trade: measure the square footage of the walls you want to paper. Divide by 36. The result will be the number of single rolls you'll need.

Most paper is sold in double rolls, so divide your result by 2 for the number of double rolls needed. (Deduct 1 single roll for every 2 ordinary size windows or doors and each 50 square foot opening.)

2. How much paint will you need?

For a room: Multiply the width of each wall by its height in feet. Add these and divide by 600. The answer is the number of gallons you'll need for one coat.

For the outside of your house: Figure as above but divide by 400. The answer is the

[1] Compiled by Churchill, Judith Chase, "A Woman's Tricks," *The Woman's Home Companion*, March 1950, pp. 32–33. Copyright 1950, Crowell-Collier Co. Reprinted by permission from the publishers.

number of gallons you'll need for two coats of outside paint.

For porches and floors: Multiply the width by the length and divide the result by 250 for two coats of paint.

3. How much wax will you need for your floor?

Measure the length and width of your floor in feet, and multiply the two numbers.

For liquid wax: Divide the answer by 300 for the number of quarts you'll need.

For paste wax: Divide by 400 and the answer will be in pounds of wax needed.

4. How much cloth does a woman need to make a dress?

The following directions apply for an average dress being cut from silk, rayon, or cotton material which averages 36 to 39 inches in width.

Measure in feet your height from the neck down. To this, add the length of your arm from shoulder to finger tips. Multiply by 2 and divide by 3. This will be the amount of material you'll need in yards.

In the light of the above directions, is the pattern a factor in the amount of cloth needed?

5. What is your proper weight?

First of all, run a tape measure around your chest at the level of the end of the breast bone. Now exhale. Now measure your chest circumference, pulling the tape measure quite tight.

Next, divide this measurement by 4. Take this answer and multiply it by itself. Take the result and multiply by your height in inches (without shoes). Divide by 27.69. The answer is your normal weight. (From a University of Illinois formula based on body build.)

6. How old are your trees?

Measure the tree trunk's diameter in inches at chest height. Multiply by the number below which corresponds to the tree. The answer is its age.

White elm, tulip, chestnut	2½
Black Walnut	3
Black Oak	3½
Birch, sweet gum, sycamore, chestnut, oak, red oak, scarlet oak	4
Ash, white ash	5
Beech, sour gum, sugar maple	6
Shagbark, hickory	8

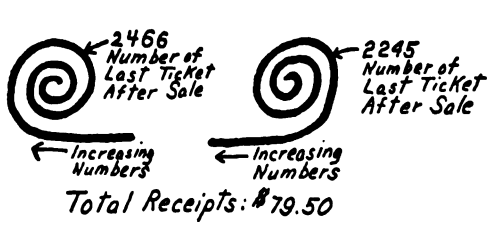
Al. 7 Gr. 9–11 ALGEBRA RESCUES A STUDENT TICKET SELLER IN DISTRESS

We are indebted to Eugene P. Smith for the following real-life anecdote. Mr. Smith teaches mathematics at the University School, Ohio State University.

One morning before school a most distraught, young high school girl rushed into Mr. Smith's office and woefully exclaimed: "Oh, Mr. Smith, what will I do! I forgot to take down the number where I broke a roll of tickets into two rolls—one for adults and one for students, and now I don't know how many of each kind I sold. I simply must find out, so that I can fill in the tax form!"

"Well, Mary," said Mr. Smith calmly, show me all the data you have on your ticket sale last night. Perhaps we can use algebra to find out."

With this encouragement, Mary spread the following sketch of the data before Mr. Smith.



"Now let's see, Mary. Can you find out how many tickets you sold altogether?" asked Mr. Smith.

"Why sure. I subtract 2245 from 2465 and get 220 tickets sold."

"Now, according to your figures, you know the total number of tickets sold, the price of each kind, and the total amount of receipts. Does that remind you of anything?"

With a squeal of delight, Mary exclaimed, "Why that's just like some of those verbal problems we've been having lately in algebra! Hmm. If I let "s" be the number of student tickets and "a" be the number of adult tickets, I could write the equation: $s+a=220$. But I can't find s and a from that equation. We always had two equations when we had two letters. I wonder if the prices and receipts would help any? I've got it! The money we took in on student tickets could be represented by $.25s$ and the money from adult tickets by $.60a$. The sum of these would be my total receipts. I can now write the other equation: $.25s+.60a=79.50$."

Mary then wrote down the two equations and proceeded to solve them as follows:

$$.25s+.60a=79.50$$

$$s+a=220 \qquad s=220-a$$

$$.25(220-a)+.60a=79.50$$

$$55-.25a+.60a=79.50$$

$$.35a=24.50$$

$$a=\ 70 \text{ adult tickets}$$

$$s=220-a=150 \text{ student tickets}$$

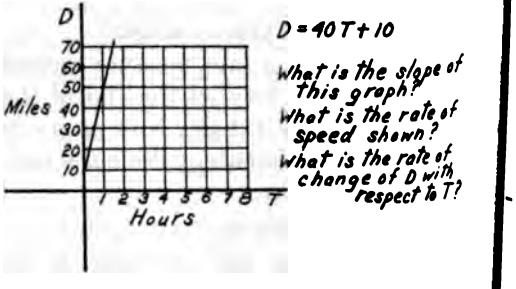
After checking these figures with prices to see if they gave the correct receipts, Mary dashed happily off to the business office to fill out the tax form.

Al. 8 Gr. 9–12 Some Well-Known Rates Illustrated in Algebra

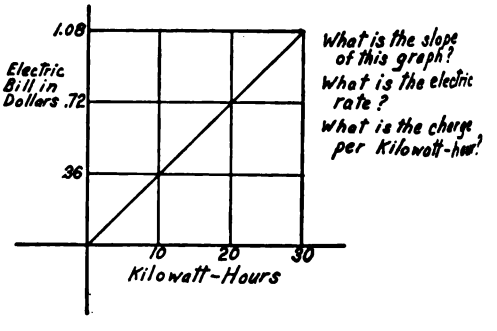
The word "rate" must be understood physically and algebraically for an adequate grasp of the function concept in algebra. The slope-intercept concept of the linear function can be successfully taught in beginning algebra and can provide a powerful motivation for students studying the subject. The word "rate" can early be associated in student thinking with the slope "m" in the general form: $Y=mX+b$. "m" is variously defined for the student as $\Delta Y/\Delta X$ and as the change in the vertical variable divided by the corresponding change in the horizontal variable. If Y increases as X increases, the slope is positive; if Y decreases as X increases, the slope is negative. Some of the sophisticated students might define the slope "m" as the tangent of the angle that the linear graph makes with the horizontal axis. The word "rate" might also be defined verbally as follows: "The rate of change of variable A with respect to variable B is the amount A changes for a unit change in B." The values of physical rates, therefore, depend on the dimensions in which the variables are measured.

It is our opinion that the foregoing ideas could not be taught adequately unless they were developed in the context of many illustrations within the experience of high school boys and girls. Following are some illustrations of rates used by the writer for a number of years in beginning algebra.

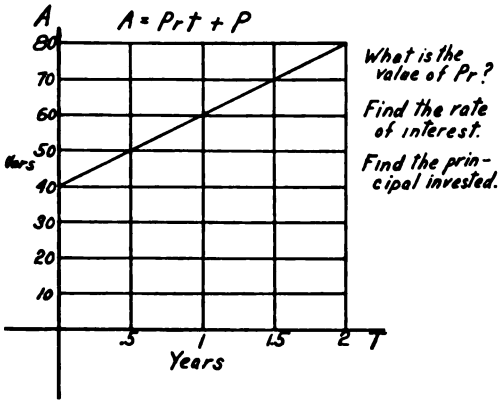
8.1 Car Moving at Constant Speed



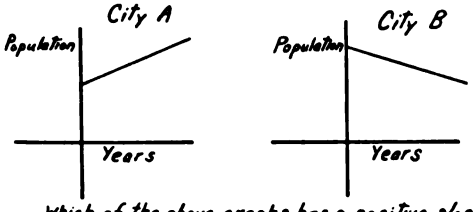
8.2 Your Electric Bill



8.3 Growth in Principal Invested at Simple nterest

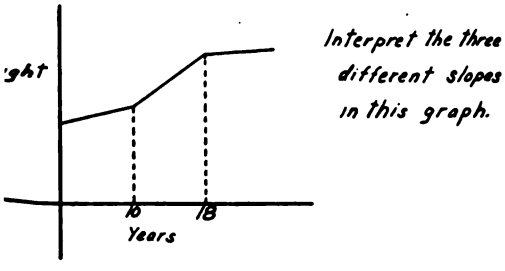


8.4 Population Change in Two Cities



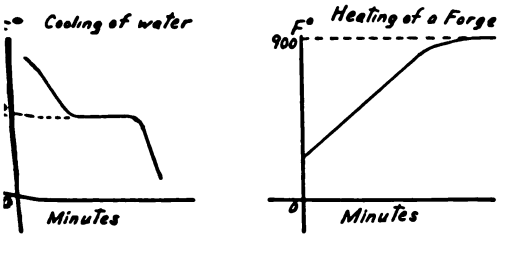
Which of the above graphs has a positive slope and which a negative slope?
What is the interpretation of positive and negative slopes in the graphs above?

8.5 Growth of Jim



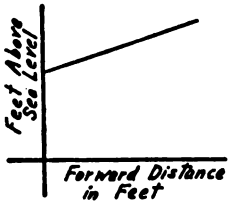
·6 Cooling of Water

8.7 Heating of a Forge

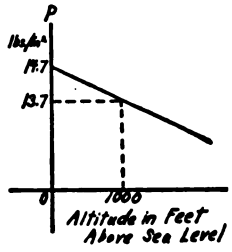


What is the meaning and cause of the zero slope in the left graph?
Why does the slope on the right graph reduce to zero at 900° F?

8.8 Climbing Airplane

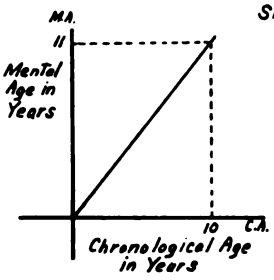


8.9 Change in Air Pressure With Rise in Altitude



Interpret the slopes of the two graphs above.

8.10 The IQ, or Intelligence Quotient



Slope = $\frac{\Delta MA}{\Delta C.A.} = \frac{11}{10} = 1.1$
1.1 × 100 = 110%
or IQ of 110

The IQ, or Intelligence Quotient, has been defined as the per cent the M.A. is of the C.A. Here it can also be interpreted as a rate of mental growth.

While the above approach to the topic of rates can make a significant contribution to the general education of students, it also lays a sound foundation for the understanding of interpolation with logarithms and trigonometric functions and for the understanding of the delta process in calculus.

Sister Mary Agnes of Central Catholic High School in Great Falls, Montana, requested the solution to "P. 1 A Problem For Thinkers" in the November issue, which we have neglected to give.

Referring in parentheses to the numbered statements: the father (5) is not the instructor because the instructor has no living children (6) and is not the president because of (3). Therefore the father in (5) is the professor but cannot be named Mr. Haines or Mr. Jones because he would not confuse his own son (5). The professor is not Mr. Ross because of (1). Therefore the professor is Mr. James. If the president is not Mr. Jones (2, 3) or Mr. James, he must be Mr. Ross or Mr. Haines. The assumption that the president is Mr. Ross leads to the instructor being Mr. Jones which makes (2) and (6) contradictory. Therefore the president is Mr. Haines. If the instructor cannot be Mr. Jones (see above), Mr. Haines, or Mr. James, he must be Mr. Ross. The janitor is therefore Mr. Jones.

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, New York

THE RULE OF SIGNS IN MULTIPLICATION

THE RECENT discussion* of the rules for determining the sign of the product of two signed numbers suggested a study of the ways in which this question has been treated in algebras in the past. A problem of this type does not merit an exhaustive investigation. The question is how have various authors gone about the task of making this rule reasonable to the reader. The volumes chosen for the study were simply those that happened to be at hand: several of them were very influential; others were perhaps a bit obscure.

In some, a simple enunciation of the rules is all that is given. An example of this occurs in Sir Isaac Newton's *Universal Arithmeticke*, the quotation being from the second edition in English (London 1728).

> Simple Algebraick Terms are multiply'd by multiplying the Numbers into the Numbers, and the Species into the Species, and by making the Product Affirmative, if both the Factors are Affirmative, or both Negative; and Negative if otherwise.

François Legendre's *Arithmétique en sa Perfection* (first printed in 1656 and reprinted in many editions the last being in 1812) devoted a few pages to a summary of algebra in which the rules for the signs of the product of two quantities are called "maximes"

> Quand on multiplie P par P, il vient plus.
> Multipliant M par M, il vient P.
> Multipliant M par P ou P par M, le produit est toujours M.

Edward Hatton is more explicit in his work *An Intire System of Arithmetic* (London, 1721, 3rd edition, 1753)

> The Reason of the General Rule for Multiplication:
>
> 1st—Multiplication being nothing but the Work of many additions; it follows that $-ab$ being added to $-ab$, is $-2ab$ which is the same as multiplying $-ab$ by 2; and so $-gh-2gh-3gh$ added is equal to $-6gh$ which shows that the Negative Quantity $-gh$ by the Affirmative 6 must give the Negative $-6gh$ and the like Reason holds for any other Negative by a positive Quantity.
>
> 2nd—And as in *Logic*, two Negatives make an Affirmative; so in *Algebra* one Negative Number or Quantity multiplied by another Negative produceth an Affirmative; For the Sign − being directly contrary to +, must make a Product of a direct contrary Nature when the Multiplier is −, then (as above) when it is +; as subtracting a Negative, is adding really so much as we seem to subtract, as -2 from 4 is 6.

John Ward's *Young Mathematician's Guide* (London, 1707; 1758 ed.) goes into more detail. The author announces the rules and then says

> That + into + will produce + in the Product, is evident from *Multiplication* in Common *Arithmetick:* viz. +5 into +7 will give +35 &c. But that + into −, or − into + should produce the Sign − and that − into − should produce the Sign +, . . . may perhaps seem somewhat hard to be conceived; and requires a Demonstration.

The demonstration uses the postulates that equals multiplied by equals yield equals, and the same for addition and subtraction. One of his illustrations will suffice. To prove that $-7b$ into $+3f$ equals $-21bf$, he reasons as follows:

suppose	$4a-7b=0$
then	$4a=7b$
but	$+3f=+3f$
multiplying	$12af=21bf$
or	$12af-21bf=0.$

Consequently + into − or − into + produces −. That is, the product of $-7b$ by $+3f$ was found by first finding the product of $7b$ by $3f$.

Bonnycastle's *Introduction to Algebra*

* See Van Engen, H., *On Understanding Mathematical Methods*, THE MATHEMATICS TEACHER, December, 1950, pp. 397–444.

with Notes and Observations had a number of editions in the United States, the first being Philadelphia, 1806. In this text, the explanation of the rules is given in a footnote. The author establishes the product of two positive quantities by assuming that multiplying by $+b$ means taking the multiplicand b times. Then, the other cases are explained as follows:

2nd.—If two quantities are to be multiplied together, the result will be exactly the same in whatever order they are placed; for a times b is the same as b times a: and, therefore, when $-a$ is to be multiplied by $+b$, or $+b$ by $-a$, this is the same as taking $-a$ as many times as there are units in $+b$; and since the sum of any number of negative terms is negative; it is evident that $(-a)\times(+b)$ or $(+b)\times(-a) = -ab$.

3d.—When $-a$ is to be multiplied by $-b$: Here $a-a=0$, therefore $(a-a)\times -b$ is also $=0$ because 0 multiplied by any quantity produces 0; and since the first term of the product, or $a\times(-b)$ is, by case 2, $-ab$, the last term or $(-a)\times(-b)$ must be $+ab$, in order to make the sum $(-ab+ab)=0$ consequently $(-a)\times(-b) = +ab$.

The third New York edition was "revised, corrected, and enlarged . . . by James Ryan." It appeared in 1825. The note on the signs in multiplication differs from that of the Philadelphia edition of 1806. Its explanation is:

The above rule for the signs may be proved thus: if B, b be any two quantities, of which B is the greater, and $B-b$ is to be multiplied by a, it is plain that the product, in this case, must be less than aB, because $B-b$ is less than B; and consequently, when each of the terms of the former are multiplied by a, as above, the result will be

$$(B-b)\times a = aB-ab.$$

For if it were $aB+ab$, the product would be greater than aB, which is absurd.

Also, if B be greater than b, and A greater than a, and it is required to multiply $B-b$ by $A-a$, the result will be

$$(B-b)\times(A-a) = AB-aB-bA+ab.$$

For the product of $B-b$ by A is $A(B-b)$ or $AB-Ab$ and that of $B-b$ by $-a$ which is to be taken from the former, is $-a(B-b)$, as has been already shown; whence $B-b$ being less than B, it is evident that the part which is to be taken away must be less than aB; and consequently since the first part of this product is $-aB$, the second part must be $+ab$; for if it were $-ab$, a greater part than aB would be taken from $A(B-b)$ which is absurd.

The introductions to each of these editions speak of the "moderns" to whom "we are principally indebted for the most curious refinements of the art (algebra) and its great and extensive usefulness in every abstruse and difficult inquiry." Each edition mentions Newton, Maclaurin, Saunderson, Simpson, and Emerson. The edition of 1825 inserts the names of Clairaut, Euler, Lagrange, and Lacroix also and each edition then concludes with the statement that the men cited "have particularly excelled in this respect; and it is to their works that I would refer the young student as patterns of elegance and perfection." There may be no significance in the reference to Lacroix (1765–1843), but Lacroix was a well-known teacher and textbook writer and was successively at the École Normale, the École Polytechnique, and the Collège de France. His *Algebra* was translated "for the use of the students of the University at Cambridge, New England" by John Farrar and was published as a part of the first volume of *Cambridge Mathematics*. The quotation that follows is from the Fourth Edition (Boston, 1833).

When the multiplicand contains parts to be subtracted, the products of these parts by the multiplier must be taken from the others, or in other words, have the sign —prefixed to them, for example:

the product of	$a-b$
multiplied by	c
is	$ac-bc$;

for each time that we take the entire quantity a, which was to have been diminished by b before the multiplication, we take the quantity b too much; the product ac therefore, in which the whole of a is taken as many times as is denoted by the number c, exceeds the product sought by the quantity b, taken as many times as is denoted by the number c, that is by the product bc; we ought then to subtract bc from ac which gives as above,

$$ac-bc.$$

A little more explanation follows and then there is the conclusion:

When the multiplier has the sign +, the product has the same sign as the corresponding part of the multiplicand.

The author next discusses the product of

$a-b$ by $c-d$, which has the product

$$ac-bc$$
$$-ad+bd$$

for the product of the multiplicand by the first term of the multiplier, will be by the last example $ac-bc$; but by taking the whole of c for the multiplier instead of c diminished by d, we take the whole of $a-b$ so many times too much as is denoted by the number $-d$; so the product $ac-bc$ exceeds that sought by the product of $a-b$ by d. Now this last is, by what has been said, $ad-bd$, and in order to subtract it from the first it is necessary to change the signs. We have then $ac-bc-ad+bd$ for the result required . . . We conclude that . . . each particular product must have the same sign, as the corresponding part of the multiplicand, when the multiplier has the sign +, and the contrary sign when the individual multiplier has the sign −.

Warren Colburn's *Introduction to Algebra upon the Inductive Method of Instruction* (Boston, 1836) begins by illustrations,—A is worth a number a of dollars, B is not worth so much as A by a number b of dollars, and C is worth c times as much as B, how much is C worth? If a is greater than b, then $c(a-b)$ is positive, but if b is greater than a, then $ac-bc$ is negative.

Another example follows:

A gentleman owned a number of farms, and each farm was worth a number c of dollars, which was his whole property. He hired money and fitted out a number b of vessels and each vessel was worth as much as one of his farms. All the vessels were lost at sea. How much was he then worth? . . . Now if the number of farms exceeded the number of vessels, he still had some property, but if the number of vessels exceeded the number of farms, the quantity $ac-bc$ is negative and he owed more than he could pay. Hence if a positive quantity be multiplied by a negative the product will be negative.

The author then goes to the problem of $a-b$ multiplied by $c-d$. He shows that the product may be put in the form

$$(a-b)c+(b-a)d.$$

Let it be remembered that $a-b$ has the same numerical value as $b-a$, they differ only in sign. Suppose $a-b=-m$, then by changing all the signs $b-a=+m$. Hence

$$(a-b)c+(b-a)d=-cm+dm=m(d-c).$$

Now if d is greater than c (which is the case when $c-d$ is negative), the quantity $m(d-c)$ is positive.

Suppose both $a-b$ and $c-d$ to be negative, as before; then $b-a$ and $d-c$ will both be positive, and their product will be positive. (The multiplication is then performed). This product is precisely the same as that produced by multiplying $a-b$ by $c-d$. Therefore if two negative quantities be multiplied together, the product will be the same as that of two positive quantities of the same numerical value, and will have the positive sign.

It should be remembered that these examples are not by any means a complete collection, but they have the merit of showing a number of different approaches to the topics from Sir Isaac Newton's enunciation of the rules to the detail of the Lacroix presentation.

LETTERS

Statistics in Secondary School Mathematics—When?

Professor Burr, in his article in the January 1951 issue of THE MATHEMATICS TEACHER, recognizes that "although statistical background is important to the student both for his vocation and in his function as a citizen in a democracy, statistics is unfortunately either absent or forms only a minute part of many curricula."

The question of including statistical ideas in secondary school mathematics has been of concern to writers for at least thirty years. T. Kelley in an article published in *School and Society* in February 1920 writes "Statistics can be used for the making of a fuller knowledge of the past and present. It has values which are basic in the life of a democracy, recognising that nature provides distributions, not types without variability, and that the problem of a democracy is to utilize the talents, attitudes and training of the individuals of a distribution for the sake of raising the mean of the self-same distribution to higher levels of capacity, attitude and training, etc." Concerning the ability of secondary school pupils to cope with statistical concepts, he adds, "The basic ideas in statistics are entirely comprehensible to the high-school level in intelligence." He also adds, "What better time than the high school age; what better teachers than those trained in mathematics. . . . ?"

In 1933 a course in statistics was given at James Monroe High School in Brooklyn, New York. This is described by E. Hausle in an arti-

(*Continued on page 266*)

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

Meaning, Rigor and Discipline in Secondary Mathematics

FOR a considerable number of years now we have been greatly concerned about the slow learners in mathematics. Until the relatively recent advent of "second track" mathematics, the standard courses in algebra and geometry were gradually watered down until they no longer offered a challenge even to moderately able pupils. Nor has the somewhat checkered career of "general mathematics" helped matters very much. Isn't it about time that we gave a little thought to the more able pupils—there are some still, of course—and stop deceiving them as to the nature of mathematics? The appearance in the literature during the last two or three years of several excellent papers dealing with a more rigorous treatment of elementary mathematics would seem to suggest that the time is ripe for some changes in this direction.

We do not wish to revive the traditional courses of a quarter of a century ago, of course. Yet it seems feasible to introduce sufficient rigor and substance into mathematics for our better pupils so that they will have something to chew on intellectually.

Consider elementary algebra. How do we "rationalize" operations with signed numbers? Do we emphasize sufficiently the meaning of generalization? Of induction? How clear do we make the concept of operations? the laws governing operations? the extension of concepts, such as fractions, irrationals, and exponents? Do we really teach algebra as generalized arithmetic? There is ample justification for teaching *demonstrative algebra* instead of a more or less arbitrary algebra in which the whys and wherefores remain unexplored. Why not let the pupil realize the role played by postulates and undefined terms in algebra, even as we purport to do in teaching geometry? If it be objected that he is not yet mature enough for this in the ninth grade, which may well be true, then let us defer algebra until *after* demonstrative geometry. After all, what is so sacrosanct about teaching algebra before geometry?

And even here, in the teaching of demonstrative geometry, how successful are we? Do our pupils really understand the meaning and significance of postulational thinking? Do they understand the nature of a hypothetico-deductive system? I strongly doubt it. Most of them, to paraphrase E. T. Bell, have the experience of being black-jacked into memorizing a string of theorems without the faintest notion as to the reason for their sequence, and without the slightest inkling that we can get out of a postulational system only what we put into it—though to be sure, most of the implications are not at once apparent.

In short, our plea is that we put some teeth into high school mathematics. Let our pupils learn something about the logical notions involved; the difference between inductive and deductive thinking; the abstract nature of numbers and other symbols; the power of generalizations; the meaning of necessary and sufficient conditions; the unique significance of postulates, primitives, definitions, and implications; the nature of indirect reasoning.

It is a challenge well worth while to pupil and teacher alike.

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MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

22. Euclidean Constructions with Well-Defined Intersections

Suppose we consider a point of intersection of two loci as *ill-defined* if the two loci intersect at the point in an angle less than some given small angle θ, say $\theta=5°$, and that otherwise the point of intersection will be considered as *well-defined*. The question arises: Can any given euclidean construction be accomplished by using only well-defined intersections? Although we are posing, and shall therefore treat, this question as a purely academic one, it might be that it has a practical aspect in connection with drafting. Incomplete results, which in part contain an invalidating oversight, have been given by Hilda Hudson.[3] It is the purpose of this note to remedy these defects and to answer the question in the affirmative. It will be convenient to consider four auxiliary constructions.

CONSTRUCTION 1. *To find, by well-defined intersections, the point of intersection of two lines cutting at an angle less than θ.*

Let the two lines be m and n (see figure 1), intersecting in the ill-defined point O. At approximately equal distances from O erect perpendiculars to m cutting m at A and B and n at C and D respectively. By

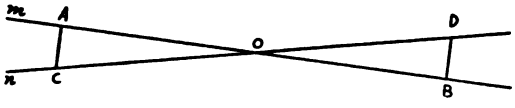

FIG. 1

similar triangles we have $AO:OB=AC:DB$, whence $AO:AB=AC:(AC+DB)$. Since AB, AC, and $AC+DB$ are well-defined distances, we may, by the familiar method of elementary geometry and by using only well-defined intersections, construct the length AO. The point O is then easily located.

CONSTRUCTION 2. *To find, by well-defined intersections, the points of intersection of a line and a circle cutting at an angle less than* s.

Let l be the line and c the circle (see figure 2), intersecting in the ill-defined points M and N. We may, using well-de-

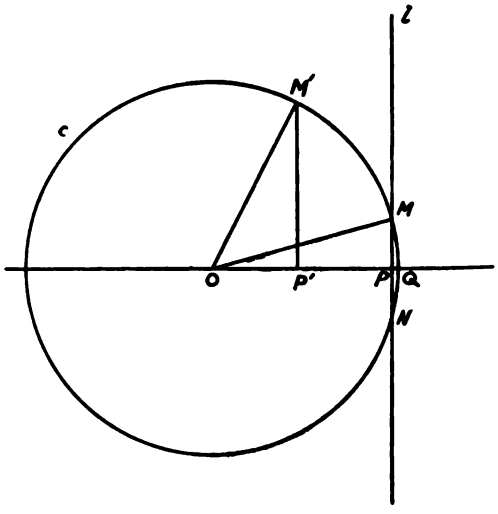

FIG. 2

fined intersections, draw the perpendicular OP to the line l from the center O of circle c. Let OP cut the arc MN in the point Q, let the radius of c be chosen as unit length, and designate angle POM by α. Then $OP=\cos\alpha$. By the familiar method of elementary geometry and by using only well-defined intersections, we may construct a length of measure $\cos^2\alpha$, as the third proportional to 1 and $\cos\alpha$. We may then readily obtain a length of measure $\cos 2\alpha=2\cos^2\alpha-1$. The process may be continued until we obtain the length $OP'=\cos(2^n\alpha)$, where $2^n\alpha$ is greater than θ. We may thus construct a central angle $QOM'=2^n\alpha$. By a succession of n bisections, which can be accomplished by well-

[3] Hilda Hudson, *Ruler and Compasses* (1916), pp. 109–112.

defined intersections, performed on arc QM', we find the point M. Point N is then easily found.

CONSTRUCTION 3. *To find, by well-defined intersections, the radical axis of two circles cutting at an angle less than θ.*

Let c and d be the two circles. Draw any circle e cutting circles c and d in well-defined points. Since the three radical axes of three circles taken in pairs are concurrent,[4] the common chord of c and e and that of d and e intersect on the radical axis of c and d. To find this point of intersection we may have to use construction 1. Another point on the radical axis of c and d can be similarly found, and the radical axis drawn.

CONSTRUCTION 4. *To find, by well-defined intersections, the point of intersection of two circles cutting at an angle less than θ.*

By construction 3 draw the radical axis of the two circles. The problem is now reduced to construction 2.

It is now evident, in view of the above auxiliary constructions, that we are able to state the

THEOREM. *Any euclidean construction may be accomplished by using only well-defined intersections.*

It is possible to prove more than the above theorem. In addition to the given small angle θ, let us be given a small linear distance d. Suppose we define a euclidean construction to be *well-defined* if it utilizes only well-defined intersections, straight lines determined by pairs of points which are a greater distance than d apart, and circles with radii greater than d. Then it can be shown that *any euclidean construction can be accomplished by a well-defined one.* The treatment of this more general theorem is too lengthy to be given here.

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[4] Cf. P. H. Daus, *College Geometry* (1941), p. 78.

23. *The Product of Two Numbers Each Ending in 5*

Let us consider the product of two numbers ending in five, each number having two digits. Let x be the ten's digit in the first and larger number, and let y be the ten's digit of the second number. Then

$$10x+5 = \text{the first number,}$$

and $10y+5 = \text{the second number.}$

$$(10x+5)(10y+5) = P,$$

the product of the two numbers.

or

$$100xy+50x+50y+25 = P.$$

or

$$50xy+50x+50xy+50y+25 = P.$$

$$100\,\frac{[(xy+x)+(xy+y)]}{2}+25 = P.$$

or

$$\frac{100[x(y+1)+y(x+1)]}{2}+25 = P.$$

The expression

$$(a) \qquad \frac{x(y+1)+y(x+1)}{2}$$

is the arithmetic mean of the products of the ten's digit of one number and one plus the ten's digit of the other.

When

$$x = y, \quad \frac{x(y+1)+y(x+1)}{2}$$

becomes

$$(b) \qquad x(y+1) \quad \text{or} \quad y(x+1).$$

When

$$x-y = 2, \quad \frac{x(y+1)+y(x+1)}{2}$$

becomes

$$(c) \qquad \frac{2xy+x+y}{2} = \frac{2y(2+y)+2+2y}{2}$$

$$= y(2+y)+1+y$$

which shows that an integral arithmetic mean is obtainable in this case. In general, when $x-y$ is any positive *even* number, $2xy+x+y$ is divisible by 2 and an integral arithmetic mean is obtainable. When

$$x-y=1,\ \frac{2xy+x+y}{2}=\frac{2y(1+y)+1+2y}{2}$$

and the mean is

(*d*) $\qquad y(1+y)+y+1/2$

which differs from an integral mean by .5. In general, when $x-y=k$ is any positive *odd* number the mean (*a*) differs from an integer by an amount equal to one half of $x-y$ and is given by

(*e*) $y(k+y)+y+k/2$, or $xy+y+k/2$.

This argument provides a short cut for calculating the products of pairs of integers ending in 5. For example:

(*f*) $35\times35=100(3)(3+1)+25=1225.$

See (*b*) above.

(*g*) $35\times15=100[3(2)+4(1)]/2+25$
$=525.$

See (*a*) above. Or,

$35\times15=100[1(3)+1+1]+25=525.$

See (*c*) above.

(*h*) $85\times75=100[7(8)+7+.5]+25$
$=6375.$

See (*d*) above.

(*i*) $85\times55=100[5(8)+5+1.5]+25$
$=4675.$

See (*e*) above.

(*j*) $95\times25=100[2(9)+2+3.5]+25$
$=2375.$

See (*e*) above.

Note the special rule for squaring a number ending in 5 as in 35×35 above. The result may be obtained by writing down 25 and then prefixing to it the product of the units digit (3) with one plus the units digit (4).

The most general rule (*e*) may also be expressed in words as: Multiply by 100 the sum of the product of the ten's digits plus the smaller ten's digit plus one-half the difference of the ten's digits, then add 25.

The above arguments using two digit numbers may be appropriately extended to numbers of more than two digits.

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24. The Generation and Use of Heronian Triangles

Problems which illustrate and apply numerical properties of geometric figures, as studied in geometry and in trigonometry, frequently lead to complexities in computation because of irrationality. In order to place emphasis upon the principles and processes being applied it is well to lighten the burden of computation by arranging that the quantities involved are rational, or even integral. It has already been noted in these columns that Pythagorean numbers furnish a fund of raw material for constructing such problems. [THE MATHEMATICS TEACHER, Vol. XLIII (April, 1950), p. 102; (Nov., 1950), p. 352.] Another source of rational data is the rational triangle—an oblique triangle whose area and lengths of sides are rational. Many associated quantities—altitudes, segments on the sides formed by the feet of the altitudes, circumradius, inradius, exradii, trigonometric functions of the angles—are rational. Consequently such a triangle may be taken as the basis for a variety of these problems.

An oblique triangle with integral sides and area need not have an integral altitude. Witness the triangle with sides 100, 143, 153, having area 6930; also the triangle with sides 10, 35, 39 and area 168. Neither triangle has an integral altitude. By "Heronian triangle," however, we mean a triangle that does have an integral altitude as well as integral sides and areas.

It is easy to generate Heronian triangles by compounding two Pythagorean triangles. Take, for example, the Pythagorean primitives 5, 12, 13 and 8, 15, 17. Since 24 is the L. C. M. of 12 and 8, these triangles may be suitably enlarged so that each has a leg 24. Then, placing the triangles with the equal legs in coincidence, the Heronian triangles 26, 51, 55 and 26, 35, 51 result, depending on whether or not the triangles are placed so as to overlap. Tables of Pythagorean numbers may be constructed and from them tables of Heronian triangles. There would be eight entries in the latter table for each pair in the former as shown:

Pythagorean Triangles

Hyp.	*Legs*
5	4 5
13	12 5

Heronian Triangles

	Sides		*Perimeter*	*Area*
13	15	14	42	84
13	15	4	32	24
39	25	16	80	120
39	25	56	120	420
13	20	21	54	126
13	20	11	44	66
52	25	63	140	630
52	25	33	110	330

A triangle, 100, 143, 153, with integral area but no integral altitude may be formed as follows. Take the right triangles 65, 63, 16 and 85, 77, 36, whose hypotenuses have the common factor 5. Compound them into a Heronian by matching the legs 63 and 77, taking their L. C. M., $7 \cdot 9 \cdot 11$, to be the altitude. The oblique sides are $11 \cdot 65$ and $9 \cdot 85$. The base is $11 \cdot 16 + 9 \cdot 36 = 500$, also having 5 as a factor. The desired triangle results when the common factor is removed.

Various properties and some special classes of Heronian triangles can be investigated by elementary methods. These furnish interesting topics for club meetings or other activity periods. As an example, consider Heronian triangles whose sides are consecutive integers, say $2m-1$, $2m$, $2m+1$. It turns out that the general form of $2m$ is given by

$$2m = (2+\sqrt{3})^n + (2-\sqrt{3})^n, \; n \text{ an integer.}$$

Successive values of n give rise to the following series of triangles:

Sides	*Area*
1, 2, 3	0
3, 4, 5	6
13, 14, 15	84
51, 52, 53	1,170
193, 194, 195	16,296
723, 724, 725	226,974
etc.	

Readers who wish to pursue the subject further may consult the following:

Carmichael, R. D., *Diophantine Analysis.* New York: John Wiley & Sons Inc., 1915, pp. 8–13.

Alliston, N., *Mathematical Snack Bar.* Cambridge (England): W. Heffer & Sons, Ltd., 1936. pp. 116–125.

Dickson, L. E., "Rational Triangles and Quadrilaterals." *American Mathematical Monthly*, vol. 28 (June-July, 1921), p. 244.

Cheney, W. F., Jr., "Heronian Triangles." *American Mathematical Monthly*, vol. 36 (Jan., 1929), p. 22.

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25. The Improper Use of "=," and a "Pet Peeve"

In response to an invitation issued earlier in this column Helen G. Crozier, Head of the Mathematics Department of Fortuna Union High School, Fortuna, California, writes:

"My pet peeve is the misuse of the equality sign. In substituting in formulas, pupils often do this sort of thing:

$$A = 1/2bh, \quad b = 8, \quad h = 12$$

$$A = 1/2 \cdot 8 = 4 \cdot 12 = 48.$$

To avoid this I have them substitute all values at once, writing the numerical values right under the letters, and writing only one equals sign on a line, thus:

$A = 1/2bh, \quad b = 8, \quad h = 12$

$A = 1/2 \cdot 8 \cdot 12$

$A = 4 \cdot 12$

$A = 48.$

Also in checking algebraic equations, I have students use a '?' above the equal sign until the last step when an identity is obtained, then use a check mark ($\surd$) over the equal sign. In this way, I try to have students realize and recognize what 'equals' really means."

The editor of "Miscellanea" cannot resist adding in this same vein his "peeve" against the use of "=" in trigonometric identities which are to be proved. This unfortunate use of a symbol is a major reason for our trouble with students who in these proofs operate on both sides as if we had an equation. The editor uses the question mark device with much verbal emphasis and insistence upon it to indicate the supposedly unsettled status of these trigonometric problems. Incidentally, should we not include in the problems for this section of trigonometry some statements which are not true? For the students to be able to discover and explain the conditions for equality to exist would be far more valuable (and harder) than for them to "prove the identity" when told to do so and to "solve the equation" when so instructed.

Letters

(*Continued from page 258*)

cle appearing in the January 1937 edition of THE MATHEMATICS TEACHER. The experiment was conducted for the purpose of answering the question, "Can a course in statistics be given to secondary school pupils?" All the pupils enrolled in the course had completed Intermediate Algebra and were taught not only statistical concepts but operational statistics as well. According to Hausle, "comments of the pupils who took the course indicate that they derived a real benefit and enjoyment from it because they could see a meaningful application of their mathematics." As for Hausle's personal opinion, she feels that such a course should become a unit in every high school wherever possible.

In 1946, a Committee on Collecting and Analyzing data at Duke University (see MATHEMATICS TEACHER, April 1946) came out strongly in favor of including statistics in high school. "The study of statistics offers unlimited opportunities for applications of mathematics in fields where social problems exist; business, industry, government, agriculture, medicine, psychology, sociology and education are enormously rich in problems, data and opportunities. Statistics enters into surveys, analyses, norms, testing, etc. The committee recommended: (1) that at least some statistical training be given to all high school students, and (2) that opportunities for broadening experiences with statistical points of view and techniques be made available for those high school students who wish them.

The three references cited represent the general feelings of most writers on the subject of statistics in the secondary school. A survey by the writer of all the literature that could be found on the subject reveals almost unanimous agreement on the following points:

1. There is much of value to be obtained through the study of statistics at the secondary school level and that mathematics is the subject best equipped for this purpose.

2. Pupils of secondary school mathematics are capable of mastering many statistical ideas.

3. The study of descriptive statistics with emphasis on concepts is of greater value than the study of statistics from an operational point of view.

Having outlined briefly the firm position of writers in the past on the matter of statistics as a secondary school tool, we now feel justified in asking, "Why the delay during all these years in introducing statistics in the secondary school mathematics curriculum?" The writer does not profess to know the answer. One possibility is that the value of the study of statistics has never, as far as is known, actually been put to test by experimentation, to warrant its being placed in the mathematics curriculum at the expense of some of the traditional topics in mathematics. It should be remembered that the experiment conducted at James Monroe High School in 1933, though reflecting favorably upon the study of statistics, was restricted to students comparatively advanced in secondary school mathematics. An investigation into the content and placement of statistical ideas for all students at the secondary school level has been long overdue and would represent a significant educational contribution. It could help to make curriculum committees more aware of the value of incorporating more statistical ideas in the mathematics curriculum.

As a parting word, we wish to mention that a recent publication by the Board of Education of the City of New York, (*General Mathematics for*

(*Continued on page 270*)

INSTITUTES, WORKSHOPS, CONFERENCES

Duke University, Durham, North Carolina, Institute for Teachers of Mathematics, Eleventh Annual Session, August 6–17, 1951.

Theme: "Mathematics at Work." The following persons are scheduled to lecture to the Institute: Ralph M. Berry, Mathematician, Coast and Geodetic Survey; Paul Boder, Scientist, American Optical Company; C. G. Brenecke, Chairman, Electrical Engineering Department, North Carolina State College; J. W. Cell, Professor of Mathematics, North Carolina State College; H. A. Fisher, Chairman, Department of Mathematics, North Carolina State College; J. J. Gergen, Chairman, Department of Mathematics, Duke University; Bain Griffith, Designing Engineer, Chevrolet Company; Capt. R. Hatcher, Bureau of Aeronautics, U. S. Navy; Philip O. Johnson, Designing Engineer, Fisher Body Company; Phillip S. Jones, Department of Mathematics, University of Michigan; E. S. Lee, General Electric Company; W. D. Merrifield, Industrial Education, Chrysler Corporation; Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin; W. W. Rankin, Department of Mathematics, Duke University; Robert Schilling, Engineer, General Motors Research Laboratory; H. A. Shannon, North Carolina State Education Department; Col. R. B. White, President, Baltimore and Ohio Railroad; W. M. Whyburn, Chairman, Department of Mathematics, University of North Carolina; and C. A. Wright, Statistician, Standard Oil Company.

The following study groups will meet daily:

I. Aids in the Study of Algebra (Jr. and Sr. H.S.). Leader: Frances Johnson, Oneonta High School, Oneonta, New York.

II. Mobilization and the Mathematics Curriculum. Leader: Veryl Schult, Director of Mathematics, Washington, D. C.

III. Algebra (H.S. and Col.). Leader: J. J. Gergen, Chairman, Department of Mathematics, Duke University.

IV. The Slow Student in Mathematics (Jr. and Sr. H.S.). Leader: Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin.

V. General Mathematics (Jr. and Sr. H.S.). Leader: Nanette R. Blackiston, Supervisor of Mathematics, Baltimore, Maryland.

VI. Trigonometry and Analytic Geometry (H.S. and Col.). Leader: J. W. Cell, Professor of Mathematics, North Carolina State College.

VII. History—Philosophy—Psychology of Mathematics (Jr. and Sr. H.S. and Col.). Leader: Phillip S. Jones, Assistant Professor of Mathematics, University of Michigan.

VIII. Uses of Mathematical Instruments (Jr. and Sr. H.S.).

IX. Laboratory Mathematics (Jr. and Sr. H.S.). Leaders: Ida Mae Bernhard, Laboratory School, San Marcos, Texas; and Allene Archer, Jefferson High School, Richmond, Virginia.

X. Laboratory Mathematics (H.S. and Col.). Leaders: Amelia Richardson, McKeesport High School, McKeesport, Pa., and Phillip S. Jones, University of Michigan.

On the social side, there will be one banquet, three dinner meetings, one reception, one tea, one organ recital, one carillon recital, and a watermelon party.

For further information write: Professor W. W. Rankin, Director, Duke Mathematics Institute, Duke University, Durham, North Carolina.

University of Houston, Houston, Texas. First Mathematics Institute. July 30 through August 2, 1951. This institute is being held under the joint sponsorship of the College of Education, the Department of Mathematics, and the College of Business Administration. Each day's program will begin with a general session. On Monday, July 30, Dr. M. M. Slotnick, Chief Mathematician of the Humble Oil and Refining Company, will address the first session. Dr. W. A. Brownell, Dean of the School of Education at the University of California will be in charge of sections dealing with elementary mathematics, Dr. E. H. C. Hildebrandt of Northwestern University will be in charge of sections dealing with the problems of high school teachers and Dr. E. T. Bell, Professor of Mathematics at California Institute of Technology will direct the college section. Each of these men will address a general session also. Afternoons will be devoted to small study groups for teachers at the various levels of teaching.

All meetings will be held in air-conditioned rooms. Rooms will be available in air-conditioned dormitories at $1.50 per night. The fee for the entire period of the Institute will be $10.00 and a certificate of attendance will be issued to those participating.

Additional information will be sent to those writing to Dr. Esther F. Gibney, Box 573, University of Houston, 3801 Cullen, Houston, Texas.

Indiana Workshop for Mathematics Teachers. Indiana University, in cooperation with Purdue University, will hold its Fourth Annual Workshop for Mathematics Teachers on the Bloomington campus beginning **Monday, June 25** and closing **Saturday, July 7.** The Workshop will include opportunity for discussion of your particular problems, a Visual Aids Laboratory where you may try your hand at making things, speakers from industry and business, as well as specialists in the field of mathematics. The program should be available around May 15. Write to Philip Peak, Indiana University, Bloomington, Indiana.

Northwestern University Workshop in Multi-Sensory Aids in the Teaching of Mathematics, August 6–August 24, 1951. This will be a discussion and laboratory course on instructional and learning aids which may be used to

(Continued on page 277)

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* PAUL M. WEISEL
The University of Wisconsin, Madison, Wisconsin

A TABLE summary of the 15 replies to the questions on Mathematics Enrollments, published in this section in the November, 1950, number of THE MATHEMATICS TEACHER is given below. Since the small number of replies does not provide sufficient information on which to base general conclusions on what is going on in our schools, this first set of questions is repeated with the hope that additional replies can be had. The questions are:

A. Mathematics Enrollments
   1. How many years of sequential mathematics (in grades 9–12) is it possible for a graduate of your school to take?
   2. What is the mathematics requirement for graduation from your high school?
   3. How long has this requirement been in effect?
   4. What per cent of your graduates of 1950 completed
      a. one year of mathematics in grades 9–12?
      b. two years of mathematics in grades 9–12?
      c. three years of mathematics in grades 9–12?
      d. four years of mathematics in grades 9–12?
   5. What per cent of your graduates of 1950 entered college this fall?

Please send the answers to these questions for your school to John R. Mayor, North Hall, Madison 6, Wisconsin by May 15, 1951. A summary of replies will be published in the October issue. We believe that this section can be most effective if replies are sent in promptly and promptly reported.

Readers of THE MATHEMATICS TEACHER are also invited to submit questions for publication in future issues.

In replying please give your name, the name and locality of your school, and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades 10–12; junior high school, grades 7–9; etc. The questions may be answered by number without use of a particular form or repetition of the questions.

In reports no reference will be made to names of teachers or schools.

The 15 replies to the questions on page one, received by December 15, came from 13 different states: Illinois, Iowa, Massachusetts, Michigan, Minnesota, Nebraska, New Hampshire, North Carolina, Oregon, Pennsylvania, Rhode Island, Texas, and Wisconsin. Twelve of the reports are from public schools and three from private schools.

While no conclusions can be drawn from this table it will be interesting for schools to compare their own registrations with that of other schools of similar organization and size. To facilitate one kind of comparison the schools have been listed in order of per cent of graduates of 1950 entering college.

Another interesting comparison for these schools is had by observing the effect, if any, of a requirement of one or two years of mathematics on the enrollment in the third and fourth year courses.

The fact that all except one of the schools reporting offered at least four years of sequential mathematics would, of course, indicate one characteristic of this group of schools which unfortunately is not true of all schools even as large as these. Some might suppose that those schools which had the best record in terms of per cent of students, who continue the study of mathematics through and beyond the second year, would be most apt to report, but this should not be so since names and teachers are not to be revealed. For similar questions, the information in this table is in reasonable agreement with that reported by Schorling

(THE MATHEMATICS TEACHER, April, 1948). In the Schorling survey nearly 300 replies were received, but a much longer period was allowed for collection of data.

MATHEMATICS ENROLLMENTS IN 15 SECONDARY SCHOOLS

(The numbers at the tops of the columns are the numbers of the questions above; the answers to questions 1, 2, and 3 are in years; the answers to questions 4a, 4b, 4c, 4d, and 5 are in per cents.)

School	Grade Organization	Enrollment	1	2	3	4a	4b	4c	4d	5
A	9–12	740	4	3	50	100	100	100	80	100
B	9–12	112	4	2	—	100	100	10	0	100
C	7–12	325	4	1	20	100	86	48	20	73
D	Senior	—	4	1	10	100	90	20	10	70
E	9–12	400	6	2	15	100	100	40	16	54
F	7–9, 10–12	450	4	2	10	100	100	40	20	45
G	Senior	—	4	0	10	100	36	17	14	40
H	9–12	610	4	2	5	100	89	30	9	30
I	9–12	—	3½	2	10	100	100	27	9	30
J	9–12	900	4	—	—	100	100	30	10	25
K	10–12	786	4	2	20	100	100	20	10	25
L	9–12	1297	4	0	—	79	62	34	11	24
M	9–12	485	4	2	20	100	97	24	6	19
N	9–12	325	4	1	—	86	49	10	—	17
O	7–12	245	4	1	10	100	64	44	0	16

MATHEMATICAL RECREATIONS

Edited by AARON BAKST

School of Education, New York University, New York 3, N. Y.

THE Christmas greeting extended by this department to the readers of THE MATHEMATICS TEACHER elicited several interesting responses. MR. FRANK J. HARDMAN of the Southern Junior-Senior High School, Baltimore, Maryland submitted the following greetings:

$$\frac{(90°-C_1)(90°-C_2)(90°-C_3)\cdots(90°-C_n)}{2u}$$

which is deciphered as: COMPLEMENTS OF THE *C*'S TO YOU.

$$\{[(X+A)(X-A)M+A^2M]E\}\cdot\left\{\left[\frac{\frac{R+X}{X}+\frac{X}{R-X}}{\frac{X}{R-X}}\right]-\left[\frac{4AS}{3X[(E+Y)^2-(E-Y)^2]}+\frac{4AS}{3E[(Y+X)^2-(Y-X)^2]}+\frac{4AS}{3Y[(X+E)^2-(X-E)^2]}\right]\right\}Y.$$

The pupils who will have patience enough to perform all the indicated operations will be rewarded with

$$MER^2Y-ASXM=MERRY\ XMAS.$$

This problem is suitable for an Intermediate Algebra class.

This department wishes to offer its apologies to MISS HAZEL DULING who is responsible for the origin of the problem "A MATHEMATICAL ROMANCE" which was published in the December issue of THE MATHEMATICS TEACHER. MISS MARGARET JOSEPH called the attention of this department to the fact that the idea of this problem originated with MISS DULING.

The solution of this problem is given

below. The missing words are listed in the order of their appearance.

1. major 2. premise 3. axis 4. powers 5. degree 6. slope 7. obliquely 8. erect 9. point 10. survey 11. area 12. base 13. cone 14. symmetry 15. sign 16. inclined 17. isosceles 18. acute 19. triangle 20. general 21. triangle 22. sum 23. plane 24. line 25. height 26. similar 27. regular 28. consecutive (or alternate) 29. functions, 30. complementary 31. chords 32. tangent 33. extreme 34. radical 35. irregular 36. left 37. right 38. correspond 39. distance 40. proposition 41. postulate (or assumption) 42. extended 43. assumed 44. erect 45. quadrangle 46. adjacent 47. space 48. section 49. period 50. acute 51. converse 52. pi 53. space 54. loci 55. inscribed 56. triangle 57. straight 58. exterior 59. revolve 60. sphere 61. premise 62. obtuse

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Geometric recreations are generally difficult to obtain. This department would like to secure problems which would lead to original work by the pupils. For example, here is a problem which might be generalized into a series of additional problems. Given a right triangle ABC with C as the right angle. Draw the altitude CD. The sum of the radii of the circles which are inscribed in the triangles ABD, ACD and BCD is equal to the length of the altitude CD. (This should be proved.)

Suppose we take any triangle ABC (this time not a right triangle). Draw the altitude CD. Determine the sum of the radii of the circles which are inscribed in the triangles ABC, ACD and BCD.

We may generalize this problem further. Suppose that we draw the three altitudes. Let the orthocenter be O. We then have three triangles AOB, AOC and BOC. What will be the relation of the sum of the radii of the circles which are inscribed in these three triangles to the radius of the circle which is inscribed in triangle ABC?

The same situation may be examined if we replace the altitudes by the medians or by the angle bisectors of the triangle.

Solutions by the readers will be welcome and will be published in this department. The greatest advantage which may be derived from the study of geometry is associated with the opportunity of variation of certain given conditions stated in a problem, whether this is an original or a construction. If the pupils are encouraged to vary the conditions, they will eventually develop the tendency to "imagine" and to formulate problems of their own. We often hear so much about "motivation" in instruction. The only "motivation" which is effective is the "self-motivation" by the pupils.

Letters

(*Continued from page 266*)

the 9th Year, Curriculum Bulletin, 1949–50 Series, p. 35) does recommend several statistical ideas for use in connection with a unit on "Graphic Presentation." However many of the topics are listed as optional. In addition, the publication is limited for use with 9th year mathematics pupils "whose interest in mathematics is of a general nature and who will not have the need for preparing for more advanced courses in mathematics."

MARTIN HIRSCH
Junior High 227
Brooklyn, New York

Topics of Interest

I read with interest the article in the February issue of THE MATHEMATICS TEACHER on "Topics of Interest to Mathematics Teachers." But how is one going to secure them to read? I should think you would have a large market for a book comprising all of those articles.

Should you decide to undertake the task, consider this my order for a copy.

BEA BUZZETTI
Bremerton, Washington

AIDS TO TEACHING

Edited by

HENRY W. SYER
School of Education
Boston University
Boston, Massachusetts

and

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.57—Mathamerica

Markilo Manufacturers, 3633 S. Racine Avenue, Chicago 9, Ill.

Booklet; 4½″×7″; 44 pp; Single copies —$.25.

Description: The booklet is concerned with the simplifications which would result from replacing our decimal number system by one whose base is twelve rather than ten. Perry cites advantages in merchandising, mensuration, and general simplification of operation. Disadvantages of our number system are also mentioned. Solutions are then given to problems in counting and performing the four fundamental operations.

Appraisal: The booklet is primarily useful in stimulating thought concerning our decimal number system. Reference to the duodecimal system serves to bring to the light many concepts which would be missed in our system because they seem so fundamental and unquestionable.

The author develops his thesis well but is almost religiously enthusiastic in recommending adoption of the radix twelve. Operations are developed psychologically —from the simple to the difficult. The vocabulary and level of the concepts developed make the booklet suitable for superior pupils only. It might prove very interesting and enlightening to elaborate upon the idea by means of class discussion of advantages and disadvantages of the duodecimal systems. Bases other than ten and twelve may also be studied.

The blue-covered booklet is printed upon poor paper and the printing is unattractive. The general appearance of the booklet is poor and the staples which serve as a binding are not sufficiently substantial to survive many readings.

The booklet represents a worthy addition to the classroom library regardless of its poor physical qualities. (Reviewed by Bernard Singer, Hyannis, Mass.)

B.58—Streamlined Methods of Computing with Slide Rule and Mathematical Tables

Donald V. Mitchell, 12345 Sand Point Way, Seattle 5, Wash.

Booklet; 5½″×8½″; 80 pp; $1.50.

Description: The booklet, written by Mitchell, is a notebook insert designed for purposes of mathematical practice and review. It considers topics required in structural shop-work. Among the subjects discussed are the art of speedy computation, the use of formulas in determining square and cube roots and solving quadratic equations, use of the slide rule, and computing with logarithms. Facility in the use of tables and slide rule is considered essential in solving problems arising in the shop.

Appraisal: The information is well arranged but the printing is dark, poorly spaced, and difficult to read. The pages and cardboard cover are of good quality.

Material discussed in the booklet is appropriate because it is practical rather than theoretical; students of shop mathematics are more interested in the instrumental value of the subject than in its logical or theoretical implications.

The price is within reason if the texts being used do not include the subjects presented in the booklet. It is worthy of mention that a 47-page book of tables may be purchased as a supplement to the booklet discussed above; it is entitled "Six-Place Tables for Precision Computing" and costs $1.00. (Reviewed by Bernard Singer, Hyannis, Mass.)

CHARTS

C. 24—Dimensions of Natural Objects (71972)

Central Scientific Co., 1700 Irving Park Road, Chicago 13, Ill.

Chart; 28″×45″; $1.00

Description: This chart is a black lithograph reproduction of a mural which was painted for the Hall of Science at the 1933 Century of Progress Exposition held in Chicago. It indicates the relative dimensions of eleven natural phenomena ranging in size from the proton to the galaxy. A simple diagram of each object appears beside the dimension which describes it. The dimensions are so arranged that the decimal points lie in a vertical line; numerical comparisons are thus easily made and relative sizes are almost immediately obvious.

The printing consists of a large number of tiny black dots which fuse, when viewed from a distance, to form the desired diagrams and lettering. Close study is ineffective however, for the dots are then plainly visible.

Appraisal: The chart may be used effectively to introduce logarithms and the scientific exponential notation whose usefulness in expressing extreme values can not be minimized. It may likewise be used to supply interesting data for exercises which require conversion from one set of units to another—from miles to light years, from inches to yards, etc.

The vertical arrangement of the numerical values is excellent for making comparisons but the chart does not have any other exceptional physical qualities. The price seems to be sufficiently reasonable to justify its purchase. (Reviewed by Bernard Singer, Hyannis, Mass.)

EQUIPMENT

E. 38—Spinno

The John C. Winston Company, Philadelphia, Pennsylvania

Game-drill device; Square base 6″×6″; 8 disks 6″ radius; shields and spinner; $1.65 plus postage.

Description: The base of this device is of heavy cardboard and contains a pivot in the center. The eight disks, printed on both sides, contain four sets of practice examples for each of the four fundamental processes: addition, subtraction, multiplication and division; there are 25 combinations on each side. The four sides for each combination are graded from easy to difficult. The answers are printed under each combination but the smaller circular shields fitting over the pivot allow only one answer to be seen, and the probability that it would be the one on which the pointer stopped is small; in fact, by putting on both shields all answers could be covered. A booklet of directions of four pages (5½″×5½″) is included with suggestions for drill games.

Appraisal: The manufacturer suggests that the game is suitable for grades 3 and 4 for learning and for grades 5 through 8 for remedial work. This distinction is moot, but the general estimate seems fair. The graded exercises seem well chosen, and cover a sufficient range of material. Zero facts are included right from the beginning. The physical appearance is colorful and sturdy. When a device consists of separate parts there is always the danger that some will be lost or bent, but since all parts can be kept on the center pivot at all times, this danger should be reduced. There seems little reason for supplying these in quantity to the whole class, but a growing collection of the interesting drill-games constantly appearing should certainly include this one.

FILMSTRIPS

FS. 81–89—Decimals and Percentage

Eye Gate House, 330 W. 42nd Street, New York 18, N. Y.

Filmstrip; 35 mm; color; silent; $4.00 each ($22.50 for the set).

Description of FS. 81—Introduction to Decimals (25 frames): The filmstrip begins with pictures showing a few great

advances in mathematics. The familiar "pie graph" is used to present fractions as a way of expressing things that are smaller than one. A problem is then given to show that operations with fractions are sometimes difficult. It is proposed that this be overcome by the use of fractions with 10, 100, or 1000 as a denominator. Two frames are used to present the idea of the decimal point. Following this is the decimal use in finance. The superiority of decimals over common fractions in business is shown by placing price tags on various pieces of athletic equipment. A final picture shows several ways of using decimals.

Appraisal: The approach is good but, aside from the first four frames, most of the material might be illustrated as well on the blackboard. Frames 22–23 are good for showing the difference of writing prices, fractions, and decimal forms.

Description of FS. 82—Decimals and Common Fractions (25 frames): Frames 3–6 show John measuring a bicycle seat, a knife, and a pencil with a meter stick. Next, John takes a paper divided into 100 squares and paints 10 squares—showing the idea of one-tenth. A penny and a dime are shown to be fractional parts of a dollar. Frame 11 presents positional notation after which several problems are given for illustration. Finally, a candy bar is used to demonstrate fractional parts—both common and decimal fractions are given.

Appraisal: There is some question in my mind as to whether or not students of this age are familiar with the metric system. There is hardly a thing shown in this filmstrip that could not be illustrated better by use of the actual object, or by use of the blackboard.

Description of FS. 83—Adding and Subtracting Decimals (25 frames): This filmstrip begins with problems and examples of adding and subtracting decimals. A considerable portion of the film is used to emphasize the importance of "placing the decimal point."

Appraisal: The problem in Frame 12 is rather confusing in that it shows the smaller boy weighing 117.6 pounds and the larger boy weighing 105.9 pounds. Frames 18 and 20 are excellent for showing the difference made by misplaced decimal points. A similar attempt is made in Frames 21–22 but the inference may be a little vague to children of this age. Frame 23 shows Tom weighing himself and he doesn't know whether he weighs 9.98 pounds or 98.7 pounds. This would probably be more effective if the same digits were used. Much of the material might be shown on the blackboard.

Description of FS. 84—Multiplying Decimals (26 frames): The first three frames of this filmstrip show John multiplying common fractions with denominators of ten and one hundred. Next, John multiplies various decimals, and in Frame 10 he writes a rule for locating the decimal point in a product. Following this are a number of problems, including the use of "dollar-and-cents," and automobile mileages. Ten frames are used to demonstrate "rounding off" numbers.

Appraisal: The frequent appearance of the boy John, who is able to work these problems may appeal to people of this age, but most of this material can be shown as effectively on the blackboard.

Description of FS. 85— Dividing Decimals (27 frames): The first twenty-one frames deal with the various types of division involving decimals. John is shown moving the decimal point to its proper position in the quotient. Frames 22–26 show three boys dividing the money they earned by mowing lawns.

Appraisal: The method shown here for placing the decimal point is purely a mechanical method. The material presented here can be shown very well by using the blackboard and play money.

Description of FS. 86—Comparing Decimals (27 frames): The first fifteen frames are used for comparing various decimals by use of pies, money, and weights in a balance. The remainder of the filmstrip shows how batting averages are calculated from the number of "hits" a player gets in a certain number of times at bat.

Appraisal: Here again, the blackboard is just as valuable as a filmstrip in writing numbers to show the class. If a majority in the class are interested in baseball, the section on baseball averages should be useful. The pictures of the baseball players and the listing of data on score cards should be a motivating factor in studying decimals.

Description of FS. 87—Introduction to Percentages (27 frames): Frames 2–4 show situations where percentage is used. The remainder of the filmstrip is given to various examples of percentage—using the percentage of milk in a bottle, percentage of girls in a school, percentage of "red heads" in a class, etc. Percentage is explained as "hundredth of something."

Appraisal: This filmstrip contains a number of pictures that would be more difficult to show otherwise. Some of the examples will be useful for solution in class. Frames 20–23, however, are not clear enough to permit ready understanding of the problem.

Description of FS. 88—Using Percentage (25 Frames): The first seven frames show objects with certain percentages taken away, or remaining. Following this, examples are given of the different types of percentage problems: to find the percent of a number, a number is what percent of another; and find the percent of change between two numbers.

Appraisal: Although a picture accompanies each problem, the pupil must read the problem. There seems to be some doubt in my mind, therefore, whether or not this illustration justifies the price of the filmstrip. There is the possibility of novelty in presenting the problems on the screen.

Description of FS. 89—Problems in Percentage (27 frames): This filmstrip is, as the name implies, a series of problems about percentage.

Appraisal: Most of the pictures seem to add little to the solution of the problems. Frame 19 is probably the best—showing a calendar page representing *September*. The days that were sunny are represented by small suns, whereas the rainy days are represented by clouds. What percentage of the days were rainy? Projection is an excellent means for showing problems of this type.

General Appraisal of FS. 81 through FS. 89: According to the producer, this series of filmstrips is directed at the intermediate and upper elementary grades. The development of ideas is good and a number of illustrative examples are given. The color is generally good and the pictures are clear. The captions, however, are a little fuzzy, and sometimes difficult to read. (Reviewed by Paul J. Eby, Boston University, General College)

INSTRUMENTS

I. 81—Industrial Drawing Kit (No. 2)

J. L. Hammett Company, Kendall Square Cambridge 42, Mass.

13″×19″; $3.50.

Description: The Industrial Drawing Kit contains a white wooden drawing board, a T-square, and wooden 45- and 30-degree right triangles. The board is cut from a single piece of wood and its dimensions are 5/16″×13″×19″. Wooden cleats attached to the ends by means of countersunk screws prevent the board from warping. The board is of good quality but the T-square and triangles are not transparent. It is often difficult to draw lines correctly when the figure beneath the triangle is hidden. Furthermore, continual rubbing of a pencil against the wooden auxiliary instruments will cause smudges to appear on the drawing paper because carbon adheres to unfinished wood more readily than to celluloid.

When not in use, the drawing board is inverted. There are spaces between the cleats for storing the T-square and triangles.

Appraisal: The kit is quite useful during the study of scales in grades eight and general mathematics courses. It is of use in making accurate geometric and trigonometric diagrams.

The set is sufficiently small to fit on the

student's desk; opaqueness of the auxiliary tools is not a very great disadvantage in the mathematics class, for the drawings required are seldom as complex as those encountered in mechanical drawing courses.

This kit is not designed for portability and is therefore not very appropriate for use during field work despite its convenient size. As a classroom set, however, its price is reasonable; one set for every four or five students is not only a sufficient number but tends to minimize the financial and storage problems. (Reviewed by Bernard Singer, Hyannis, Mass.)

MODELS

M. 19—Circle

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Disk; 30″ in circumference; ¼″ plywood; $2.00.

Description: This circle is a ¼″ plywood disk with 30″ circumference, a painted diameter and two movable radii. The disk is encircled by a tape measure permanently attached at one end and snap fastened at the other.

Appraisal: The two movable radii may be placed along the diameter to show that the radius is one-half of the diameter. The tape measure that encircles the disk may be loosened at one end to show the distance that a wheel moves in one revolution is the circumference of the circle. Fastened firmly at one end of the diameter the tape may be folded back and forth along the diameter to show that the circumference is a little more than three times the diameter. It is a simple but valuable concept builder when teaching the circle. (Reviewed by Alice M. Phillipson, Mathematics Consultant, Los Angeles)

M. 20—Linkage

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Models; 6 steel linkages; $4.00.

Description: Six spring steel linkages, all with a 30″ perimeter, comprise the set of polygons; scalene, isosceles and equilateral triangles, square, rectangle and trapezoid. In a given figure, sides of equal length are the same color, and all may be opened at one vertex.

Appraisal: The linkages each have the same perimeter as the circle to show that the area of regular polygons having the same perimeter increases with the number of sides. These linkages show the rigidity of the three-sided figures and the nonrigidity of the four-sided figures.

The fact that the area of a parallelogram depends upon the base and altitude and not the length of the sides may be shown by changing the linkage from a rectangle to an oblique parallelogram. In each linkage, bars of equal length are painted the same color so that pupils can easily recognize the line value. By opening the linkage so that the sides form a straight line, the perimeter formula is developed. Almost unbreakable, securely fastened at the vertices, easily opened or closed, and occupying little storage space, they are a valuable aid in developing concepts both in mensuration and in geometry. (Reviewed by Alice M. Phillipson, Mathematics Consultant, Los Angeles)

M. 21—Plane Geometry Figures

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Models; 14 cardboard figures; $2.00.

Description: The set of plane geometric figures is made of cardboard covered with velour paper. They are designed for use on the flocked demonstration board of the Visualizer. The set consists of the following figures all having the same altitude and area; three rectangles 3″×5″, four parallelograms and four trapezoids. Also included are three squares and eighteen square inches. One of each of these groups of figures is cut on the diagonal. The area of a rectangle or square may be developed by the use of square inches. Two of the parallelograms are cut perpendicular to the base so that they may be changed to rectangles to show that the

area is the same. One trapezoid is cut perpendicular to the base through the mid-point of each of the non-parallel sides, so that it may be changed to the original rectangle. The two rectangles with two squares, one 3″ and the other 5″, are designed as a unit to demonstrate $(a+b)^2$. A special color combination is used to make it easy to quickly identify the pieces that belong together. A specially treated demonstration board is required on which to use this set. The company designed a small case for the use of those who do not have a Visualizer available.

Appraisal: These plane figures may be placed on the flocked demonstration board where all can see them. They may be moved by teacher or pupil during the development of the problem. Matching pairs of triangles show the area and congruency of triangles and are valuable in teaching overlapping triangle theorems. Pupils quickly see that some triangles are congruent while others are not. This is a correlated set of basic figures designed to demonstrate visually all of the area formulas. Colorful, attractive, inexpensive, easy to store, it is a valuable addition to the supply of teaching aids of any mathematics teacher. (Reviewed by Alice M. Phillipson, Los Angeles)

M. 22—Cubic Foot

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Model; 1′×1′×1′; ¼″ plywood; $5.00.

Description: This cubic foot is made of ¼″ plywood. One side may be removed to permit the inside of the cube to be used for storage. Three sides that have a common vertex are in the plain natural finish. The other three are marked in inch-squares, alternate squares being painted yellow. One view presents the concept of the cubic foot, the other of the cubic inches in the cubic foot.

Appraisal: This cubic foot is well built and the squares are accurately painted. Used with the Visualizer and a set of inch-*and half-inch-cubes*, the basic concepts of volume may be developed and the ratio of the various units be shown. The removable side permits smaller units such as quarts, gallons, etc., to be placed inside for comparison. (Reviewed by Alice M. Phillipson, Los Angeles)

This section on "Aids to Teaching" has now appeared in the THE MATHEMATICS TEACHER for three years. Your response to the evaluations has been most gratifying. You have given us much help by writing evaluations, by giving us leads to material, and by encouraging us with kind words.

When we began writing this series, we expected to have enough material to keep the section going for about a year. We now find that we are further behind the supply of material than ever before. However, we still need your help. Send us information about material, write reviews on material that you have used in your teaching, let us know about reviews that are not in agreement with your experience. We will be glad to give you credit for your contributions. We want to hear from *you!*

Late in 1950 a letter was sent to all companies whose materials had been reviewed in this section to obtain information as to the continued availability of the material. The results of this survey show that many items are increasing in price—too many to be listed here. The supply of many items has been exhausted because of the demand resulting from reviews appearing in this column. Unfortunately many companies did not reply to the request for information. No survey of films and filmstrips was made since these items are in continuous supply. However, the following materials reviewed in this section are no longer available.

B. 5—Mathematics and The Laws of Nature
B. 8—Navigation
B. 15—Taxes
C. 7—Trigonometric Functions
C. 14—Astronomical Geography Chart
E. 2—Air-Age Research Maps and Charts
E. 28—Add-0
M. 9—Cone, Sphere and Cylinder
P. 6—Portraits of Mathematicians and Pictures of Mathematical Instruments
R. 1—A Tentative Resource Unit on Insurance

The status of the following materials has changed.

B. 22—Computation with Approximate Data. Now available from the Yoder Instrument Co., East Palestine, Ohio
B. 30—An Excursion in Numbers. Orders in excess of one filled at 10 cents a copy.
E. 6—Magic Number Games. Now distributed by Dydo Publishing Co., 1321 Arch Street, Philadelphia, Pa.
E. 26—Calculation. Now distributed by Miss Claire Walker, 260 Oakwood Drive, Wood Dale, Illinois
P. 7—The University Prints. No order filled for less than 25 cents.

NEWS NOTES

A High School Mathematics Conference was held at Millersville State Teachers College, Millersville, Pennsylvania, March 10, 1951. Professor Howard F. Fehr of Teachers College, Columbia University, spoke at the opening morning session on "Implementing a Philosophy of Basic Mathematics." Five demonstration lessons were held at 11:00 A.M. as follows:

I. Basic Mathematics: Grade 9
   Topic: Graphs as Related to Every Day Life
   Teacher: Anna D. Stotz, Columbia High School
   Moderator: Carl Payne, Columbia High School
II. Basic Mathematics: Grades 8–12
   Topic: Mechanical Teaching Aids with Emphasis on Geometry
   Teacher: George R. Anderson, Millersville State Teachers College
   Moderator: Dan W. Witmer, Manheim Borough High School
III. Basic Mathematics: Grades 10–11
   Topic: Visual Aids with Emphasis on Reasoning and Mathematics in Art
   Teacher: Lee E. Boyer, Millersville State Teachers College
   Moderator: V. Anthony Champa, Millersville State Teachers College
IV. Basic Mathematics: Grade 12
   Topic: Writing Checks and Types of Endorsements
   Teacher: Henry U. Walker, Manor-Millersville High School
   Moderator: Rebecca C. Leaman, Manheim Township High School
V. Specialized Mathematics: Grades 10–11
   Topic: Indirect Proof in Geometry
   Teacher: Norman B. Bucher, Lebanon High School
   Moderator: Charles R. Eshleman, Jr., McCaskey High School, Lancaster

Mr. H. W. Charlesworth, President of the National Council of Teachers of Mathematics, spoke at the luncheon meeting and Dr. Catherine A. V. Lyons presented a progress report of the Pennsylvania Council of Teachers of Mathematics at the afternoon session.

The twenty-eighth annual joint meeting of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics, was held at Mississippi State College on February 16 and 17, 1951. Eighty people registered for the meetings of whom twenty-seven were members of the National Council.

Dr. A. S. Householder of the Oak Ridge National Laboratory was the guest speaker at the banquet on the topic "Mathematics in Oak Ridge." A second lecture was on the subject "Nervous Systems and Computing Machines." Additional papers were presented as follows:

"Speak to Me in Calculus." Mrs. W. C. Snipes, Delta State Teachers College
"Mathematics in Music." Mary Ann Turner, Greenwood (Miss.) High School
"General Mathematics for Life Adjustment in the South." Robert W. Lambuth, Jr., Mississippi Southern College
"Motivation in Secondary School Mathematics through History." Robert C. Brown, Southeastern Louisiana College
"The Basic Mathematics Program at Mississippi State." Arthur Ollivier, Mississippi State College
"Social Applications of Eighth Grade Arithmetic." Lucy Douglass, Cleveland (Miss.) High School
"Treatment of Originals in Plane Geometry." Mrs. H. P. Warden, Delhi (La.) High School
"Evaluating a Mathematics Program." Mrs. Margaret Prather, West Tallatchie High School, Webb, Miss.

Newly elected officers for 1951–52 are as follows:

Lester M. Garrison, Louisiana Polytechnic Institute, Chairman
Mary Emma Fancher, Hinds County Agricultural High School, Raymond, Miss., Vice-Chairman
Alva Selman, Holmes Junior College, Goodman, Miss., Secretary
Houston T. Karnes, Louisiana State University, Recorder
Jessie Mae Hoag, Southwestern Louisiana Institute, Louisiana Representative
Virginia Felder, Mississippi Southern College, Mississippi Representative

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Institutes, Workshops, Conferences

(*Continued from page 267*)

enrich and improve the teaching of mathematics (3 weeks, 4½ quarter-hours credit). Write to A. C. VanDusen, Director of the Summer Sessions, Northwestern University, Evanston, Illinois.

University of Wisconsin, Madison, Third Annual Conference on Teaching Mathematics, Grades 1–12, July 2–6, 1951. The program includes addresses and study groups on mathematics and on topics in the teaching of mathematics, curriculum study reports, a mathematics laboratory, and recreation. Housing will be available in university residence halls. If interested, please write to Professor J. R. Mayor, North Hall, Madison 6, Wisconsin for additional information and the complete program.

BOOK SECTION

Edited by JOSEPH STIPANOWICH

Western Illinois State College, Macomb, Illinois

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is being arranged whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

ELEMENTARY SCHOOL

1. Supplementary Material

Arithmetic Readiness Cards, Set 2: The Number System, by Maurice L. Hartung, Henry Van Engen, Helen Palmer, 54 picture cards $6\frac{1}{2}'' \times 8\frac{1}{2}''$, with pictures on both sides in color for the numbers 10–100, and 12-page Teacher's Guidebook. 1950 Scott, Foresman and Company, 433 East Erie Street, Chicago 11, Ill. $3.60.

HIGH SCHOOL

1. General Mathematics

Mathematics, A First Course, by Myron F. Rosskopf, Syracuse University; Harold D. Aten, formerly supervising teacher of mathematics and counselor in the public schools of Oakland, California; and William D. Reeve, Teachers College, Columbia University. Cloth, 472 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $2.60.

2. Algebra

Self-Help Algebra Workbook, by L. S. Walker, Niles High School, Niles, Michigan; and George E. Hawkins, Lyons Twp. High School and Junior College, La Grange, Illinois. Paper, 80 pages, 1950. Scott, Foresman and Company, 433 East Erie Street, Chicago 11, Ill. $0.80.

COLLEGE

1. Algebra

College Algebra, by Herman K. Fulmer and Walter Reynolds, Georgia Institute of Technology. Cloth, vi+204+xiv pages, 1951. Ginn and Company, Statler Building, Boston 17, Massachusetts. $2.85.

2. Advanced Mathematics

Advanced Mathematics for Technical Students Part II, by H. V. Lowry, Woolwich Polytechnic; and H. A. Hayden, Battersea Polytechnic. Cloth, ix+422 pages, 1951. Longmans, Green and Co., 55 Fifth Avenue, New York. $3.75.

Coefficient Regions for Schlicht Functions, Colloquium Publication, Vol. 35, by A. C. Schaeffer, Purdue University; and D. C. Spencer, Stanford University. Cloth, xi+311 pages, 1950. American Mathematical Society, 531 West 116th Street, New York. $6.00.

MISCELLANEOUS

Better Than Rating, New Approaches to Appraisal of Teaching Services, by the Commission on Teacher Evaluation of the Association for Supervision and Curriculum Development, N.E.A., 1201 Sixteenth Street, N.W., Washington 6, D. C. Paper, 83 pages, 1950. $1.25.

Life Insurance Mathematics, by Robert E. Larson and Erwin A. Gaumnitz, School of Commerce, University of Wisconsin. Cloth, vii+184 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.75.

Mathematics, Queen and Servant of Science, E. T. Bell, California Institute of Technology. Cloth, xx+437 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $5.00.

Methods and Materials for Teaching General and Physical Science, by John S. Richardson and G. P. Cahoon, Ohio State University. Cloth, viii+485 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $4.50.

BOOK REVIEWS

Workbook in Geometry. Virgil S. Mallory and Zaidee T. Horsfall. Chicago, Benjamin H. Sanborn and Company, 1949. Paper, 176 pp., $1.00.

Although this workbook has been prepared to supplement a geometry text, it is organized in a manner that makes it readily adaptable to various teaching methods and materials. This arrangement facilitates quick application to other texts as its eight units are completely subdivided in the table of contents.

Students are provided with an easily available reference, located on the inside of the front cover, containing abbreviations and symbols. Any need for remedial work can be determined with the help of frequent reviews supplemented by a few tests. The continuity of geometry can be emphasized with the use of the large assortment of assumptions, definitions, theorems and corollaries that are enforced by many exercises pertaining to each. The development of meaning is made possible by furnishing numerous applications. The problem of loci, as it appears in both the algebra and geometry courses is considered and proofs for a number of cases are developed.—RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Everyday Arithmetic Junior Books (I and II). Harl R. Douglass, Lucien B. Kinney, and Donald W. Lentz. New York, Henry Holt and Company, 1950. ix+488 pp. and ix+502 pp., $1.88.

These texts, suited to grades seven and eight, may be considered a valuable contribution to the growing list of materials that aim toward better understanding of number and in turn toward the development of skill in dealing effectively with quantitative situations.

Emphasis is put on the functional use of numbers. The content deals with real-life problems about the home, the school, and various activities of the community. The program offers a wide variety of problem-solving experiences, such as, making problems, finishing problems, estimating answers, matching exercises, discarding irrelevant data, and working on special projects.

Provision is made for cumulative review and for practice on new processes to insure retention of skill. Each chapter includes much practice material, inventory tests to point out need for "extra workouts," "shock absorber" tests before each inventory test, problem-solving helps and problem-solving scales. Individual differences are cared for by means of optional exercises and by use of special projects which should provide rich experience and do much to stimulate interest and enthusiasm.

The pupil of average or above average reading skill will have every reason to find the program very interesting, stimulating, challenging and self-instructing. The development of subject matter is easy to follow, the frequent checks on skills and the clear and concise summaries at the end of each chapter should do much to aid the learner to evaluate understandings and to develop skill in dealing effectively with quantitative situations.—Olive G. Wear, Fort Wayne Public Schools, Fort Wayne, Indiana.

Albert Einstein: Philosopher—Scientist. Paul Arthur Schilpp, Editor. Evanston, Illinois, Library of Living Philosophers, 1949. xvi +781 pages. $8.50.

This volume, like the others in this series, is divided into four parts. First there occurs an intellectual autobiography by Einstein, given both in the original German and in the English translation by Schilpp. The second part consists of evaluations of various phases of Einstein's life and work by twenty-five outstanding physicists, mathematicians, and philosophers. This part occupies about two-thirds of the volume. Thirdly there are remarks by Einstein regarding the articles in the second part, and finally there is an exhaustive bibliography of the writings of Albert Einstein by Margaret C. Shields. The book has that characteristic of careful work—a very complete index.

Throughout the book one sees evidence that the modern physicist must be a philosopher at heart, and that the successful philosopher must have an understanding of modern physics. Einstein's autobiography is to a large extent philosophical, although it is primarily a description of his work in developing the special and general theories of relativity. It is definitely not an introduction to these theories, however, and probably cannot be read with understanding by anyone who is not already somewhat familiar with his work.

This same remark applies to most of the articles in the second part. This should not be interpreted to mean, however, that one cannot obtain much interesting information and many new ideas even if one cannot follow the technical aspects of these articles.

For a general, non-technical discussion of Einstein's work the article by Louis de Broglie is to be recommended. Niels Bohr's article consists of discussions which he has had with Einstein concerning problems in epistemology as influenced by developments in atomic physics. It contains descriptions, with illustrations, of physical experiments having bearing on these problems. For Einstein's general philosophy of physics the reviewer recommends the articles of Phillip Frank, Hans Reichenbach, and F. S. C. Northrop.

Those interested in the geometry which has been found useful in Relativity Theory should enjoy the articles by K. Menger and H. P. Robertson. Menger's article is actually a survey of old and new ideas in geometry, and it can hardly fail to be interesting to a mathematician regardless of his degree of interest in the Relativity Theory. Robertson's article is mostly a discussion of the problem of constructing a geometry applicable to the physical world, and it is in this sense that the title "Geometry as a Branch of Physics" must be interpreted.

Any attempt to mention all the articles in the book would swell this review beyond its bounds. The articles mentioned above were those which appealed particularly to the reviewer, and are not meant to be representative in any meaningful sense.

I believe this volume is the best available storehouse of argument for and against the ideas at the forefront of much of today's physics and philosophy. It belongs on the book-shelf of everyone seriously interested in these ideas.—Morton L. Curtis, Northwestern University, Evanston, Illinois.

Instructional Tests in Plane Geometry, Revised Edition. Florence C. Bishop and Manley E. Irwin. Yonkers-on-Hudson, World Book Company, 1950. x+68 pp., $.56.

This booklet consists of forty-five tests which can be used throughout the year with any standard textbook in plane geometry. Although the tests are arranged to follow the general order of topics in geometry textbooks, they may be used in any order to correspond with a particular course of study, text, or manner of presenting a logical and sequential development of the subject based on many fundamental facts and principles.

Each test covers a definite phase of work. Objective-type questions are used in testing the more than one thousand selected items involving exercises and applications related to definitions, theorems, and corollaries. Answers may be written in the booklet. Time for each test ranges from ten to twenty-five minutes.

The coefficient of reliability of most of the tests was about .70. However, the authors prepared the tests to assist in the teaching and the learning of the fundamentals of geometry, to provide a determinant for the amount and kind of remedial work needed. They are not purposed to be a measure of achievement. Validity was obtained "through the judicious selection of criteria to be used in the preparation of each test item, the tryout, and the careful revision."

The booklet includes both graphic and tabular record charts for continuous record of individual and group progress.—E. GLENADINE GIBB, Iowa State Teachers College, Cedar Falls.

Schulreform und Mathematischer Unterricht. Walther Lietzmann. Heidelberg, Quelle und Meyer, 1949. 127 pp., $1.50.

Albeit brief and unpretentious, this is nevertheless a significant and interesting monograph for several reasons: (1) the author is an eminent European educator; (2) he is well-known to many American teachers of mathematics; (3) recent trends in post-war Germany would seem to indicate that many of the problems and needs of mathematical education abroad have become rather similar to ours, and even more interesting to note, are apparently being solved and provided for in much the same way as in the United States.

For example, in discussing the objectives of mathematical education (Chapter 5), Lietzmann points out that it is not the primary purpose of secondary schools to train future mathematicians; that while pupils with mathematical aptitude are not to be neglected, it is incumbent upon the school to meet the cultural needs of less gifted pupils ("usually the non-interested") by getting them to appreciate the significance of mathematics in modern civilization. Furthermore, inductive thinking should be emphasized as well as deductive reasoning. Included among other important objectives are skills in observation; appreciation and understanding of measurement, precision, dimensions, rates, and related ideas; practical applications of mathematics; ethical and esthetic implications of mathematics.

As for methodology (Chapter 6), Lietzmann discusses the advantages and disadvantages of the lecture method, the heuristic method, and the laboratory method; suggesting, among other things, that the lecture method, in modified form, is not to be dismissed as being completely without value.

The next four chapters deal with the selection and organization of curriculum content; with various learning aids, such as textbooks, mathematical instruments, slide rule, drawing instruments, mathematical models, and mathematical exhibits; with examinations, promotions and assigned work. The important principles of curriculum construction are as follows: (1) the psychological (propaedeutic) principle; (2) the integration (fusion) principle; (3) the principle of using practical applications and "real life" situations.

Earlier chapters include a historical sketch, followed by intriguing discussions of the psychology of learning, the factors affecting success in learning, the nature of mathematics, and the education of mathematics teachers. Taken all in all, one is as much impressed with the similarity in mathematical education in the United States and Germany today as with the contrast between them forty years ago. The volume should be on the shelves of the mathematics department of every teachers college and school of education.—WILLIAM L. SCHAAF, Brooklyn College, Brooklyn, New York.

Basic Mathematical Analysis. H. Glenn Ayre. New York, McGraw-Hill Book Company, 1950. xvi+584 pp., $5.00.

Since 1917 when the first freshman college text appeared with the title mathematical analysis, there have been more than a score with the same title. One might well ask why there should be another text in this field. That there is a need for a new vision is evident in the changing concept of collegiate education, and the growth of the junior college, the community college, the technical institutes, and others. This text was written specifically to meet the requirements of these newer type educational institutions.

The usual topics are treated in simple, semi-rigorous developments beginning with number and computation, and continuing through the study of algebraic and trigonometric functions. A chapter on the derivative of functions, ending with applications to maxima and minima problems, is included. The results of this chapter are seldom mentioned in the remainder of the text. The analytic geometry of the straight line and conics is treated in the traditional manner, as well as an introduction to three-dimensional geometry of the plane and quadric surfaces. Polar and parametric representation are included. The exponential function is extended to include complex arguments and hyperbolic functions. Tables of powers and roots, common and natural logarithms, and trigonometric, exponential, and hyperbolic functions are appended.

The presentation is clear and simple. The author has managed to select topics that are pertinent to present or needed for the future study of mathematics and to omit highly difficult or unnecessary phases of the various topics. The illustrations and the applications are lucid and pertinent. The text is clearly written for the student, not the instructor. The format and printing make the book easy to read. The wide range of topics enables the instructor to make a choice to suit his mathematical preferences and to adapt to the time allotted to the course.—HOWARD F. FEHR, Teachers College, Columbia University, New York, N. Y.

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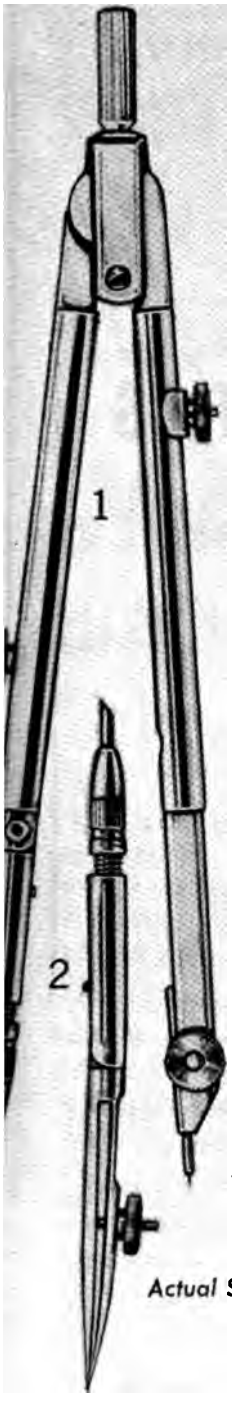

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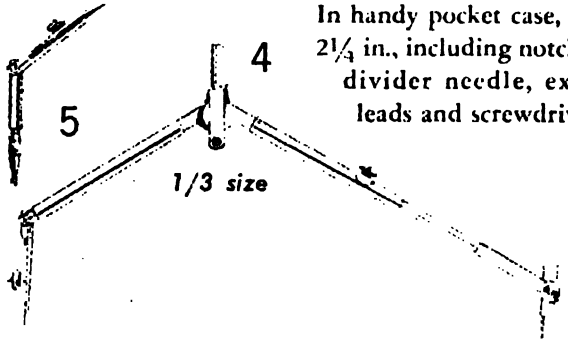

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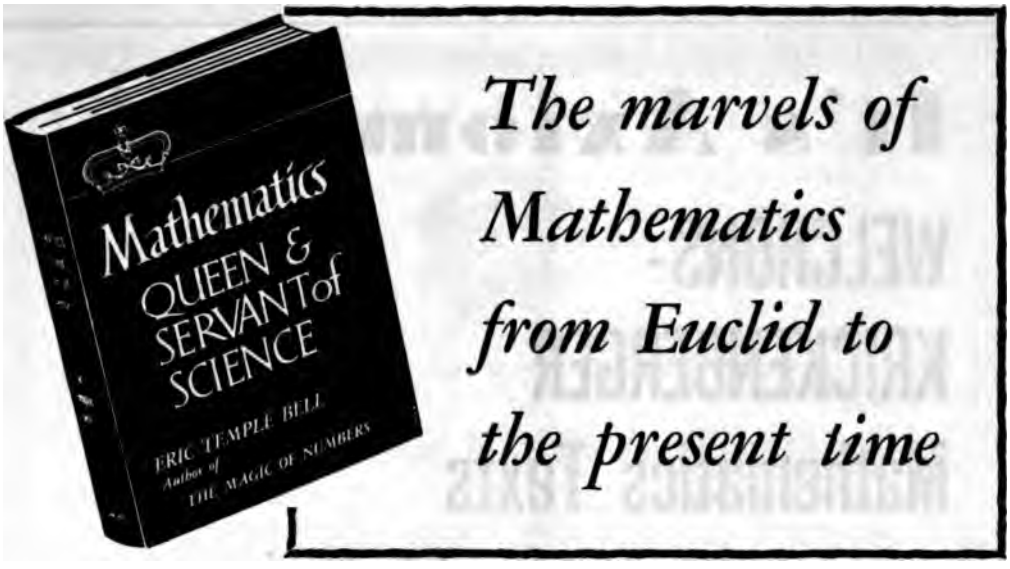

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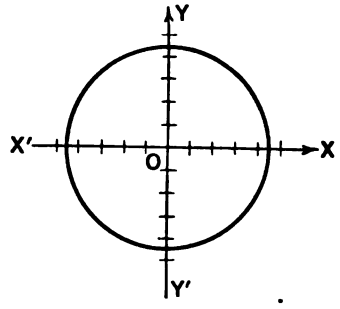

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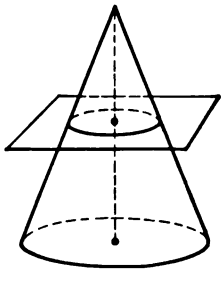

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THE MATHEMATICS TEACHER

Volume XLIV Number 5

An Arithmetic Bulletin for Parents

By ELIZABETH J. ROUDEBUSH
Director of Mathematics, Seattle Public Schools, Seattle, Washington

AT THE SUGGESTION of the Superintendent of Schools, the curriculum staff of the Seattle Public Schools has initiated a plan to inform parents of the ways by which their children learn the three R's. Four committees were formed to devise bulletins describing the methods for teaching arithmetic, reading, spelling and writing in the Seattle schools. The arithmetic bulletin given in this article was sent with the report card to the parents of each child in our elementary schools in November, 1950.

The preparation of the bulletin was a major task. A committee of six curriculum consultants and assistants first set down the questions that parents ask most frequently. Then answers were written in simple direct language. Every effort was made to avoid theoretical expressions or pedagogical lingo. The questions and answers were then studied by the entire curriculum staff. This group suggested many changes, additions and deletions. The committee, the Mathematics Director, and the Mathematics Assistant served as an editorial group to formulate a revised edition. This copy was again studied and improved by the curriculum staff.

The next step was to send copies of the material in mimeographed form to every elementary school. Principals and teachers studied the text and sent in many comments and some suggestions for further change. The committee incorporated these suggestions from the teachers in the final bulletin.

The consultant in charge of publications, the Director of Art, and the Director of Mathematics worked together in planning the format, drawing the sketches, laying out the copy, and supervising the printing of the bulletin.

Parent reaction to the bulletin was good. The most frequent comments were, "This is the kind of information we like to have the schools furnish," and "There should be more of this type of bulletin." Elementary teachers and principals have shown increased interest in the arithmetic program since the bulletin was issued.

HOW WE TEACH ARITHMETIC

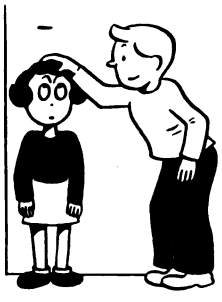
When does your child begin to learn arithmetic? When does your child get

specific drill in arithmetic? How is your child taught subtraction? Do we teach the multiplication tables? Which comes first—long or short division? When are fractions taught? Why is your child often asked to estimate an answer before working a problem? Of what value are story problems? Should your child "prove" problems? Who checks the arithmetic papers? Do we require homework? What is "good work" in arithmetic? How can parents help?

THE ANSWERS . . .

When does your child begin to learn arithmetic?

The use of arithmetic begins in the home when your child knows how old he is, when he can tell coins apart, or when



he recognizes that he is taller than Mary but shorter than Jim. In the kindergarten he probably learns to count objects from one to twenty, to recognize small groups of objects, and to know the meanings of such words as *few*, *many*, *up*, *down*, *thick*, *thin*, *short*, and *tall*. The actual reading and writing of numbers begins in the first grade.

When does your child get specific drill in arithmetic?

Specific drill begins in the second grade after your child learns to add and subtract through the use of actual objects such as books, pencils, and money. Drill speeds up addition, subtraction, multiplication, and division. It is important, however, that numbers really mean something to the child. Arithmetic problems in such everyday things as attendance, game scores, collection of lunch and milk money,

school banking, and telling time provide real experience with numbers. Then we give our practice problems.

How is your child taught subtraction?

The need for subtraction arises when your child meets a question such as "How many are left?" or "How many more?" With pictures, objects, stories, and dramatizations your child takes a smaller group from a larger group and finds how many remain. The "take-away-borrowing" method of subtraction is used with numbers of two or more figures. However, if your child has learned some other method of subtraction, he is not asked to change.

Do we teach the multiplication tables?

Yes, we teach all of the multiplication facts. In the third grade your child sees the need for multiplication and learns

some of the facts. By the time he has finished the fourth grade he probably will know "nine times nine." Understanding is most important. Instead of teaching "three times five" first, Johnny decides that a family of five needs fifteen sandwiches if each one is to have three on a picnic. Practice with multiplication tables continues through junior high school and even into senior high school if necessary.

Which comes first—long or short division?

Long division with a single-figure divisor comes first because through that longer method your child can follow each step as he thinks it through. He may use short division as soon as he is sure of these steps. Short division is first presented in the fifth grade, retaught in the sixth grade, and used extensively in the seventh grade. Short division, after all, is merely a short cut.

When are fractions taught?

The use of fractions occurs in every grade. Your child begins using fractions

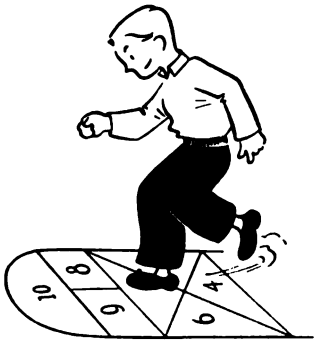
when he does such things as distributing or sharing materials, and folding paper into equal parts. In grade five he begins adding and subtracting fractions by using parts of real objects before he adds and subtracts with fractional numbers. By the end of the sixth grade he will have had multiplication and division of fractions and some preliminary work with decimals. In the seventh grade the emphasis is on decimal fractions and percentage.

Why is your child often asked to estimate an answer before working a problem?

Your child ought to know if his answer makes sense. Since estimating helps him to recognize a reasonable answer, approximating is encouraged. Estimating is not a substitute for working the problem. Its purpose is to develop the child's judgment.

Of what value are story problems?

Your child uses numbers many times during the day—in keeping scores, playing, buying, and banking. It is on these real needs that his arithmetic work is based. He is more interested in learning and remembering number facts when he uses them in connection with his work and play. Story problems help him see how important addition, subtraction, multiplication, and division are to him.



Should your child "prove" problems?

"Proving" the answer to the problem affords the child a feeling of success. The amount of written "proving" that each child needs to do is determined by the skill and accuracy with which he works.

Who checks the arithmetic papers?

There are a number of ways of checking errors on arithmetic papers. For example, a paper may be checked by the child himself, by another child, by a committee, or by the teacher and the child. The teacher makes the final check after the child has corrected his errors.

Do we require homework?

Homework is not often required. Usually enough work time is provided during the school day. When homework is necessary, the teacher plans it with the child.

What is "good work" in arithmetic?

Understanding of arithmetic comes to different children in different ways and at different ages. In each grade, material previously presented is retaught because all children do not master the fundamentals of arithmetic at exactly the same time. However, each child should work up to the level of his ability. "Good work" in arithmetic means the development of systematic, orderly habits of accomplishment as well as the mastery of the fundamental skills and understandings.

How can parents help?

Parents can best help by including the child in the many daily situations in which

numbers are used such as shopping, measuring, counting, sharing, budgeting, banking, and playing games. Number games such as dominoes or "bean bags" are fun for the whole family and will improve your child's skill in arithmetic. Often a family experience will provide a background for an original story problem which the child will enjoy presenting to his class.

Mathematically Trained Personnel Needed in the Government Service

By DANIEL B. LLOYD

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IN THE present era of national mobilization and rearmament the problem of personnel selection for civilian federal employment becomes increasingly important. It is proposed to discuss herein the latest trends in employment policies and practices in government service with emphasis on implications for teachers and counselors in mathematics. The main source of the data was the United States Civil Service Commission which holds examinations in hundreds of cities throughout the country, covering jobs in all grades and classifications. Information desired concerning a particular kind of job, or examination therefore, should be requested from them in Washington, D.C., or from one of their other 13 regional offices.

The author has interviewed a large number of government officials and specialists responsible for the preparation of civil service examinations. The view prevails among them that a larger proportion of questions involving mathematics, and the training that mathematics provides, will be included in the examinations. This applies in general to all types of

examinations, not merely to those of a mathematical nature. The reason for this is that precision techniques or precision instruments are being utilized increasingly in all lines of work, and the need for precision workers is likewise increasing.

More and more the examinations will require precision and accuracy in computations, in reading of verbal problems, in interpretation of meaning, in translation from verbal to mathematical symbols, and setting down in clear and methodical form the given data. The ability to reason logically and systematically in problems and problem situations, ability to observe relations existing in or deducible from the data, and good judgment and skill in the choice and use of techniques applicable to their solution, will be searchingly tested. Officials asserted that those applicants having "blind spots" in mathematics will not be wanted in government work. Skill in mathematical techniques they regarded as an index of their judgment and clear thinking in the widest possible range of positions, whether technical, clerical, or skilled and semi-skilled trades.

Teachers and employers, in the opinion of Civil Service officials, are coming more and more to a common belief that thorough mathematical training is essential to success and advancement in a wide field of occupations and professions.

Civil Service announcements are periodically issued calling for applicants for a wide range of trade jobs, from air-conditioning mechanic to awning maker, and from machinist to tailor. For all of these, at least accuracy in arithmetic is needed, and higher mathematical proficiency definitely improves the applicant's chances for appointment. The greatest present demand for tradesmen is in the building trades,—electricians, plumbers, carpenters, and painters.

Junior engineers, scientists, and technicians in many fields are also in frequent demand. Mathematics and science loom as vital needs for these along with special training or experience, or both, in the specific field chosen. Sub-professional grades, such as scientific aide or technician, requires no college training, but placement on the list of eligibles is on a relative basis and the higher the qualifications the better the applicant's chances for appointment. Graduation from high school in a course including three credits in mathematics and science is evaluated as a year's practical experience.

On the college level there is a present demand for physicists, mathematicians, electronic scientists, draftsmen, and metallurgists. Announcement or examination for other scientific fields is also made periodically. Formal graduation from college is no longer *technically* required, but training or experience *equivalent* thereto is expected.

Many young people with business training enter the government service through junior commercial positions, such as typist, stenographer, clerk, and bookkeeper. Computational skill, reading of tables, use of simple formulas and geometry, as well as specialized skill in the particular field, are essentials for such work. Such beginning jobs are frequently opening wedges to higher administrative positions.

Mathematics used in government service may be considered as of two types: (1) that used by technical workers, including the professional mathematician; and (2) that used by the "Jack of all Trades" who uses any mathematics appropriate to his field of work. The subprofessional grades generally use arithmetic, algebra, geometry and sometimes trigonometry. The professional grades might use mathematics through the calculus, and other higher branches depending on their particular specialty. A survey of a few of the bureaus show that the Naval Observatory uses astronomy and spherical trigonometry; the Geodetic Survey uses map projection, least squares, probability, and complex variable analysis; the Bureau of Standards uses quantum mechanics, differential equations, and various branches of mathematical physics.

Trades and Allied Positions

Positions ranging from unskilled laborer to the highly specialized trades constitute a wide range of opportunity. Outside of Washington far more of these positions are available than any other type. Most persons enter as trade apprentices and advance through successive grades of their chosen trade. The apprenticeship program has been increasing for some time. The Government Printing Office has long had an apprenticeship program for bookbinders, machinists, electricians, compositors, printers, and others. The Navy Yards and other agencies of the Department of Defense, such as Air Force and Army Ordnance, offer apprenticeships for boat builders, shipfitters, coppersmiths, molders, pipefitters, toolmakers, machinists, instrument makers, sheet metal workers, joiners, pattern makers, and electricians.

The apprenticeship usually lasts four years and includes evening classes in related theory as well as a wide and varied program of job experience. After satisfactory service in this, the apprentice becomes a journeyman, i.e., an experienced "regular" in the trade. The minimum entrance age for apprentices is 16, and a regular journeyman 18. Some journeymen enter the government service, also, without further apprenticeship. They are not given an entrance test, but submit their work experience record.

All apprentice tests include arithmetic and practical science. The arithmetic test emphasizes fundamental operations with fractions and decimals (division by decimals is a stumbling block for many); operations with denominate numbers, and their application to measurement problems; use of the metric system; conversion of units; and percentage. The science questions cover simple machines, such as gears, pulleys, levers, as well as automobile repairs, and miscellaneous mechanical information. Formerly pictures were used in this section of the test, but now most of the questions are in the form of verbal descriptions and instructions.

In the algebra, geometry, and trigonometry questions, which are required for the more technical trades, a broad coverage of fundamentals is required; a few fractional exponents, as well as radicals; equations and formulas, solving for any letter; variation and proportion; and verbal problems of the simpler type, are included.

The entire composition of the test battery is as follows:

Subject	No. of Questions
Arithmetic	60
Practical Science	50
Algebra	40
Geometry	15
Form Board	25
U. S. History	15
English Usage (grammar and spelling)	35

Applicants have shown much more weakness in mathematics than in science. Those having had geometry excel on the Form Board test. Scores are superior from areas where teachers are best paid. The validity of this test, as measured by correlation with supervisors' subsequent ratings on the job, is highest with the science section, and next highest with the mathematics.

The government wishes all apprentices trained in algebra, as is required by large private concerns, but according to officials does not find enough so trained. They say that a boy going into apprentice work having had trigonometry would be successful to an extent comparable with a man entering the business world 50 years ago with a college degree.

Sub-professional Positions

These positions are open to those having an elementary knowledge of some specialized professional field. Appointees are assigned to office, field, or laboratory to assist in carrying on certain professional work. Typical of these are:

1. Engineering Draftsman
2. Engineering Aide
3. Scientific Aide, with options in
   a. Chemistry
   b. Geology
   c. Mathematics

d. Metallurgy
e. Meteorology
f. Physics
g. Radio
4. Medical Technician
5. Graduate Nurse
6. Staff Dietician
7. Student Dietician
8. Occupational Therapy Aide
9. Social Case Worker
10. Library Assistant
11. Agricultural Aide

Adequate training of a specified amount in the option selected is necessary. College or specialized technical training is often helpful to qualify. Mathematics on a level useful to the field of work is required.

As these positions can lead the appointee to higher professional positions, it is important that the beginner shall have had all college preparatory mathematics, i.e., trigonometry, solid geometry, and college algebra. While on the job he can then take college or other technical training on a part-time study basis, thus equipping himself for more rapid upgrading.

A typical test battery for this range of positions is composed as follows:

(1) Elementary science
(2) Mathematics, including 20% algebra, and the rest computations, interpreting and reading of tables and graphs.
(3) Scale, gauge, and instrument reading.
(4) Spatial visualization and perception.
(5) Vocabulary.

Mathematics is the common core in all such technical examinations as well as in general aptitude type questions and in some specialized questions in the particular field. Sample examination questions for the positions sought can be secured from the Commission on request.

A high school graduate would possibly rate GS-1 on the civil service pay classification schedule. This was formerly called SP-2 (Sub-professional 2). He could ultimately rise to GS-7, commensurate in pay with the beginning professional grades. (*GS* means General Schedule.)

Certain technical agencies have openings at times for *Trainees* who later become eligible for regular positions.

Professional Positions

A college degree, or its equivalent in training or experience is required for the professional grades, starting with GS-5 on the pay schedule. Over half of the professional positions are filled in this lowest professional grade. Promotion to GS-14 may ultimately be attained while in service. The highest grades GS-14 to 18 are limited by legislation. Fewer than 10% of government employees now actually make more than $6400 annually (Maximum of GS-11 grade).

Experience in Sub-professional positions does not in itself qualify for promotions to professional status, except in unusual cases. However, additional night school or extension courses on the college level, leading to a degree, will enable employees to attain professional status. Thus, the Scientific Aide in Mathematics could attain the professional grade of Junior Mathematician.

Professional scientific positions, besides mathematician, include physicist, chemist, electronic scientist, metallurgist, geologist, meteorologist, astronomer, statistician, architect, geographer, geophysicist, engineer (various branches), physician, analyst and technologist in various fields, and in social sciences such as law, economics, library service, teaching (Indian Service only), business management, and so on. Thorough written examinations in the content of the particular field are given for the junior positions. Applicants for the higher grades (GS-7 and above) are rated only on credentials of training and experience submitted.

Last year's announcement for Mathematician is quoted below, in summary:

Mathematician ($3825–$10,000; grades GS-7 GS-15) (no written examination)
Optional Fields Administrative, Applied, Theoretical, General.
Requirements: (A) a bachelor's degree including 24 semester hours in mathematics, and 12 hours in engineering or the physical sciences; or, (B) four years of progressive experience of an appropriate and valuable quality; or (C) an equivalent combination of (A) and (B). In addition, practical ex-

perience of from one to four years is required, of progressive breadth and degree of responsibility for appointment to the higher grades.

Description of Work: Mathematicians appointed to these positions will plan, direct, perform, or assist in performing (a) research in basic mathematical theory or related theoretical analytic or evaluation studies; or, (b) mathematical calculations and computations incident to investigative, developmental and research work in the scientific fields, such as engineering, physics, astronomy, etc. Their duties will include mathematical research, mathematical analyses of observational data, computation of scientific tables, preparation of graphs and charts, and the writing of scientific reports, all involving a thorough knowledge of basic mathematics and in most cases involving a familiarity with the physical sciences or with engineering practices. The difficulty of the work performed and the responsibility assumed will vary with the grade of the position.

Typical duties: 1. Applying methods of mathematical analysis to problems relating to chemistry and physics, including least squares reduction of data; solving equations relating to heat conduction, electrical circuits, chemical reading and stress analyses; making power and Fourier series analyses of empirical curves; computing magnetic fields of models to obtain fits with observed fields; reviewing, analyzing and assisting in the mathematical formulation of physical problems arising in other government agencies or in industry.

2. Planning and initiating research in theory of pure and applied mathematics aimed primarily at developing methods of analysis which will permit the most efficient and general use of high speed automatic electronic computing machinery; studying and formulating requirements for internal organization of such machinery; developing performance specifications for such machinery.

3. Making necessary computations in reduction of data from tide and current records and correlation of resulting data; computing tidal and current differences and constants, and harmonic constants for prediction.

4. Planning and initiating fundamental theoretical investigations in trajectory theory, aerodynamics and hydrodynamics of missiles and aircraft, the effect of control surfaces on the motion of long-range and high velocity guided missiles and aircraft, and advanced theories of optics and radar as applied to ballistic instrumentation.

5. Computing, adjusting by the method of least squares and making analyses of geodetic observations consisting of those for triangulation, leveling and astronomy; computing and making analyses of observation made for the determination of the intensity of gravity and of the isostatic reduction of gravity.

General Data

Some short-cuts are now being used to expedite examining procedure in certain areas, due to urgent needs. More examinations are unassembled and even when written, emphasis is shifted from subject matter content to practical intelligence and general aptitude for job performance. Work habits and skills, keenness of observation, and perception are emphasized.

At the present time, appointments are being made on an "indefinite" basis, due to unsettled conditions. Many of these positions will become permanent in the future, however. Information on current openings can be obtained from the U. S. Civil Service Commission in Washington, D. C. Sample questions for various examinations are available. Also Pamphlet 11, May 1948, "Specimen Questions from U.S. Civil Service Examinations" is on sale for 20¢ from the U. S. Government Printing Office, Washington, D. C. "The Government Employees Exchange" published bi-weekly, 10¢ a copy, at 1740 K Street, N. W., Washington, D. C., is a privately published news bulletin of government jobs. It is an unofficial, but fairly accurate journal. Announcements of openings are posted in postoffices, first and second class, throughout the country.

ANNUAL N.C.T.M. SUMMER CONFERENCE WITH N.E.A.

San Francisco, California, July 2, 1951

Theme: *Improving the Teaching of Mathematics to Meet the Emergency*

See announcement of program on page 329 of the April issue.

Luncheon Reservations: ($2.25 each) should be mailed to Dr. A. J. Hall, San Francisco State College, San Francisco, Calif., not later than June 15, 1951.

Functional Mathematics—Grades Seven Through Twelve*

By WILLIAM A. GAGER
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IN NOVEMBER 1947 the Florida State Department of Education and the college of Arts and Sciences at the University of Florida sponsored a study under my direction to determine ways of improving certain parts of the present secondary mathematics curriculum. Thirty-six secondary mathematics teachers, representing all areas of subject matter, all types of schools, and all sections of the state were selected to make the study. Work on the project was begun at the University of Florida on June 14, 1948.

Impressed by the surprisingly large number of Courses of Study already available on traditional secondary mathematics the Workshop Group thought it wise not to concern itself with further study of the traditional courses. Instead the Group set as its objectives:

1. To isolate from the traditional mathematics courses, and other sources, those mathematical concepts and principles most essential for effective living in this modern age.
2. To select content materials of interest and value to the respective age groups but which definitely involve the essential mathematical concepts and principles.
3. To organize the materials into a sequence of, functionally worthwhile, mathematically sound courses for grades seven through twelve.

On August 5, 1949, this project was brought to a close and the materials were submitted to the State Department of Education. After a careful study and upon the recommendation of the Florida Courses of Study Committee the State Department of Education placed its stamp of approval upon the study and authorized its publication under the title of *Functional Mathematics in the Secondary Schools*—Bulletin No. 36.

With the basic concepts considered to be of first importance, the Workshop Group set as its first assignment the task of isolating the concepts which it proposed to include in the sequence of functional courses. As one might expect, the personnel of the Group found it very baffling to react concisely and precisely to ideas such as angle, area, equation, locus, similarity, slope, and volume. The fascination of the job however more than compensated for the difficulties encountered.

After numerous revisions there was common agreement that the following 61 concepts would provide adequate stability and flexibility for the proposed functional courses. The list shows that only 35 of the 61 concepts are to be taught in seventh grade and that some of these have been taught in earlier grades; the eighth grade covers 44 concepts with only 9 new ones; the ninth grade 55 concepts with 11 new ones; the tenth grade 60 concepts with only 5 new ones; and the eleventh and twelfth grades 61 concepts with but one new one.

With the realization that concepts by themselves were not enough, the next step taken was the preparation of a list of basic working principles through which the concepts could function. These principles as listed below are those which each student must understand if he hopes to find any meaning in the mathematical processes which follow. One might add that there is little hope that the student will understand these principles in a way that he can effectively use them unless each teacher of mathematics is continuously alert to them and really knows how to teach them for understanding.

* Presented at the Twenty-Eighth Annual Meeting of the National Council of Teachers of Mathematics, Chicago, April 15, 1950.

by the highest sum of the exponents in any term of an algebraic equation in standard form.
2. An "n^{th}" degree equation has "n" roots.

XII. The same *operation* may be performed *on* both members of an *equation* without changing the balance.
1. If both members of an equation are increased, decreased, multiplied or divided by the same number, or equal numbers, the results are equal.
2. Like powers or roots of both members of an equation are equal.

XIII. To every pair of *rectangular coordinates* there corresponds one and only one point in a plane, and, to every point in a plane there corresponds one and only one pair of rectangular coordinates, with respect to a fixed pair of axes.

XIV. *Comparison* between quantities may be expressed by subtraction or division.
1. Only like quantities can be compared.
2. Ratio is a comparison by division, and is an abstract number.
3. The ratio of the circumference of a circle to its diameter is constant.
4. The ratio of the diagonal of a square to its side is constant.
5. In similar figures the perimeters are in the same ratio as any corresponding sides, the areas as the square of any corresponding sides, and the volumes as the cube of any corresponding sides.

XV. The idea of *locus* involves two interpretations: (a) locus as the path of a moving point so as to satisfy certain given conditions; (b) locus as the composite of many fixed points which satisfy certain given conditions.
1. All points on a locus satisfy the given conditions.
2. All points that satisfy the given conditions are on the locus.

XVI. When the variable V approaches the constant C so that their difference $V-C$ becomes and remains less than any positive number, no matter how small, named in advance, then the variable V is said to approach the constant C as a *limit*.
1. If a variable V approaches a limit C, and each is multiplied by K, then KV approaches KC as a limit.
2. If two variables are always equal while approaching their limits, then the two limits are equal.

XVII. Mathematical *infinity* is the concept of becoming large beyond any known bound and may be represented by the symbol ∞.
1. Division by zero is impossible. However it is sometimes expressed symbolically as $\lim_{x \to 0} \frac{1}{x} = \infty$, where ∞ is understood to represent an idea but not a number.

In order to have some assurance that *the concepts* and principles herein listed were involved in the proposed materials, so that there would be an acceptable vertical spread of them within each grade and a horizontal unfolding of their meaning from grade to grade, a Grade Placement Chart was prepared to cover the six grades.

The Workshop Group, in selecting materials for functional courses for seventh and eighth grades, did not greatly alter the present offerings. Keeping in mind the varied interests of seventh and eighth graders and the fact that their mathematical needs are actually very few, much effort was expended to build the content around pupil interests and to introduce new concepts and review old ones through activities. It was hoped that this approach would permit the pupils to work with the materials in such ways that they might feel the elation that could come from their own discovery of workable procedures.

The present ninth grade experiences in general mathematics brought the Workshop Group face to face with a difficult problem. In studying the general mathematics program over the last twenty-five years it was found that general mathematics had been introduced into the ninth grade as a substitute for algebra with the hope that: (1) the number of failures in algebra would be reduced; (2) general mathematics would provide materials within the interest and understanding of the poorer students; (3) general mathematics would be a course of special value to those pupils who would not take mathematics after the ninth grade.

It was also found that an alarmingly large number of teachers considered general mathematics in the ninth grade a failure. The most prominent reasons given for this situation were: (1) the unfavorable and rebellious attitude of many mathematics teachers and administrators toward the subject; (2) the over-socialization of the course to the point of crowding out much of the essential mathematical frame work; (3) the over-loading of the

BASIC PRINCIPLES

I. In *integers and decimal fractions* the position of the digits in a number determine the value of the number.
   1. The name given to a decimal fraction is determined by the place value of the last digit of the number.
   2. The value of a decimal fraction is not changed by annexing or removing zeros at the right end of the number.
   3. Integers and decimal fractions should be carried out one more place than is desired in an approximate answer.

II. The numerator and denominator of a *common fraction* may be multiplied by the same number (except zero) without changing the value of the fraction.
   1. In final form common fractions may be reduced to lowest form.
   2. Of the three signs, sign of the numerator, sign of the denominator, and the sign before the fraction, any two may be changed without changing the value of the fraction.

III. In *common and decimal fractions* the same quantity may be expressed as a common fraction, as a decimal fraction, and as a per cent of a number.

IV. *Numbers* can be combined by addition, subtraction, multiplication and division.
   1. Like quantities can be added or subtracted. The sum or difference of unlike quantities can be indicated only.
      a. In finding the sums and differences of decimal fractions, the digits of the same place value must be added or subtracted.
      b. Common fractions can be added or subtracted only if they have a common denominator.
      c. Quantities expressed in the same unit of measure can be added or subtracted.
   2. The numbers to be added or multiplied may be combined in any order without changing the result. (Commutative Law)
   3. The numbers to be added or multiplied may be grouped in any manner without changing the value of the result. (Associative Law)
   4. Multiplication may be performed by separating one of the factors into any number of parts, multiplying each part by the other factor and adding the partial products. (Distributive Law)
   5. If one factor of a product is multiplied or divided by a number, the product is multiplied or divided by that number.
   6. If a product is zero, then one or more of its factors is zero.

V. The following *order* is recommended *for performing all operations:*
   1. Perform all operations indicated in the parentheses.
   2. Perform the indicated multiplications and divisions, before the additions and subtractions, in the order in which they occur.
   3. Perform the additions and subtractions in the order in which they occur.

VI. A *ratio* may be expressed as a common fraction, a decimal fraction, and as a per cent.
   1. Per cent should be changed to fractional form before multiplying or dividing.

VII. *Measurement* is the comparison of any magnitude with a unit of measure by which it can be measured.
   1. All measurements are approximations.
   2. The largest allowable error in measurement is one-half the unit of measure used.
   3. Measurements must be expressed in the same unit of measure before computations can be made.
   4. In addition and subtraction of approximate data the result should claim no greater precision than the least precise unit used.
   5. The accuracy of a product or a quotient is no greater than the accuracy of the least accurate factor involved.

VIII. A *quantity* is *equal* to, *greater than*, or *less than* another quantity of the same kind.
   1. A quantity may be substituted for an equal quantity in an expression without changing the value of the expression.
      a. Corresponding parts of congruent figures are equal.
      b. Corresponding angles of similar figures are equal.
   2. If three quantities are so related that the first is greater than the second and the second is greater than the third, then the first is greater than the third.
   3. The whole is equal to the sum of its parts and is greater than either of them.

IX. *Distance* between two points implies shortest distance.
   1. In a plane or space a straight line is the shortest distance between two points.
   2. The perpendicular is the shortest distance from a point to a line or to a plane, or between two parallel lines or two parallel planes.
   3. The minor arc of a great circle is the least distance between two points on a sphere.
   4. The square on the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

X. When one quantity depends upon another for its value the two quantities are said to be *functionally related.*

XI. An *equation* is a statement that two expressions are equal.
   1. The degree of an equation is expressed

A Cooperative Plan to Share Objective-Type Test Items

By HENRY W. SYER
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TEACHERS have long been advised to build and use objective-type test items in their classroom tests in addition to the use of standardized tests which have been constructed and sold by outside authorities. Our own classroom tests reflect the particular topics which we, as teachers, and our students have found interesting and important for our particular class during a particular year. Purchased tests can never fully replace the class tests made and used by the teacher who has carried the class along through the day-by-day development of the subject. However, all who have tried know the time and energy which is required to formulate worthwhile test items in mathematics. Sometimes items which seemed good do not work out in practice at all. If we all had time the ideal procedure would be to use, analyze, revise, use, analyze and discard items in a growing file which would thus be constantly refined and improved. Few teachers have time to follow through such a procedure individually. The purpose of this report is to indicate a procedure which might facilitate the exchange of items concocted by individual teachers of secondary mathematics through the country so that these items could be used by others. The suggested plan is to establish a regular department in THE MATHEMATICS TEACHER which will collect, classify and publish items supplied by teachers who have written and used them. There is no thought of standardizing the topics or procedure in the teaching of mathematics; the items will be displayed for use, but no teacher is urged to use them if they do not meet the objectives of a particular class. As time goes by this pool of items may contain many which test the same concepts, skills or other objectives. This is all to the good for the bigger the selection, the more interesting the shopping tour.

Eventually, the best items may be skimmed off and published as separate collections in booklets for teachers to purchase, but this will be possible only if the cooperative plan described appeals to teachers and a steady flow of items is maintained. The National Council for the Social Studies has published such booklets concerning American History, World History, Economics, American Government and Study Skills and has found that they were used and appreciated by teachers.

Many of the considerations which are important when a complete test in a subject is being examined are not possible or appropriate when individual items are being collected. The most important is probably reliability. It would be very difficult to attach a meaning to the reliability of an individual item out of the context of a completed test. The other items, the position of the item in the test, and other factors influence the reliability. Also, we are usually interested in the reliability of the whole test and thus we leave this consideration to each teacher who chooses items from our pool to construct classroom tests. Other considerations which would be important for a whole test which we cannot discuss include the following: ease of administration, scoring and interpretation; and time and supplies needed. Specifically, there is no use in trying to call an item an "aptitude," "achievement" or "diagnostic" measure, since the same item may be used in various ways; this again depends upon the purpose and form of the test as a whole.

The following classification and symbolism of items will look awkward and

forbidding at first glance. It becomes simpler with use, and is intended to compress a great deal of information about each item into a small space. It will thus become apparent that the only items to be included are those which have been used and analyzed to some small extent. This means a bit of work for each person who submits items, but the whole purpose of this cooperative plan is to standardize such work and to share it enough with others to justify the time.

TABLE I

Subject Matter of Items

Code Letter	Subject Matter
G	General mathematics
C	Computation in arithmetic
R	Reasoning in arithmetic
E	Elementary (first year) algebra
A	Intermediate (second year) algebra
P	Plane geometry
S	Solid geometry
T	Trigonometry
V	Advanced mathematics
B	Business arithmetic
H	Shop or allied mathematics

In Table I appears a classification for the items to show the major field in which the item will probably be used. The code letter will be the first part of a code designation attached to each item printed. After some thought it was decided to make the primary classification of items in terms of the subject matter since this is the most common way which teachers will be searching for items to use. This is not to be interpreted to mean that it encourages water-tight courses organized around these areas, but is a simple admission that years of usage have given meaning to the subjects listed and for purposes of *classification* of items they are very useful; for purposes of *teaching* they may not be so useful.

All the meanings are self-evident except possibly G, V, B and H. *General mathematics* will mean any item which tends to overlap several fields combining, for example, material from algebra and geometry in one item. There will also be found many of the informational items of social mathematics now used in grades seven through nine. It is not necessary that the item be used in a course entitled "general mathematics" in order to be placed in this category. *Advanced mathematics* is a useful category for topics which are appropriate to a senior mathematics course, but are not solid geometry or trigonometry. Topics from third-year algebra (permutations, determinants, etc), topics from the analytic geometry and calculus which some find appropriate for the secondary schools will be placed here. *Business arithmetic* includes the items designed for courses of that name, and are fairly specialized items. Many which could be used here will appear under "general mathematics" because of their broader usefulness. Of course, such a classification as this we are attempting is neither perfect nor inviolate; any teacher forming a test will look through all categories which may possibly be appropriate. *Shop mathematics*, similarly, consists of specialized items useful in those course taught in conjunction with mechanical and trade courses. Many of the items listed under arithmetic will also be very useful in these courses.

Table II lists the various forms in which items may be cast. In mathematics the types most commonly used are completion, multiple-choice, computation, and true-false items, but there is no reason why we

TABLE II

Forms of Objective-Type Test Items

Code Letter	Form
A	Analogies
C	Completion
D	Deductions
E	Essays
F	Forms corrected for errors
G	Matching
H	Homologs
I	Identification
M	Multiple-choice
N	Constructions
O	Omission by cross-outs
P	Reproduction from memory
R	Rearrangements
S	Computations
T	True-false

should not explore and utilize less common types if they prove applicable. *Analogies* are the types of items which resemble mathematical proportions using concepts instead of numbers. For example: "Polygon: perimeter = circle: ?" Such questions require the formulation of a relationship and then the finding of a concept which has that relationship. *Completion* items require the filling in of one or more blanks with words, symbols or numbers. *Deductions* are items which list a body of informational facts, usually in paragraph form, and then either ask for deductions which could be made from the facts, or supply deductions to be evaluated as possible or not possible. More subtle forms have more possibilities: absolutely deductible, possibly deductible, unable to determine whether deductible, possibly not deductible, absolutely not deductible, or some similar choices. *Essay* questions may be keyed so that they can be corrected in an objective manner, but this is neither a common nor a very satisfactory practice. *Forms corrected for errors* are statements which require the location and correction of errors within the statements; some, of course, in the set may be completely correct. An example might be, "A cube has 6 faces, 12 vertices, 8 edges, 4 diagonals and 16 diagonals of faces." *Matching* items consist of two columns of concepts, usually unequal in number, which are matched by association of meaning. *Homologs* may be association tests, or synonym-antonym items. In mathematics we might have one such as the following: "Give the word which means the opposite of each of the following: negative, rational, parallel, convex, and infinite." *Identification* items require classification of material given either by means of a list given, or from some classification retained in the memory. We might have in mathematics: "Which of the following could be regular polygons: square, isosceles triangle, parallelogram, circle, equilateral triangle, rhombus, right triangle, pentagon, trapezoid, and tetrahedron." *Multiple-choice* items are familiar. Since it is awkward having different numbers of choices for different multiple choice items within the same test, we will establish a ground rule that all multiple-choice items submitted will have exactly four choices. *Constructions* have the usual meaning of plane geometry, ruler and compass constructions, but they must be cast in the form of objective questions, with definite directions to the student and definite instructions to the person who will grade them. *Omission by cross-outs* would contain incorrect or redundant words or phrases which may be crossed out leaving the remainder correct. For example: "Cross out the words which make the statement incorrect: In geometry we are careful to define the following words: circle, diagonal, point, hypotenuse, parallel, ratio, triangle, and polygon." Or, for redundancies: "Cross out any redundant portions of this definition: The logarithm of a number n which is used to multiply and divide numbers is the exponent, usually written above and to the right of a number, which another number a, called the base, and usually taken as 10, or e (an irrational number), is raised to, to give the first number n." *Reproductions from memory* are seldom needed in mathematics, but might include the derivation of the quadratic formula or reproduction of complicated construction or similar facts. *Rearrangements* consist of a set of facts or ideas which have a natural, logical order, but are not in that order and must be rearranged. *Computations* should contain just one clear answer in order to beome objective-type items. *True-false* items are so familar that they need no discussion.

In Table III we find the classification of the types of materials which can be tested in the items. Since it is probably desirable to have only one clear-cut reason for each objective-type item, there should be no question of overlapping of objectives. It must not be thought that undefined words, definitions, unproved statements, and proved statements are reserved for the use of geometry courses; and rules,

formulas and skills, the special province of algebra. A moment's reflection will show that they all permeate all of mathematics. There is no doubt that the teaching as well as testing of applications, attitudes, appreciations, judgments and discriminations are deficient in the usual course in mathematics. They are included on the list for completeness, and to encourage the formulation of items for their evaluation.

TABLE III

Objective to be Tested by Item

Code Letter	Material
	Facts
U	Undefined words
D	Definitions
P	Unproved statements (postulates)
T	Proved statements (theorems)
R	Rules
F	Formulas
S	Skills
A	Applications
Q	Attitudes
N	Appreciations
J	Judgments and discriminations

So that teachers who wish to choose items from our stock may know what performance to expect from the item there should be some pertinent data attached to each. These should be supplied with the items by the teachers who send in contributions. Table IV tabulates the information needed. Originally, each item was doubtless part of a test constructed, administered, corrected and analyzed by a classroom teacher. The first bit of information needed is the total number of students who answered the item on the test. This should be distinguished from the number who took the test, and all who did not answer the item should be disregarded. In order to collect data on enough cases only items which have been answered by 100 or more students should be analyzed and submitted for publication. This should not be difficult, for it represents about three or four classes taking most tests. Next, we need the number of students who answered the item correctly. In order to describe the type of student who is represented by this group there need be three facts given: the age, grade and I.Q. of the group. This will enable other teachers to determine whether they wish to use the item with their groups. The validity of the particular item will be described in terms of the achievement of the bottom and top quarter of the whole group taking the test. By reporting the per cent (0 to 100%) of those in each quarter who answered the item correctly a fairly accurate indication can be given. Note that this must be the lower and upper quarter of the group who answered that particular item (correctly or incorrectly) and does not include those who omitted the item. This makes it more difficult to compute, than if it were the upper and lower quarters of the group taking the test, for many items may have been omitted for lack of time rather than ignorance. The computation of these figures will be greatly simplified if the students taking the test are encouraged to answer each item,—"guessing" they call it—and time enough is allowed for this to be done.

TABLE IV

Information Needed on Each Item

Code Letter	Information
N	Number of students answering item.
R	Number of students answering item correctly.
	Type of students answering item
A	Mean age and standard deviation of age
G	Mean grade and standard deviation
I	Mean I.Q. and standard deviation
	Validity information
Vb	The per cent of those in the lowest quarter (on the basis of the whole test) who answered this item correctly.
Vt	The per cent of those in the highest quarter (on the basis of the whole test) who answered this item correctly.

It may now be wise to give three examples of how individual items would be written up using the system described above. The items are new, and the data are fictitious.

AMD.0 [N—106, R—56, A—16.4 (±1.1), G—11.0 (±0.0), I—113 (±6); Vb—20, Vt—92]

Ans: (d)

$16^{-1/2}$ equals (a) $\frac{1}{8}$ (b) -8 (c) 4 (d) $\frac{1}{4}$

"AMD" means (from Tables I, II and III) that the item concerns intermediate algebra, that it is a multiple-choice item, and it is testing definitions. The number after the point will be a serial number telling how many of that type have been published. (Since this is a fictitious item, the number 0 is used.) All the information within the square brackets is identified in Table IV. It will be noticed that years and grades (and their standard deviations) are given to the nearest tenth, while I.Q. is given to the nearest whole number. These would of course be found by computing one more digit and rounding back (five and above would raise the digit). The validity per cents are given to the nearest per cent only. After the square brackets will be found the answer to the item. Here are two more sample items:

TRS.0 [N—221, R—144, A—18.2 (±0.3), G—118 (±0.1), I—121 (±2); Vb—40, Vt—90]

Ans: b e a c d.

Arrange in order of magnitude with the smallest value first:

(a) $\log_{10} 20$ (b) $\sin 25°$ (c) $\log_e 10$ (d) $(\sin 10°)^{-1}$ (e) $\sin^{-1/2}$ (in radians)

It will be noticed here that separate answers are identified by letters rather than numbers since there is less danger of confusing them with the answer itself.

PCN.0 [N—117, R—94, A—16.5 (±2.4), G—10.5 (±1.1), I—130 (±15); Vb—65, Vt—98]

Ans: Rectangle.

The most pleasing type of quadrilateral to use in art is the————.

There is the plan. If enough people are interested to make it a going proposition, the pool of test items should prove to be a tremendous help to teachers. It will not work if every one sits back and waits for others to supply him with items. Dig out the items you have, add to them, and let's get the department rolling. In order to utilize test items which may not yet have the information called for by Table III, it is possible to send in items without that information, if you wish to do so. However, this is not very desirable, since it precludes the possibility that they will be used in any later collection.

Items may be sent either to Mr. John H. Haynes, Acton High School, West Acton, Massachusetts, or the author of this article, who will act as joint editors.

The Signal Corps Posters

(see pages 327–330)

EDITOR'S NOTE: On the center pages of this issue we are reproducing four of the twenty posters prepared and distributed in 1943 through the cooperative effort of the Signal Corps Schools of the Sixth Service Command, the Men's Mathematics Club of Chicago and the Women's Mathematics Club of Chicago and Vicinity. Unfortunately the supply was exhausted within a few months and the cost of preparing reprints in two colors at that time was prohibitive.

Shall we include the remaining sixteen posters in issues next fall? Our decision depends on the number of requests (a postcard will do) we receive from our readers, before June 15. Are we correct in estimating that 20% of our members will read this note and that 50% of those who read it may favor reprinting the other sixteen posters? Our question: How many responses will reach The Editor of THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Ill., before June 15?

NOTES ON THE HISTORY OF MATHEMATICS

Edited By VERA SANFORD
State Teachers College, Oneonta, New York

THE RULE OF FALSE POSITION

THE RULE of False Position, Single and Double, is a method of solving equations of the first degree in one unknown by guessing the answer and then adjusting the guess in terms of the conditions of the problem. As the name suggests, there might be one such guess or there might be two. The word *position* in this case is used in the sense of a "thing on which one takes a stand"—in its meaning here it is allied to the idea of letting or allowing, forming a tenet or proposition on which one expects to act. *Supposition* would be closer to our thinking.

The Rule of Single False is sometimes utilized implicitly by pupils who reason out a solution to problems involving simple equations rather than working with an unfamiliar algebraic notation. Such an instance occurred in an eighth grade class when the pupils were asked to find how the sum of $35.00 should be divided among three boys if it was understood that Tom should have twice as much as Dick and that Jim should have twice as much as Tom. One solution was that if Dick were given one dollar, then Tom would have two and Jim four. But under those circumstances, the total amount would be seven dollars not 35. But since five times seven is 35, the correct division would be to give each boy five times as much as had been computed in the first place—Dick would have $5.00, Tom would have $10.00 and Jim would have $20.00. The sum would be $35.00 and the amounts would be in the proper proportion. An instance such as this makes the Rule of Single False of interest to teachers of elementary algebra.

The Rule of Double False is important from the historical point of view because it furnished one of the important uses of signed numbers and because it illustrates clearly the way in which an understanding of computation with signed numbers enables a person to replace several rules by a single one.

The Rule of Single False appears implicitly in the Ahmes Papyrus (*c.* 1650 B.C.). There is reason to suppose that the two rules were used by the Hindus. They certainly were used by the Arab mathematicians. They were prominent features of mathematical works in Europe from Fibonnacci's *Liber Abaci* (1202) to the arithmetics of the sixteenth century. As algebraic symbolism developed, the Rules of False disappeared from the more advanced texts. For example, the rules were used in the arithmetics of Cocker and Leybourne in the late seventeenth century, but not in John Ward's *Young Mathematicians' Guide* (1706). The rules are given in Bonnycastle's *Scholar's Guide to Arithmetic* (1780) but not in the same author's *Introduction to Algebra*, an indication that the author considered it an arithmetic rule, not an algebraic one. Nicolas Pike's *New and Complete System of Arithmetic for the Use of the Citizens of the United States* (1788) gives a clear explanation of the matter and puts the algebraic proof in footnotes.

At first glance, the Rule of False is uncanny. It is reported that Robert Recorde (*c.* 1542) astonished his friends by proposing difficult questions and finding the true result from the chance answers of "suche children or yedotes as happened to be in the place." Humphrey Baker explained the name of the rule in his *Well spring of Sciences* (1568) in these terms:

"The Rule of falsehood is so named not for that it teacheth any deceit or false-

hood, but that by fained numbers taken at all adventures, it teacheth to find out the true number that is demanded. And this (of all the vulgar rules which are in practice) is the most excellent. . . . Those questions which are done by false positions, have their operations in a manner like unto that of the rule of three: but only that in the rule of three we have three numbers known, and here in this Rule, we have but 1 number that cometh in use to work by."

Nicolas Pike defines the Rule of Single Position in these terms: "Single Position teaches to resolve those questions, whose results are proportional to their suppositions (i.e., their guessed or assumed values). Such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times.

"Rule 1. Take any number, and perform the same operations with it as are described to be performed in the question.

"Then say, As the sum of the errors is to the given sum; so is the supposed number, to the true one required. *Proof.* Add the several parts of the sum together, and if it agrees with the sum, it is right." Pike then gives an example. A school master, being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I should then have 435. Of what number did his school consist?

(*See top of next column*)

This should be compared with the solution given in the Ahmes Payprus of the question to find a number such that the sum of the number and its seventh part is 19. In this case, the number is assumed to be 7, then the number and its seventh part would be 8, but it should be 19. Accordingly as many times as 8 is multiplied to give 19, so many times 7 must be multiplied to give the required number.

Problems solved by the Rule of Single False are varied. A popular one stated that the head of a fish weighs $\frac{1}{9}$ of the whole fish, the tail $\frac{1}{4}$ and the body weighs 30 ounces. What is the weight of the fish?

Suppose he had	80
As many	80
$\frac{3}{4}$ as many	60
$\frac{1}{2}$ as many	40
$\frac{1}{4}$ as many	20
$\frac{1}{8}$ as many	10
	290

As 290:435::80

80

290) 3480 0(120 *Answer*

29

58

58

0

120

120

90

60

30

15

435 *Proof.*

The problem of the pipes filling a cistern could be solved by this scheme and so too could the problem of the men doing a piece of work.

The Rule of Double False is more involved. Baker puts it this way:

"The summe of this Rule of Two false positions is thus, when any question is proposed appertaining to this Rule, first you must imagine any number at your pleasure, which you shall name the first position, and with the same you shall worke in stead of the true number, as the question doth import: and if you see that you have missed of the true number that you do seeke: then is the last number of the worke, either too great or too little: the which number, you shall note with the signe of more or lesse, for that is the first error, in the which you have failed, the which signes of more and lesse, shall be noted with these figures X—, This figure X, betokeneth more: and this plaine line—

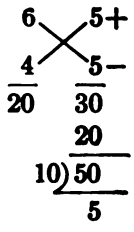
signifieth lesse: that it to say, the one signifieth too much, and the other too little: then you must begin againe, and take another number, which shall be the second position, and work by the question as before: if you have failed againe note the excesse or want, for that is the second error. Then shall you multiply the first position by the second error crosse-wise, and againe the second position by the first error (and this must alwaies be observed) and you must keepe the two products: then if the signs be both alike, that is to say, either both too much, or both too little, you shall abate the lesser product from the greater, and likewise you shall substract (*sic.*) the lesser error from the greater, and by the remaine of those errors, you shall divide the residue of the products, the quotient shall be the true number that you seeke. But if the signs be unlike, that is to say, the one too much and the other too little, then you shall adde those products together, and likewise you must adde both the errors together, and by the Sum of those errors, divide the totall sum of both the products; the quotient shall be the true number that you seeke."

Leybourne states that the Rule of False Position (and he restricts himself to the Rule of Double False) is suited to questions "which are not presently fit for the Golden Rule." Pike is more explicit—

"Those questions, in which the results are not proportional to their positions, (i.e. the guesses) belong to this rule: Such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required." Pike's illustration will help to clarify the rule quoted from Baker.

"A lady bought damask for a gown at 8 *s* per yard, and, lining for it at 3 *s* per yard: the gown and lining contained 15 yards, and the price of the whole was 3 *l.* 10 *s*; How many yards were there of each?"

Pike supposes first that there were 6 yards of damask, value 48 *s*. Then there would have been 9 yards of lining with value 27 *s* but this adds to 75 *s* which is 5 *s* too much. Then if there had been 4 yards of damask value 32 *s* and 11 yards of lining value 33 *s*. the sum would have been 65 *s* which is 5 *s* too little. Then he places the guesses at the left and the errors at the right as is indicated here:



He multiplies on the cross lines and adds getting 50. The sum of the errors is 10. Dividing 50 by 10 he reaches the conclusion that there were 5 yards of damask and 10 of lining.

The problem of deciding how to adjust the guesses and errors led to a number of memoriter devices. Cuthbert Tonstall (1522) went to his friend Sir Thomas More who wrote a stanza for him. He says "Et quo facilius has praeceptiones studiosi memoria tenerent, Morus, rogatu nostro, redegit eas in hoc carmen

> "A plure deme plusculum,
> Minus minori subtrahe:
> Pluri minus coniungito,
> Atque ad minus plus adiice."

Cocker in 1677 put it this way:

> "When Errors are of unlike Kinds
> Addition doth ensue
> But if alike, Subtraction finds
> Dividing work for you."

In brief, the cross products of guesses and errors are found and their difference is divided by the difference between the guesses. Pike's explanation of this occurs in a footnote to which earlier reference has been made. If a and b are the guesses and r and s the errors, N the given number and A and B the numbers that are produced by the guesses a and b, and x the number that is sought, then r is the difference between N and A; s is the difference between N and B, then $r:s::x-$

$a:x-b$ from which x is equal to $rb-sa/r-s$ "and if r and s be both negative, the Theorem is the same, and if r or s be negative x will be equal to $rb+sa/r+s$."

The operation of the Rule of Double False seems to have suggested recreational problems in which the results of two guesses were given. For an example of this there is a problem given by Calandri (1491).

"A school master has so many scholars that if each pays 8 pence he will lack 10 soldi of paying his rent, but if each pays 10 pence, he will have 20 soldi too much. How many scholars were there and what was the rent?" Clavius (1608) had a somewhat similar case in a problem which he attributed to Euclid,—If a mule takes one measure of wine from an ass's load, the mule will have double the ass's burden, but if instead the mule gives one measure from his load to the ass their burdens will be the same. How much was each carrying?

The need of indicating whether the errors were in excess or defect was the occasion for the introduction of the signs for positive and negative numbers into arithmetics printed in the sixteenth century. (It will be noted from the example given from Humphrey Baker that he used an X instead of the minus sign to which we are accustomed.) There is reason to believe that Widman who was the first to use the signs + and − in print introduced them into his arithmetic because he wished to have his students familiar with them for their use in the application of this rule. Humphrey Baker could claim that the Rule of False was the most excellent of all the common Rules, but that was in 1566. Today the rules are curios made obsolete by algebraic symbolism.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES

University of Michigan, Ann Arbor, Michigan

26. More About Nedians

NORMAN ANNING of the University of Michigan generalizes as follows the concept of *nedian* as defined by John Satterly in THE MATHEMATICS TEACHER for January, 1951 (vol. XLIV, pp. 46–48).

Assume that the points D,E,F on the sides BC, CA, AB of triangle ABC (Fig. 1) divide these sides cyclically so that the segments of each side are in the ratio $p:q$.

It then is not hard to show (he says) that

$$(1)\qquad \frac{\Delta XYZ}{\Delta ABC}=\frac{(p-q)^2}{p^2+pq+q^2}$$

$$(2)\qquad \frac{\Delta DEF}{\Delta ABC}=\frac{p^2-pq+q^2}{(p+q)^2}$$

$$(3)\qquad \frac{\Delta XYZ}{\Delta DEF}=\frac{(p^2-q^2)^2}{p^4+p^2q^2+q^4}.$$

Of course (3) follows from (1) and (2). When (3) is compared with (1) one observes it to be merely (1) with p replaced by p^2 and q replaced by q^2. This leads to the conclusion that YZ cuts EF, ZX cuts FD, XY cuts DE in the ratio $p^2:q^2$ and thus suggests that within ABC there is a readily constructable hierarchy of triangles having AD, BE, and CF as nedians and ΔXYZ as a limit.

Professor Anning's formula (1) specializes to Professor Satterly's formula (3) for the area of the nedian triangle. Let $p=1$ and $q=N-1$, then $BD=(1/N)BC$ and

$$\frac{\Delta XYZ}{\Delta ABC}=\frac{(1-N+1)^2}{1+N-1+(N-1)^2}$$

or

$$\Delta XYZ=\frac{(N-2)^2}{N^2-N+1}\cdot\Delta ABC.$$

Please note that this also constitutes a correction of a sign error in the denominator of the Satterly formula as previously printed.

A second and voluminous note came from MERTON T. GOODRICH of Keene Teachers College, Keene, New Hampshire. Professor Goodrich had been working independently on essentially the same idea for some time prior to the appearance of Professor Satterly's note. Goodrich had invented the term *redian* drawing the r from *ratio*. He has extended the concept from triangles to parallelograms, and polygons in general. He states that he has obtained such theorems as that the central *redian* figure of any polygon is the same kind as the original polygon, and that in any triangle, parallelogram, or regular polygon what he terms the *redian* triangles, one for each vertex, are all equal.

For our Figure 1 the *redian* triangles are:

$$\Delta AFY = \Delta BDZ = \Delta CEX = \frac{1}{N(N^2-N+1)} \cdot \Delta ABC.$$

Further for the quadrilaterals:

$$FBZY = DCXZ = EAYX = \frac{N^2-N-1}{N(N^2-N+1)} \cdot \Delta ABC.$$

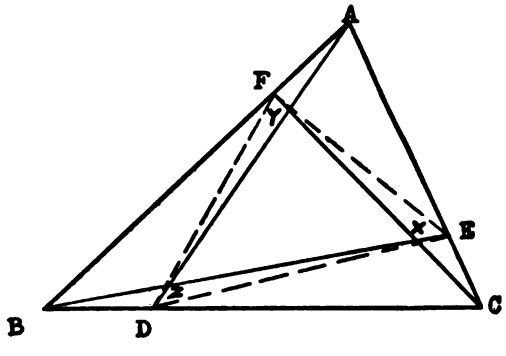


FIG. 1

Interestingly, these same formulas were also relayed to us from Professor Petrie in a letter from John Satterly.

We hope to see more of Professor Goodrich's interesting work in print later on. Perhaps other readers, too, have further results which they will send in.

27. Geometric Progressions—Two Notes

KARL S. KALMAN of Abraham Lincoln High School, Philadelphia, Pennsylvania writes that in addition to the usual more rigorous derivation of the formula for the sum of an infinite geometric progression, $S = a/1-r$, using the limit concept, he finds the following presentation adds interest and conviction for his students.

Let $S = a+ar+ar^2+ar^3+ \cdots$ where a is the first term, r is the constant ratio, $r<1$, S is the "sum" (which it is assumed exists) then

$$S = a+r(a+ar^2+ar^3+ \cdots)$$

$$S = a+rS$$

$$S = \frac{a}{1-r}.$$

To add further insight into and "meaning" for this theorem Mr. Kalman uses the device of specialization. He calls the attention of his students to the case $a=1$, when $S=(1/1-r)=1+r+r^2+r^3+ \cdots$. It is particularly evident in this case that if $r=1$ then the limit of the series does not exist. This is beautifully consistent with the fact that the fraction $1/1-r$ is not defined for $r=1$. Further, if $r>1$, the formula clearly gives a negative value for S while the series is just as clearly positive. With this heuristic treatment students will not only recall longer the condition $r<1$, but will also feel it to be more intuitively sensible as well as mathematically necessary.

A. R. JERBERT of the University of Washington, Seattle, Washington contributes the following note which he titles *Clock Problems—Achilles and the Tortoise:*

It is no longer the fashion to include clock problems in beginning algebra texts. The reason can't be that they are

unpractical because that is the case with nearly all the problem material which is available to a first course in algebra. In the matter of difficulty, too, they compare favorably with work problems, time-rate problems, and a host of others.

A typical clock problem is to determine "at what time between 9 and 10 o'clock are the hands of a clock opposite to each other?"* Evidently the minute hand must *gain* 15 minute spaces. Thus,

let x = number of spaces traversed by the minute hand,

and $x/12$ = number of spaces traversed by the hour hand, so that

$$x - x/12 = 15$$

whence,

$$x = 15(12/11) = 16\tfrac{4}{11}.$$

For the student who is familiar with the concept and formula it is just as expeditious to "sum" an infinite geometric progression. Thus we can argue that the minute hand must cover 15 spaces (at least). However, while this is going on, the hour hand moves up 15/12 spaces. The minute hand must therefore cover this many spaces in addition. Continuing in this manner, the minute hand must cover in all

$$15 + 15/12 + 15/12^2 + \cdots,$$

spaces. Since $r = (1/12) < 1$,

$$S = 15/(1 - 1/12) = 15 \cdot (12/11), = 16\tfrac{4}{11}$$

as before.

If we substitute Achilles and the tortoise for minute and hour hand respectively, we have essentially the classical problem which Zeno proposed as a paradox to confound rival Greek philosophers. Rephrase it and we have: "Assuming that Achilles travels 10 times as fast as the tortoise, how long will it require for him to overtake the tortoise if the latter has a 100 yard start?"

Analysis and solution: While Achilles is running the 100 yards which separates him from the tortoise, the latter moves ahead another 10 yards. In the time that it takes for Achilles to cover the additional distance, the tortoise moves forward another yard. Thus Achilles must run successively,

$$100, 10, .1, .01, \cdots,$$

yards. Since this series runs on indefinitely, it would (and did) seem evident that Achilles would never catch up with the tortoise. As with the clock problem, however, we have merely to employ our formula to obtain $(10/9) \cdot (100)$ as the total number of yards which Achilles must run to overtake the tortoise.†

28. Card Sorting and the Binary System‡

A widely used application of the binary number system occurs in connection with punched card systems of the McBee Keysort type (Fig. 2). The application arises, of course, from the fact that there are just two choices available for a given

* The writer recalls a "perennial student" in a beginning algebra class, who was essaying the course for the third or fourth time. When clock problems came around, this man (he was 20 or more) pulled out a gold watch and solved them empirically. Unfortunately, the expensive watch failed to "turn up" the odd elevenths which are yielded by a little pencil work.

† *Editor's Note:* Gregory St. Vincent in his *Opis Geometricum quadriturae circuli*, (Antwerp: 1647) was the first to apply this procedure to the "Achilles" problem stated by Zeno circa 450 B.C. according to Florian Cajori, *A History of Mathematics* (New York: 1938), p. 182.

‡ Historical and recreational applications of binary notation have been discussed by Vera Sanford in "Notes on the History of Mathematics," THE MATHEMATICS TEACHER Vol. XLIV (January, 1951), pp. 29–30. A treatment of uses of binary notation in electronic computers may be found in L. N. Ridenour's article, "Mechanical Brains," *Fortune* Vol. XXXIX (May, 1949), pp. 109–118. In the appendix of this article Ridenour applies the binary notation to the mathematical theory of the game of Nim. The "Russian peasant" multiplication procedure, the theory of which rests upon the binary system, is described in Oystein Ore, *Number Theory and Its History*, (McGraw-Hill Book Co., Inc., 1948), pp. 38–39 and Sir T. L. Heath, *A Manual of Greek Mathematics*, (Oxford, 1931), p. 29.

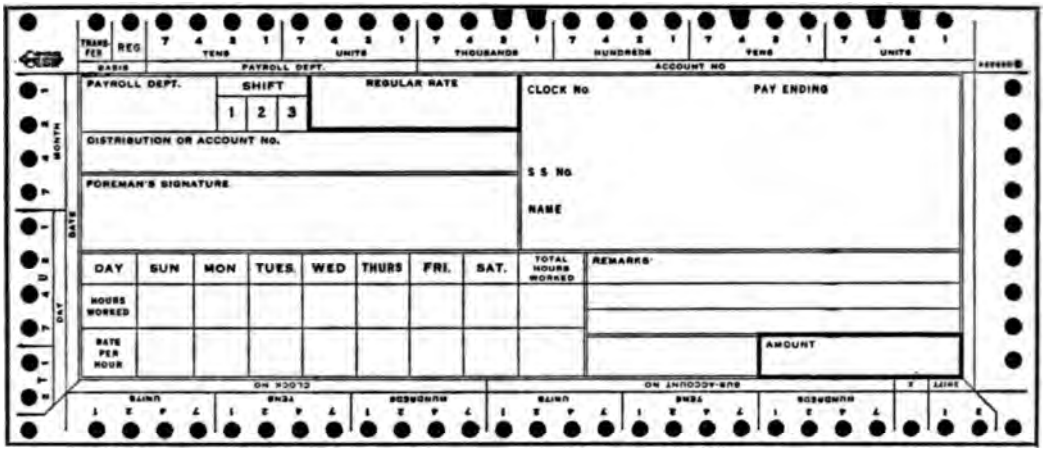


FIG. 2. A McBee Keysort Card.* Manufactured by the McBee Company, Athens, Ohio.

position on the card—there may or may not be a hole punched there.

By the use of what is called a "sequence code," the cards in a pack are coded serially. The serial number for each card is designated in binary notation, with two modifications: (1) the decimal *place* system is retained, there being portions of the card set aside for the units, the tens, the hundreds, etc., places of the serial number with the binary system being used merely to specify the digits from 0 through 9 which should appear in that place; and (2) the binary system itself is altered so that instead of the first four places (right to left) representing $2^0=1$, $2^1=2$, $2^2=4$, $2^3=8$ respectively, the digits used for coding are 1, 2, 4, and 7.

The 7 is used in place of the 8 because this substitution makes it possible to specify any of the numbers from 0 to 9 by not more than two code numbers. Standard binary notation for these numbers would have been:

0— 0
1— 1
2— 10
3— 11
4— 100
5— 101
6— 110
7— 111
8—1000
9—1001

In this list, only the number 7 requires three non-zero digits (and hence three punches on the card) for its representation. With the altered notation (the fourth place representing 7 rather than 8), the digits 7, 8, and 9 become respectively 1000, 1001, and 1010. This departure from strict binary notation, it may be noted, results only in the loss of the ability to designate the number 15, an unimportant defect here since we need to represent only the digits 0 through 9.

In the Keysort system, the cards as furnished by the manufacturer have holes punched around the borders. Information is written on the face of the card, and a hole on the border is designated to refer to each item of information. When a card contains a certain item, that fact is indicated by punching out a V-shaped notch at the position of the corresponding hole, so that the hole is replaced by a notch in the edge of the card. For example, Figure 2 pictures a card notched to correspond to the number 3046. When it is desired to select, out of a pack of cards, those which contain a particular item, a long needle is run through the pack at the position of the hole for that item. The needle is then lifted, and those cards

* Reprinted by permission from the manufacturer.

which have been punched at that position may be made to fall out, while the rest of the cards remain on the needle. This is the basic sorting procedure, and the procedures for performing the more complicated types of analyses are merely extensions of it, involving somewhat more elaborate equipment.

As the cards are used for various purposes, they are likely to get out of their serial order. The restoration of this order is easily accomplished, requiring only 4 sorts for each digit of decimal notation. For example, suppose a pack of the cards are in jumbled order, and it is desired to place them in serial order. The sorting is first performed in the units portion of the card. The units digits will have been coded as follows:

0—no punch
1—1
2—2
3—2, 1
4—4
5—4, 1
6—4, 2
7—7
8—7, 1
9—7, 2.

All cards coded with a 1 are first sorted out and put at the back of the pack. This will divide the pack into two parts. In the first part will be the cards with units digits 0, 2, 4, 6, 7, or 9; and in the second, those with units digits 1, 3, 5, or 8. This situation may be indicated:

0, 2, 4, 6, 7, 9—1, 3, 5, 8,

and it should be kept in mind that the cards are in jumbled order in each group. It is not necessary, during the sorting process, to keep the groups separate.

A sort is now made by the digit 2, and the cards sorted out placed at the back of the pack. This will give the following grouping:

0, 4, 7—1, 5, 8—2, 6, 9—3

and it may be observed that all cards coded 3 are together.

Sorting by 4 gives:

0, 7—1, 8—2,9—3—4—5—6.

The 4's and 5's and 6's are all together because each came from a different group out of the preceding sort.

Finally, sorting by 7 will result in all the cards being in proper order according to their units digits:

0—1—2—3—4—5—6—7—8—9.

The same sorting procedure is next followed using the code numbers for the tens digits. It may easily be verified that at the end of the process the cards will all be in order according to their last two digits. The process of four sorts per digit is then repeated as many times as there are digits in use on any of the cards.

A parlor-trick type of illustration of this sorting technique may be performed with a deck of ordinary playing cards. Assigning the values 11, 12, 13 to the Jack, Queen and King, and using the code digits 1, 2, 4, 8 makes it possible to arrange the cards in order of value in 4 sorts, sorting the cards into two piles each time.

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University of Michigan
Ann Arbor, Michigan

29. Sorting Jumbled Cards into Numerical Sequence

The previous note recalled to the editor an experience of the bankrupt days of the 1929's when he was employed by a receiver to sort into a numerical sequence a large number of invoices which had been filed alphabetically. In his youthful innocence he started to sort into piles by hundred thousands, then these into subpiles, etc. Working space was soon exhausted and confusion reigned. An efficient procedure would have been to sort into ten piles by the units digit, assemble the piles in order into a single bunch and re-sort into ten piles by the tens digit, repeat this process until all the places in use had served as a basis for a sort. At this time the invoices would then be in proper serial order.

AIDS TO TEACHING

Edited by

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and

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College of Education
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of this department are invited to
ation about new or recent "Aids to
ither to Dr. Syer or to Dr. Johnson.
about items which have not been
date is especially welcome but sug-
other available items are needed
sible, include name and address of
or publisher, cost, grade level at
rial might be used, and any com-
the effectiveness of this material
n the classroom.
terial reviewed is checked with the
determine whether it will be avail-
nth when the review appears; some-
itions change before publication,

ompanies prefer to send one free
material to a teacher and not to
ıtity for a whole class. Since this
on the number of requests made, it
vell to ask whether a quantity is

BOOKLETS

Outlook for Women in Mathe-
l Statistics

ıdent of Documents, U. S.
nt Printing Office, Washington

"×9", 21 pages; U. S. Depart-
abor, Women's Bureau, Bulle-
er 223–4; $.10.

ion: This booklet is primarily
with the changes and trends in
f work and the supply and de-
women in mathematics and
It includes a large amount of
ving the number of persons
ıd working in these fields. It
earnings and opportunities for
ent. On the basis of the new
esulting from our technological
the booklet attempts to forsee
and opportunities for mathe-
and statisticians in the future.
al: The information for this
as culled from 800 books, arti-
cles, or pamphlets as well as from such primary sources as scientific organizations, employers and trainers of women scientists, and men and women scientists themselves. Besides giving the teacher and personnel worker valuable information for the guidance of superior mathematics students, the description of the kinds of work performed by mathematicians, statisticians and actuaries will indicate the applications of mathematics in a variety of situations. It will give the mathematics teacher useful information to show that training in mathematics is becoming increasingly important.

B.60—Geometric Constructions

New York University Bookstore, 18 Washington Place, New York 3, N.Y.

Mimeographed booklet; 8½"×11"; 59 pages; $.75.

Description: This booklet, written by Aaron Bakst, summarizes and organizes various methods of determining a solution of problems in construction. The methods are grouped under the following headings: Loci, parts of figures, similarity, algebraic method, motion, symmetry, inversion, perspective, and trigonometric method. There is a bibliography of 25 items at the end, four of them in English. A short discussion of other methods than straightedge and compasses is included at the end.

Appraisal: This is a splendid summary of the subject with many helpful examples worked out. It is far more complete than most secondary school pupils will need, and may even trouble some teachers with its completeness. Still, better to have it too complete than less complete. Without doubt more work could be done with con-

structions in classes if teachers could see the possibilities of their being considered miniature research projects, as the author of this booklet does. Unfortunately, many teachers and pupils do not have the "research mind," and the abstractness of mathematics as a field for learning about research, although theoretically ideal, does not appeal to them.

It has been noted that the types of construction problems discussed are often more difficult than we have time and interest to teach. On page four it is stated that the purpose is "to develop the system of geometric constructions along the model of a mathematical system." This development is not obvious in the rest of the booklet; nor are the fundamental constructions independent. In general however, this booklet will be interesting and valuable for teachers to read and should influence their teaching for the better.

B.61—Math Miracles

Wallace Lee's Magic Studio, P. O. Box 105, Durham, N. C.

Bound book; $5\frac{1}{2}'' \times 8\frac{1}{2}''$; 83 pages; $3.00.

Description: Here are mathematical tricks in the form of magical presentations. Most of them are familiar, but the selection is good. Enough hints are given of the way to present them so that an appealing performance can be given. Twenty topics are used; casting out nines, magic squares, and similar topics are mentioned.

Appraisal: To enliven mathematics a program of these tricks could be presented by pupils who have learned them. There is plenty of opportunity to learn mathematics by analyzing why the tricks work. Some are too difficult to analyze at the level where they are very interesting and acceptable as tricks, and could be enjoyed for their own sake at that level. After all, there is no reason why we must always pull to pieces everything mathematical; for some people the play with numbers is satisfying and beautiful without *the understanding*. Possibly the aesthetics of arithmetic deserve more attention.

The binding and illustrations are quite attractive. The hard cover should last for a long time.

CHARTS

C. 25—Charts for Arithmetic

F. A. Owen Publishing Company, Dansville, N. Y.

Ten charts; $10'' \times 13''$; $1.00.

Description: Since each of the ten sheets is printed on both sides, there are really twenty posters to display. They are printed on bright colored sheets of light cardboard and enclosed in a portfolio. The following subjects are covered: measures, a time chart, linear measure, Roman numerals, change for a dollar, kinds of subtraction, temperature, weight, liquid and dry measure, measurement by counting, decimals, fractions, interest problems, long division, and percentage. These are mostly useful in the intermediate grades.

Appraisal: The purpose seems to be both understanding and drill. The material is a heterogeneous mixture of tables of denominate numbers, problems to be solved, and graphical representations of fractions. All material could very easily be written on the blackboard, but is probably useful to have immediately at hand for display. Only five of the twenty contain anything other than numbers and words, and then only circles to represent fractional parts, or pieces of money. It is unfortunate that some enlivening illustrations could not be included. There is a need to supplement the drill with applications, or, better still, to achieve the drill through applications.

BULLETIN BOARD DISPLAYS

D. 1—Problem of the Week

Mr. Mel Lieberstein, Collinsville Township High School, Collinsville, Ill.

Mimeographed sheets, 45 in all; $8\frac{1}{2}'' \times 14''$; $2.50 per set, $6.50 for 3.

Description: This set of 22 problems is mimeographed on a large scale so that

they are all ready to mount on bulletin boards. There is one horizontal title sheet which serves as a caption. The other sheets are vertical and contain the problem on one sheet and the answer on another. There is room on the answer sheets to record the names of pupils who submitted the correct answers to the problem of the week before.

Appraisal: The idea is clever, the problems are quite appropriate for secondary school pupils, and the job of mimeographing is extremely well done. The line drawings which serve to illustrate some of the sheets are attractive and add a great deal to the interest in the display. By coloring some of the sheets by hand, even more interest may be aroused. The price charged is far too much for the material received; it should be sold at no more than $1.00. Then individual teachers could be supplied with sets to use. The best method now is to acquire one for a whole school system and circulate it for all teachers to use. By making other caption sheets by hand and sending the sheets from school to school, many teachers could use the set. Another way would be to look up similar problems in books on mathematical recreations and have sheets made in local classes. These could be mimeographed for all, or done by hand by individual pupils. If necessary to get the idea started, this set could be bought.

EQUIPMENT

E. 39—Aritho

Psychological Services, 4502 Stanford Street, Chevy Chase, Maryland

Game; 12″×13½″; $1.50.

Description: This game consists of fifteen answer cards (each 4″×5½″) with twenty-five numbers on each arranged in the form of a square. The five columns are headed +, −, ÷, × and ×. There are six problem cards which are cut up into 390 problems in addition, subtraction, multiplication and division. There are also many small squares of cardboard to cover answers on the answer cards. The game is played like Bingo. A caller picks up a small problem card and reads the problem aloud. The answer must be covered in the correct column of the answer card which any player holds. Some one keeps track on a master sheet of the answers which are used to check. Only integers are used in problems and answers.

Appraisal: This seems to be capitalizing on a familiar and fascinating kind of competition. Its purpose, is, of course, a painless method of drilling on number combinations. There is no reason why a whole closet full of these games should not be owned by the schools and used constantly. Once the meanings of the operations are taught, the drill should follow up to the desired level of mastery.

The answers to addition range from 0 to 18; those in subtraction and division from 0 to 9; and those in multiplication include 0–10, 12, 14, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 49, 54, 56, 63, 64, 72, and 81. This gives some idea of the scope of combinations included. The fundamental combinations are the chief ones used. All the addition facts from 0+0 to 9+9 are used; all the subtraction facts from 0−0 to 18−9 are used; all the multiplication from 0×0 to 9×9; and all the division from 1÷1 through 81÷9 are used. There is some doubt about the advisability of arranging the problems with the second number under the first in every case. It would be wise to have both the horizontal and the vertical arrangements for addition, subtraction and multiplication. For division the horizontal arrangement is probably more familiar than the vertical which is used. Since only the caller sees the form, however, this is probably a minor point.

This game should provide many exciting minutes in the drill of number combinations.

E. 40—Criteria Quadrilateral

W. M. Welch Scientific Co., 1515 Sedgwick Street, Chicago, Ill.

Device; aluminum; Catalog No. 7520; $6.25 each—$3.25 each for 100 or more.

Description: This device consists of two aluminum strips which rotate about a point at which is mounted a circular protractor. On each member are two fixtures which can be moved relative to the pivotal point. The aluminum members represent the vertices of a quadrilateral. Each diagonal is graduated in centimeters reading outward from the point of intersection. An elastic thread stretched around the four fixtures (vertices) represents the quadrilateral. This device is designed to show how different diagonal relations determine the different types of quadrilaterals.

E. 41—Parallel Lines Device

W. M. Welch Scientific Co., 1515 Sedgwick Street, Chicago, Ill.

Device; $14\frac{1}{2}'' \times 12''$; aluminum; Catalog No. 7515; $5.50 each—$2.75 for 100.

Description: This instrument consists of two colored aluminum strips and a transversal having circular protractors mounted at the points of intersection. Each device is equipped with a snap-on fixture which holds the lines parallel. Thus, it can be used as a parallel rule for drawing parallel lines. Lines are etched on the strips to avoid considering the strip as a line. The lines can be rotated until they appear to be parallel and the relative sizes of the various pairs of angles can be read from the protractors; or, the sizes of the angles can be controlled and the relative positions of the lines can be visually checked. The theorems relating to alternate interior angles, corresponding angles, exterior angles, interior on the same side of the transversal, etc., can be investigated or verified. With the snap-on fixture in place, all the theorems relating to parallel lines cut by a transversal can be illustrated.

Appraisal of E. 40 and E. 41: These simple devices are similar to other Schacht Dynamic Geometry Instruments previously evaluated in this section. They are well constructed devices that should make it possible for the geometry teacher to *illustrate* relationships within a quadrilateral and also between parallel lines more dynamically than with drawings. Although the price for one or two seems high, they compare favorably with the cost of equipment for the science laboratory. In addition, the rapid drop in cost as additional sets are ordered, for example, orders for three to nine units ofthe parallel lines device cost $4.00 each, should stimulate large orders so that they will be available for student manipulation. To be effective they should be used by the student to discover or verify relationships. From such an investigation only tentative statements of probable relationships can be made. After the theorems have been established, the devices provide concrete representations of them for review and emphasis.

E. 42—Quad

The Gangler-Gentry Co., Catonsville 28, Maryland

Game; $3'' \times 5'' \times 5''$; $1.00 each, $6.90 per dozen.

Description: This game consists of a plastic base and stem arising from it; on the stem are three square platforms marked into sixteen equal squares on each platform. The base is similarly marked. A cardboard box set into the base holds eighteen markers of each of three colors. The game can be played by two, three, four or six people. The side wins which can get four in a straight line first. There are seventy-six ways that this can be accomplished.

Appraisal: There is some amount of space-perception involved when a line of four counters, all on different platforms, is being arranged. This would serve as a bit of three-dimensional thinking in a plane geometry course, or as an introduction to lines in space at the beginning of solid geometry. Once the game is understood it might be well to replace it by a set of four tic-tac-toe blocks drawn side by side on a piece of paper or the blackboard. Each square would contain sixteen small squares and, by imagining them piled on top of each other, one can play

a similar game involving even more space-perception.

This game is well made and will prove of fair durability. Of course, the small markers will be lost, but can be replaced. All thoughts of teaching mathematics aside, this is an interesting and exciting game.

MODELS

M. 23—Mek-N-Ettes

The Judy Company, 310 North Second Street, Minneapolis, Minn.

Toy; Mounting board, plastic gears, bars, discs, etc; $4.50 to schools.

Description: This educational toy has the necessary gears, bars, and pins to show many modifications of motions. Two gear sizes, 1-inch and 3-inch, can be used to build gear ratios up to 1 to 243. Various gear trains can be built to show changes in speed as well as direction of motion. Elliptical gears and a straight toothed rack, as well as cranks and levers can be used to illustrate other methods of mechanical power transmission. The slotted bars can be used as connecting rods or lever arms to set up linkages which convert rotating motion to motion in a straight line. A manual of instruction gives directions and photographs of several simple mechanisms.

Appraisal: This mechanical toy will be useful for the science and mathematics teacher to show different types of machines and the formulas that apply. It will give the teacher simple, inexpensive apparatus for laboratory work in which the student can discover relationships and formulas. These principles can then be applied to show the development and operation of a great variety of every day machines. The board on which the devices are built is used like a graph in that the positions for attachment are located by coordinates. The most appropriate grade level for its use is probably junior high school, although it can frequently be used to illustrate more advanced topics such as vectors.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

A SIMPLE TRISECTION DEVICE

The trisection device illustrated in Fig. 1 is an interesting application of several important geometric theorems.[1] One of these is that arcs of a circle cut off by parallel lines are equal. Proof of the device for a problem in application will show how this theorem is employed.

A workable device may be built with four narrow strips of either wood or cardboard. For a blackboard demonstration device the dimensions of these strips should be as follows: AE and DB, each 12" long; CS and CR, each 18" long. To assemble the device find the midpoints of AE and DB, and locate A and B on CR and CS respectively so that $CA = CB = 6"$. It can readily be seen that quadrilateral $ACBO$ is a rhombus with sides of 6". If wood is used for the strips, the movable joints A, B, C, and O may be secured with small brads. If cardboard strips are used, holes should be punched at the movable

[1] Plans for construction of the trisection device described in this article were developed in the Mathematics Laboratory, Monroe High School, St. Paul, Minnesota. It is a modification of a similar device which appears in the *Eighteenth Yearbook* of the National Council of Teachers of Mathematics: Yates, Robert C., "Trisection," p. 151.

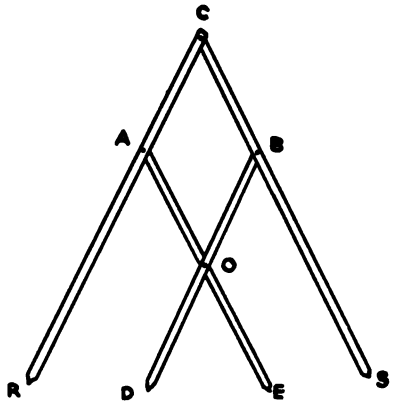


FIG. 1

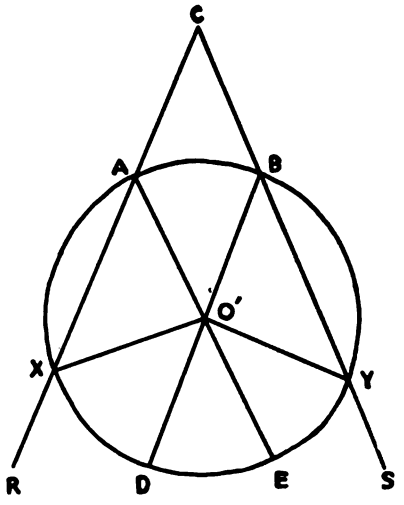


FIG. 2

joints, and paper rivets inserted.

To trisect any angle $XO'Y$ with this device, draw a circle of radius 6″ with its center at vertex O'. (See Fig. 2.) Sides $O'X$ and $O'Y$ should be extended until they intersect the circle. Place the device on the angle so that O falls on O', CR passes through X and CS through Y. Since the circle O' is a twelve-inch circle by construction, points A,B,D, and E all lie on the circle. The following proof shows that trisection of angle $XO'Y$ is immediate.

Proof

(1) Angle $AO'B$ = Angle $DO'E$.	(1) Vertical angles are equal.
(2) Arc AB = Arc DE.	(2) Equal central angles have equal arcs.
(3) Arc AB = Arc XD.	(3) Arcs of a circle cut off by parallel lines are equal.
(4) Arc AB = Arc EY.	(4) Reason (3).
(5) Angle $XO'D$ = Angle $DO'E$ = Angle $EO'Y$.	(5) Equal arcs have equal central angles.

A MULTIPLE PURPOSE DEVICE

The device to be described in the following paragraphs is a favorite of this contributor because of its simplicity and usefulness. It may be used to help clarify the differences between the medians and altitudes of a triangle, to illustrate several types of loci, to show how area and perimeter are related, and to demonstrate facts about parallel lines.

The materials needed for construction include four narrow extensible curtain rods, four large letters, four metal paper *clamps,* one screw eye, a quantity of elastic, colored cord with alternate half-inch bands of red and white, and two wooden uprights $\frac{3}{8}'' \times 1'' \times 4'$ (or long enough to fit firmly inside the frame of the blackboard).

To assemble the device fasten the curtain rods R_1, R_2, R_3, and R_4 to the uprights in such a way that all of them will be parallel to the base of the blackboard. The distances between them should be in the ratio 2:3:3. (See Fig. 3.) Next glue the letters A, B, C, and D to the four metal clamps, and clamp those bearing letters A, D, and C to rod R_3 so that AD will be equal to DC. Finally place the clamp bearing the letter B on rod R_2.

By running an elastic cord through the holes in the handles of the clamps, a triangle may be formed with the vertex angle at B. By joining B and D with an elastic cord, a median of the triangle is outlined. If the screw eye is suspended from B by means of colored cord then it can be made to serve the purpose of a plumb bob and the cord itself will outline an altitude of the triangle.

The difference between a median and an altitude of a triangle drawn from the same vertex may be demonstrated by sliding the vertex clamp along rod R_3

from one end to the other. As the triangle approaches the isosceles in shape, the altitude approaches the median. By starting with the isosceles triangle one can show the altitude leaving the median and swinging farther and farther away until it is entirely outside the triangle.

The difference between the median to side AC and the bisector of angle B can be clarified by sliding the vertex of the triangle to an extreme position.

The relationship between the altitude and the base of the different kinds of triangles can also be demonstrated with this device. If it seems desirable the median may be removed for this exercise. In the isosceles triangle the altitude from the vertex B bisects the base; in the general acute triangle it falls inside the triangle; in the right triangle it coincides with one of the legs, and in the obtuse triangle it falls outside the base. The class can see all the possible situations in rapid succession simply by sliding the vertex clamp along R_2 and allowing the triangle to change shape as desired.

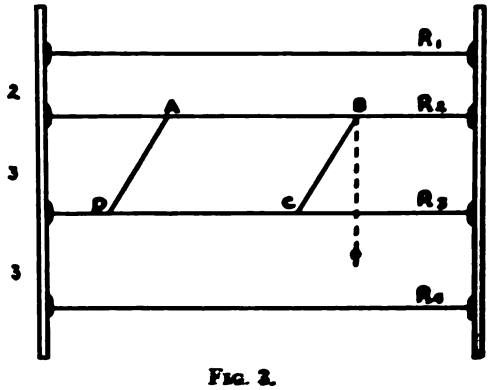


FIG. 3.

The contrast between the perimeter and area of a triangle can be illustrated rather nicely by moving the vertex clamp from the left until an isosceles triangle is formed, and then on to the extreme right. The area remains the same for all positions of the vertex on R_2 but the perimeter changes perceptibly.

Remove the elastic joining D and B, and place clamp A on rod R_2. Then connect D and A with elastic; the resulting figure is a trapezoid (see Fig. 4). By spacing the clamps so that $AB = DC$ a parallelogram is formed. The shape of this parallelogram may easily be changed by altering the positions of A and B. Thus it is possible to show that the altitude from B to the base DC may fall inside, on, or outside the figure. Following are some of the theorems that can be illustrated with the "new" device.

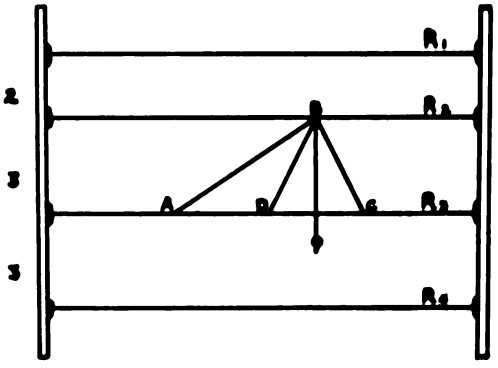


FIG. 4.

(1) The area of a rectangle is equal to the product of its dimensions.
(2) The area of a parallelogram is equal to the product of its base and altitude.
(3) The areas of all parallelograms with equal bases and altitudes are equal.

In connection with the third theorem it should be noted that the device is helpful in showing that the area of a parallelogram does not depend on its perimeter, and also that of all equal parallelograms with equal bases the rectangle has the least perimeter.

The device can also be used to demonstrate several important locus theorems:

(1) The locus of vertices of all equal triangles with a common base is two parallel lines, each parallel to the base, with one on either side of it, and at the distance of the altitude from it. (Use R_2 as the base.)
(2) The locus of points at a given distance from a given line is two lines parallel to the given line, one on either side of it, and at the given distance from the line. (Use R_2 as the given line.)
(3) The locus of points equidistant from two parallel lines is a line parallel to the two given lines and midway between them.

Remove the clamps A, B, C, and D, and four parallel bars remain. Clamp a tailor's tape measure to R_2 and R_4. The result is a transversal which is divided

two equal parts. Move the clamps to different positions on the same two lines and it can be seen that every transversal formed by the tape is divided into two equal parts. If the tape is clamped between R_1 and R_4, it is divided into segments whose ratio is 2:3:3

Louise B. Eddy (Mrs. John M.)
Chicago, Illinois

Fact Finders

The "fact finders" suggested in this article are intended for use with children of the primary grades. The two devices may be used simultaneously. The first—the simpler of the two—is a learning aid; the second is a demonstration teaching aid. Each pupil in the class should be supplied with one of the first type, but only the teacher needs to have a demonstration device. The combined use of the two devices should make possible a more meaningful presentation of the counting process and the number combinations.

For first grade pupils the learning aid consists of ten colored beads strung on a ten-inch wire which has a loop at each end. (See Fig. 5.) Coat hangers may be used as a source of supply for the wire.

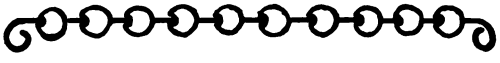


Fig. 5.

The demonstration device to be used with first graders is about 20 inches long, and the wire is mounted on a wooden frame with a shield at one end. The purpose of the shield is to hide beads that are not in use. (See Fig. 6.) When using the learning aid, the pupil should hold the beads that he is not using in one hand and manipulate the group that he is using with his other hand. If the child is permitted to keep a fact finder at his desk, he will have it available for handy reference with number stories and practice work.

The following outline is an illustration of how fact finders may be used in developing the addition fact that three and four are seven:

(1) Describe the necessary manipulations with the aid of the demonstration device.
(2) Ask the child to reproduce the set-up of the demonstration device on his own fact finder.
(3) Record the fact on the board in both of the following ways:
  (a) "Three beads and four beads are seven beads."
  (b) "Three and four are seven."
(4) Write the fact on the board vertically and horizontally in terms of the number symbols.
  (a)
$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$$
  (b) $3+4=7$.

Fact finders for use with second grade pupils should have 20 beads so that all combinations may be taught. The wooden frame for the demonstration device should be about 30 inches long in this case with the shield covering about half of the total length.

By using fact finders, pupils will be able to learn that addition is used when groups of beads are combined and that subtraction is used when groups are separated or compared. The devices can also be used to introduce pupils to the concept that combining equal groups of beads is a process of multiplication, while that of separating large groups into smaller groups is division.

Mrs. Edna Swan
Elementary Teacher
Parramore Tuberculosis Sanatorium
Lake County, Indiana

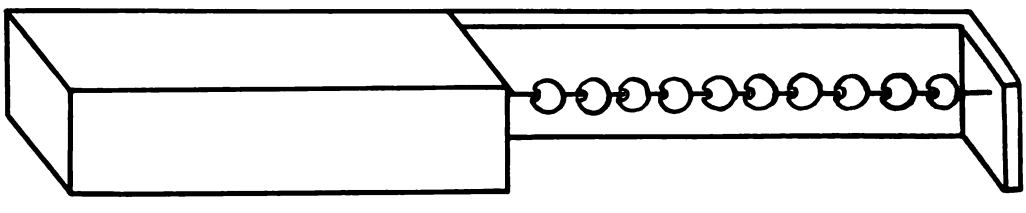


Fig. 6

APPLICATIONS

Edited by SHELDON S. MYERS
University School, Ohio State University, Columbus, Ohio

WITH the last issue of the year, the summer vacation stretches ahead. We hope you will find time in the summer months to send us fresh ideas and examples for this department.

Ar. 8 Gr. 7–9 A PUBLIC OPINION POLL

When I asked my eighth grade mathematics class recently if they would like to take an opinion poll in their grade, they responded enthusiastically in the affirmative. The class spent about thirty minutes composing the questions which they wished to ask. They found that it was not easy to compose a question so that the replies would have clear meaning. Each word and phrase had to be carefully examined for ambiguity.

Finally they were satisfied with the following list of questions:

1. Should eighth graders date each other for school affiairs?
2. Should girls be permitted to wear blue jeans to school?
3. What time should eighth graders go to bed at night?
4. How many hours sleep a night should an eighth grader get?
5. What should be the weekly allowance of eighth graders?
6. Should eighth graders perform home tasks in order to earn their allowance?
7. Who do you think will be the next football coach at Ohio State University?
8. If you could have three wishes granted, what would you wish for?

After each question on the dittoed sheet the type of answer was indicated, such as, "yes," "no," "no opinion," a numerical reply, a name, or a sentence. When the raw data were presented to the class, a discussion arose as to how to treat the data in order to make it say more. It was decided to compute percentages on the yes-no type of answers and to make frequency tables on questions 3,4 and 5. Since each sheet was anonymous but marked boy or girl, the yes-no answers had three sets of data—boys, girls, and total. Quite a debate arose as to whether to compute the per cent of boys who replied "yes" to an item by dividing the number of boys into the number who said "yes" or by dividing the total in the class into the number of boys who replied "yes." Of course, there was no "right" way. After they had made the distinction in meaning between these two per cents, one-half decided to compute it one way and the other half the other way.

While they were working at their seats on these computations, many of them made some startling discoveries. For instance, when the same total group was being divided into two numbers, if one of the numbers was an even multiple of the other, the corresponding percentages had the same relationship. When all but one of a set of percentages had been computed, this one could be found by adding the others and subtracting from 100. Some students converted all of their division problems into multiplication problems as follows: there were 32 in the class; 1/32 was changed to a decimal which was then multiplied by each of the subgroups. All of these discoveries were eventually brought before the class and shared.

About a week later I learned that six allowances had been increased when the respective parents learned that their child's allowance was below the average of the class. During the class discussion on the data, the point was made that the size of an allowance did not mean much unless it were known whether such expenses as carfare and meals had to be paid out of it.

The latter point is an example of the type of interpretive thinking which the opinion poll fostered. Besides providing many valuable arithmetic insights, the opinion poll also stimulated critical thinking, provided strong motivation, and served a guidance function.

AL. 9 GR. 9–10 A FREQUENT PROBLEM INVOLVING POINT-HOUR RATIOS

Many 11th grade, 12th grade, and university advisers tell me that students often wish to know with what point-hour ratio they must finish a given block of work in order to have an over-all point-hour ratio above a stated minimum. For those of you who are unfamiliar with the method of computing a point-hour ratio, an illustration will be provided. A boy earns A's in two 3-credit hour courses; a B in a 5-credit hour course; and C's in three 2-credit hour courses.

Credit Hrs.		*No. of Courses*		*Total Hrs.*
3	×	2	=	6
5	×	1	=	5
2	×	3	=	6
		0		0
		0		0
				17

	Grade Points		*Total Points*
×	4 (A)	=	24
×	3 (B)	=	15
×	2 (C)	=	12
	1 (D)		
	0 (E)		
			51

51 ÷ 17 = 3.0 point-hour ratio.

Here is a sample problem involving point-hour ratios which algebra handles nicely.

For 178 hours a student has maintained a point-hour of 2.6. He wishes to know what average he must maintain for his last 18 hours in order to have the over-all average of 2.5 required for graduation.

Notice that the following equation involved in the solution has about the right degree of complexity for a verbal problem in ninth grade algebra.

$$18y+(178)(2.6)=2.5(178+18)$$

$$18y+462.8=490$$

$$18y=28.2$$

$y=1.56$ point-hour ratio required.

AR. 9 GR. 7–10 ARITHMETIC IN TRACK EVENTS

The May issue is a good time to run the following anonymous guidesheet used at the University School on the spring sport of track.

1. In a mile relay, the first boy ran his quarter in 53½ seconds, the second in 52½ seconds, the third in 54½ seconds, and the last boy in 50½ seconds. How long did it take the team to run the race?
2. The American record for the mile run in 1929 was 4 minutes 12 seconds. The record for skating a mile was 2 minutes 45 seconds. How much shorter than the mile run time was this? The record time for swimming a mile was 21 minutes 46 seconds. How much longer than the mile run was this?
3. Sixteen times around he running track at Benton School make a mile. How many yards long is the running track?
4. Harry can run the 440 in 1 minute 10 seconds. How long would it take to run the mile at this rate?
5. In a contest Ray jumped 5′ 9″ while Joe jumped 6′ 2″. How many inches longer was Joe's jump than Ray's?
6. Jack won the 440 yard dash at the meet. What fraction of a mile is 440 yards?
7. In the Olympic games a man vaulted 3.9 meters. How many feet and inches was this?
8. How much difference is there between the 400 meter run and the quarter mile run? How many yards are covered in the 100 meter and the 800 meter events?
9. Which is the longer run and how much: 13 times around a quarter mile track or 7 times around a half mile track?
10. In a hop, skip, and jump event at a field meet, Donald hopped 3.1 feet, skipped 7.8 feet, and jumped 9.5 feet. What was Donald's total distance in this event?
11. In the 1936 Olympics, the 10,000-meter speed skating contest was won by Ballagrud of Norway in 17 minutes 24.3 seconds. The American record for six miles at that time was 18 minutes 38 seconds held by Johnson of Montreal. How much faster per mile was the Olympic record than the American record?
12. Bob ran 220 yards in 27.5 seconds. What was his average speed in yards per second and in miles per hour?

S. G. 2 GR. 11–14 SIMILARITY OF FORM OF EULER'S FORMULA AND GIBBS' PHASE RULE

There are many times in mathematics and science when the mind pauses in silent wonder. Duality, the principle of

isomorphism between logical calculi and their interpretations in mathematics and science, the role of the calculus in mechanics, the Euler Line and the Nine-Point or Feuerbach Circle, the fixed-point theorem in continuous geometric transformations without translation—these are some of the things which generate simultaneously a feeling of awe, admiration and humility. One of these things which is not generally known or publicized is the similarity between Euler's Formula and Gibbs' Phase Rule. Here are the two formulas:

Euler: $V=E-S+2$
Where $V=$ no. of vertices
$E=$ no. of edges
$S=$ no. of surfaces of any polyhedron.

Gibbs: $F=C-P+2$
Where $F=$ no. of degrees of freedom
$C=$ no. of components
$P=$ no. of phases of any system of equilibrium.

The Phase Rule is one of the great generalizations of modern science and represents the foundation and corner stone of thermodynamics and such industrial fields as metallurgy, oil industry, and synthetic organic chemistry. The Phase Rule deals with such systems of equilibrium as water in a tight container with vapor above it in equilibrium with the liquid. Degrees of freedom refer to such variables as temperature, volume, and pressure. Components refer to the number of distinct chemical substances present, such as water. Sugar dissolved in the water would represent a second component. Phases refer to the number of states of matter present in equilibrium, such as gas, liquid and solid. These three are the maximum number of states possible. Sometimes a substance can have several solid phases such as sulfur, which has two crystalline and one amorphous solid phases.

The Phase Rule can be used to predict the number of degrees possible in any given system of equilibrium. For example, if water and water vapor were present in equilibrium, $C=1$, $P=2$. Therefore:

$$F=1-2+2=1\cdot$$

This means that temperature, or pressure, or volume may vary and equilibrium retained, but not more than one of these.

If ice, water, and water vapor were present in equilibrium, $C=1$, $P=3$. Therefore:

$$F=1-3+2=0\cdot$$

This means that ice, water, and vapor can be present in equilibrium only at a fixed temperature and pressure and volume, because the Phase Rule predicts that there are no degrees of freedom in this system of equilibrium. The specific temperature required for the equilibrium of the three phases of water is known as the "triple point" for water.

When the two axes of a graph are allowed to represent the two degrees of freedom of a system of equilibrium and the phases in equilibrium mapped out on the coordinate system, a "phase diagram" is created. Phase diagrams for thousands of substances and mixtures have been worked out experimentally. The theory which has been developed about such diagrams is known as the "thermodynamics of plane surfaces." When three degrees of freedom are plotted on three axes, three dimensional phase diagrams are created. While on the plane diagrams the boundaries between phases are represented by curved lines, on three dimensional phase diagrams, the boundaries of phases are represented by surfaces. These surfaces intersect along edges, which come together at vertices. This would indicate that some kind of isomorphism has been established between the V, E, and S in polyhedra and F, C, and P in systems of equilibrium. The relation stated by these two formulas seems to be a topological one.

We would appreciate receiving from the readers any further information about these two formulas and their relationship.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* PAUL M. WEISEL
The University of Wisconsin, Madison, Wisconsin

SECTION A of a study on trends in teaching mathematics was published in the November issue of THE MATHEMATICS TEACHER. A summary of replies to the questions in Section A, which was concerned with mathematics enrollments, was reported in the April number. Because of the small number of replies received the questions of Section A will be repeated in THE MATHEMATICS TEACHER for October.

Section B, General Mathematics, appeared in the February number of THE MATHEMATICS TEACHER.

Questions in Section C are on content and practice in the third year of sequential mathematics. It will be interesting to determine if there is a definite trend to complete the third year in any of the various ways listed: more algebra, solid geometry, or trigonometry. Requests for information on teaching aids has been limited to films and filmstrips in keeping with the policy of this section to be brief and to include only questions on which answers can be easily reported.

It is hoped that this question-answer department will yield information on trends in teaching mathematics which will be useful in the study and research carried on by teachers everywhere. The success of this project depends on the co-operation of readers of THE MATHEMATICS TEACHER. Teachers are invited to submit questions for publication in future issues.

In replying please give your name, the name and locality of your school, and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades 10–12; junior high school, grades 7–12; etc. The questions may be answered by numbers without use of a particular form or repetition of the questions. The names of teachers or schools will not be revealed in any of the reports.

Please send the answers to these questions to John R. Mayor, North Hall, Madison 6, Wisconsin by June 10, 1951.

C. THIRD YEAR MATHEMATICS

1. What courses do you offer for the third year of sequential mathematics?
   - a. A third semester of algebra and a semester of solid geometry _____
   - b. A third semester of algebra and a semester of trigonometry _____
   - c. A second year of algebra _____
   - d. A year of plane geometry _____
   - e. Algebra and another subject in combination different from that in a) or b); if e) is checked, please indicate also _____
     - Number of weeks of algebra _____
     - Number of weeks of solid geometry _____
     - Number of weeks of trigonometry _____
   - f. Other (please describe briefly).
2. Do you have ability grouping in your third year course? _____
3. Do you have a mathematics club:
   - For juniors only? _____
   - For seniors only? _____
   - For both juniors and seniors? _____
4. Do you use films and film strips in the third year of sequential mathematics? _____
   If so, please list the two you have found most helpful.
5. Do your third year mathematics students use the library as part of the suggested study in this mathematics course? _____

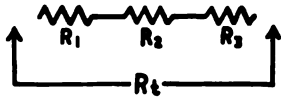
Mathematics Kits

Beginning July 1, 1951, all orders for mathematics kits should be sent to the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C. For the past two years, the kits have been distributed by Mr. M. H. Ahrendt, Anderson College, Anderson, Indiana, and he will continue to receive orders until July 1. After that date, the distribution will be handled by the Washington office of the Council.

Computation Unit No. 123 is proving very popular. Many teachers have ordered this kit in quantities for use in entire classes. Straight Line Unit No. 113 is still available. All kits sell for *50¢ each or 3 for* $1.00. Quantities may be assorted.

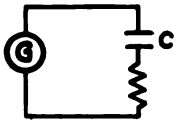
MATH IN COMMUNICATIONS —

ARITHMETIC *Finding the sum of resistances in series*



$$Rt = R_1 + R_2 + R_3$$

ALGEBRA *Finding reactance*



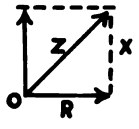
$$X_c = \frac{1}{2\pi fC}$$

TRIGONOMETRY *Finding the phase angle*

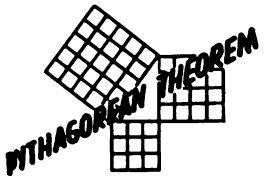
$$\text{TAN } \theta = \frac{X}{R}$$

$$\text{SIN } \theta = \frac{X}{Z}$$

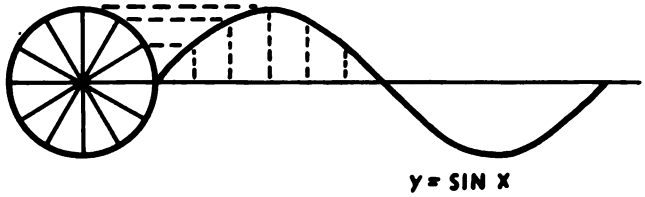
GEOMETRY *Finding impedance*



$$Z = \sqrt{R^2 + X^2}$$



PERIODIC FUNCTIONS *Graphing alternating currents*

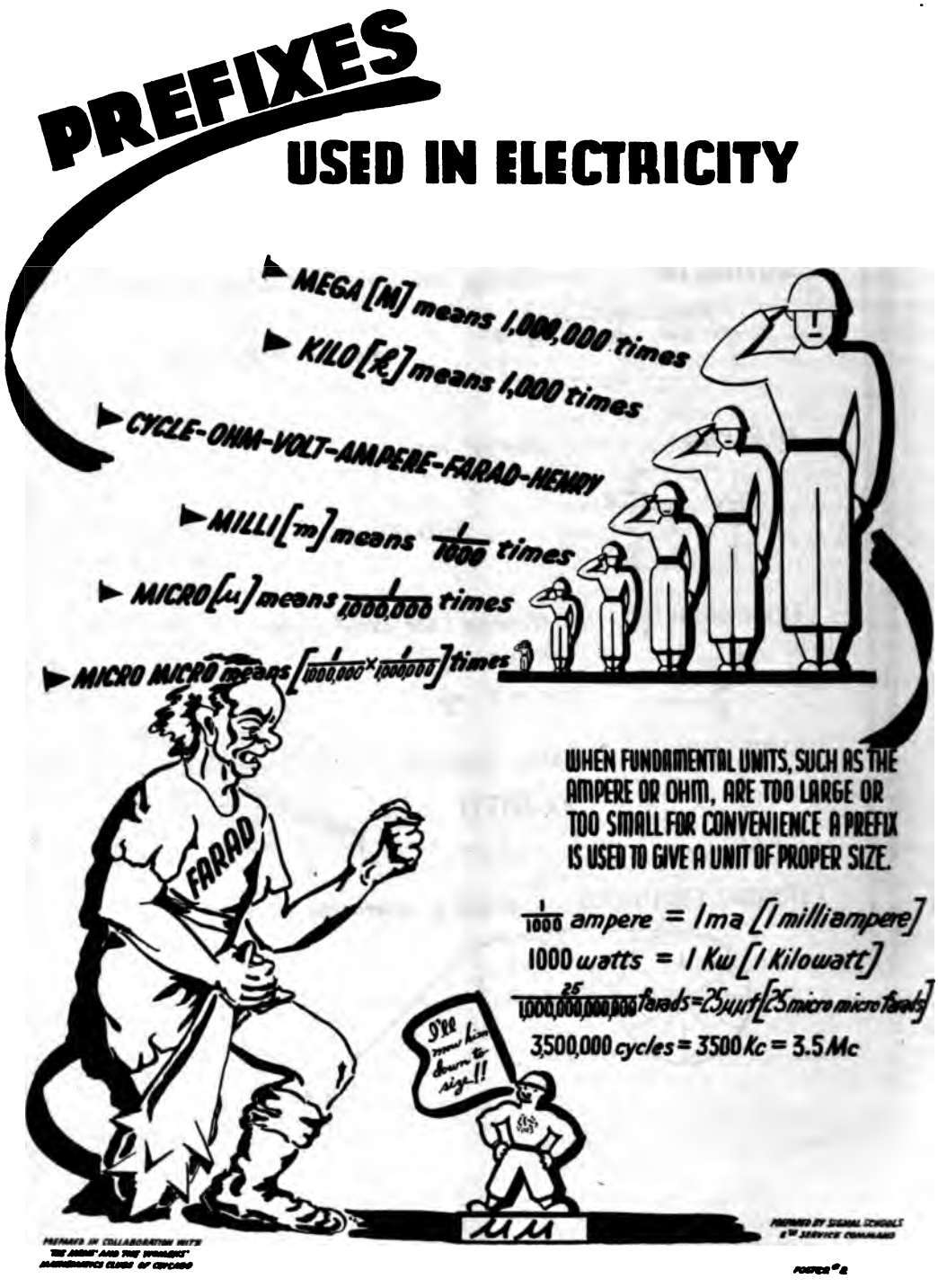


$$y = \sin X$$

PREPARED IN COLLABORATION WITH THE ILLINOIS INSTITUTE OF TECHNOLOGY

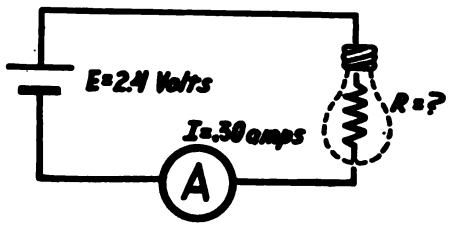
PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND

POSTER NO. 2


PREFIXES
USED IN ELECTRICITY
MEGA [M] means 1,000,000 times
KILO [K] means 1,000 times
CYCLE-OHM-VOLT-AMPERE-FARAD-HENRY
MILLI [m] means 1/1000 times
MICRO [μ] means 1/1,000,000 times
MICRO MICRO means [1/1,000,000 × 1/1,000,000] times
FARAD
I'll reduce him down to size!!
WHEN FUNDAMENTAL UNITS, SUCH AS THE AMPERE OR OHM, ARE TOO LARGE OR TOO SMALL FOR CONVENIENCE A PREFIX IS USED TO GIVE A UNIT OF PROPER SIZE.
1/1000 ampere = 1 ma [1 milliampere]
1000 watts = 1 Kw [1 Kilowatt]
25/1,000,000,000,000 farads = 25 μμf [25 micro micro farads]
3,500,000 cycles = 3500 Kc = 3.5 Mc

ALGEBRA AND ELECTRICITY

ELECTRICAL PROBLEM TO BE SOLVED: *A flashlight bulb draws .30 amperes at 2.4 volts. What is the resistance of the bulb?*



MEANING OF SYMBOLS

E = Voltage in Volts
R = Resistance in Ohms
I = Current in Amperes

PROCEDURE

MATHEMATICS NEEDED

GIVEN: *The equation* $a=\frac{b}{x}$

Where $a = .30$
$b = 2.4$
$x = ?$

APPLICATION TO THE PROBLEM

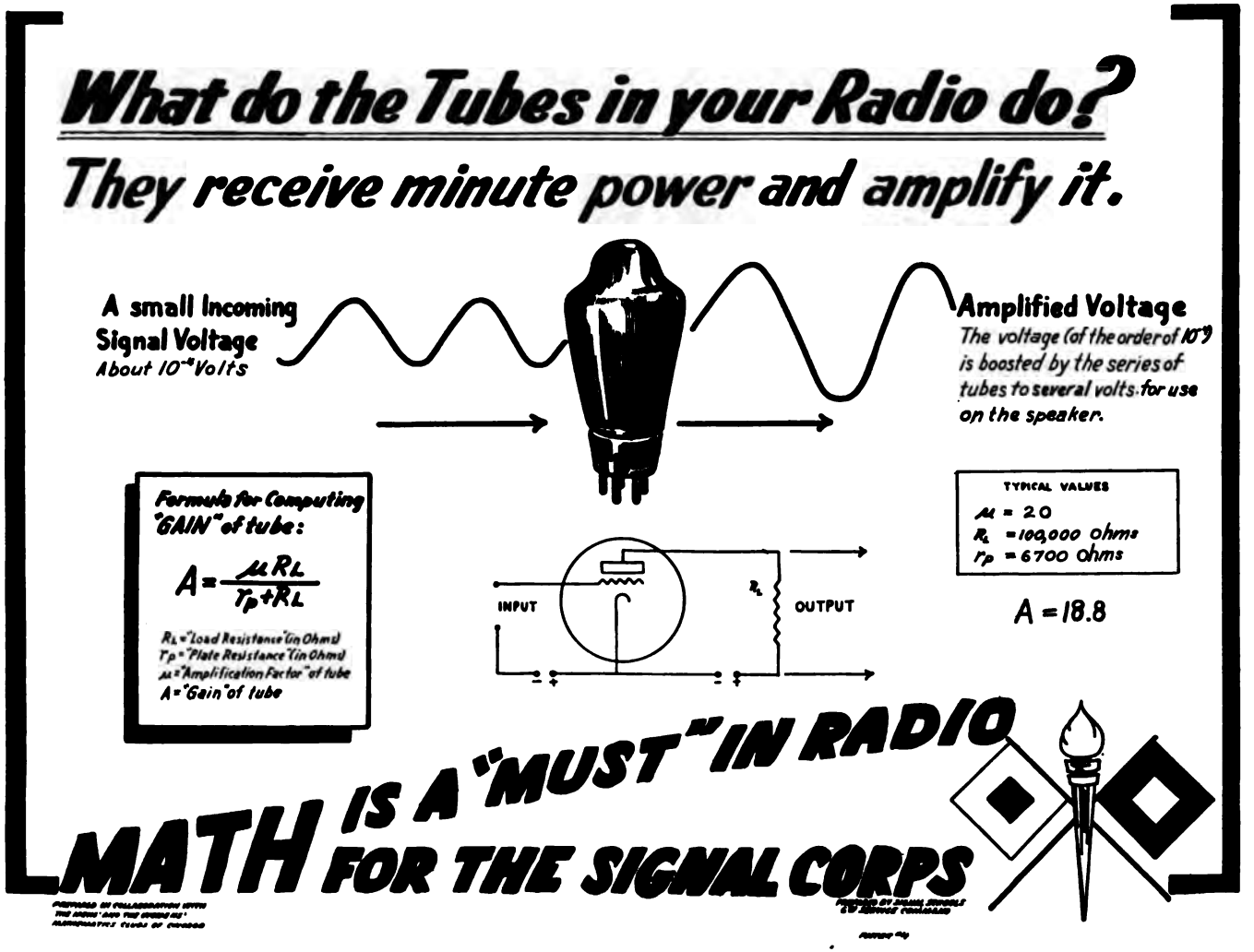
GIVEN: *Ohms law* $I=\frac{E}{R}$

Where $I = .30$ amps
$E = 2.4$ volts
$R = ?$ ohms

PREPARED IN COLLABORATION WITH THE MATHEMATICS CLUBS OF CHICAGO

PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND

POSTER #3


What do the Tubes in your Radio do?
They receive minute power and amplify it.
A small Incoming Signal Voltage
About 10^{-6} Volts
Amplified Voltage
The voltage (of the order of 10^{-6}) is boosted by the series of tubes to several volts for use on the speaker.
Formula for Computing "GAIN" of tube:
$A = \frac{\mu R_L}{r_p + R_L}$
R_L = "Load Resistance" (in Ohms)
r_p = "Plate Resistance" (in Ohms)
μ = "Amplification Factor" of tube
A = "Gain" of tube
INPUT
OUTPUT
R_L
TYPICAL VALUES
μ = 20
R_L = 100,000 ohms
r_p = 6700 ohms
A = 18.8
MATH IS A "MUST" IN RADIO FOR THE SIGNAL CORPS

THE PRESIDENT'S PAGE

WHAT are the recent developments in the mobilization of education for national security? What is the impact of mobilization on instruction in the schools? What are the implications for mathematics?

The National Conference for Mobilization of Education (MOE) was established at a meeting held in Washington on September 9 and 10 by representatives of major national educational organizations who met with representatives of the Armed Services and the Federal Government to review the experience of World War II as it relates to schools and colleges, to study the present aspects of the emergency situation with particular emphasis on the relationship to education, and to develop long-range plans for continuous coordinated educational effort in support of the national security. The purpose of MOE is to further the cooperative efforts of voluntary educational organizations in the mobilization of the nation, each participating organization to retain full freedom of action with respect to its own policies and programs. It is the aim of MOE to strengthen rather than restrict the operations of the participating organizations. Your National Council is one of the more than eighty participating organizations in MOE. Each organization has two representatives. The National Council is being represented at present by Professor Walter H. Carnahan of Purdue University and your president. The MOE is to serve as a cooperative channel of communication between organized education in the United States and the Federal Government on matters pertaining to mobilization of education.

On January 26 and 27 a work conference dealing with the impact of mobilization on instruction in the schools was held in Washington. Your president represented the National Council at this conference in which sixty school and military people participated. The purpose of the conference was to clarify objectives and to help strengthen and improve instructional programs in this mobilization period.

The following recommendation came from the conference of January 26 and 27: "This conference requests the National Conference for Mobilization of Education to recommend strongly that the Office of Defense Mobilization and Office of Defense Manpower in the allocation and assignment of personnel under the defense program give every consideration to the necessity for retaining as far as possible in elementary and secondary schools the teaching personnel necessary for the adequate education of all children and youth." And these relating to mathematics and science: "This conference believes that the programs of mathematics and science in the elementary and secondary schools that contribute best in times of peace likewise contribute best in times of stress and conflict. Therefore we urge, (1) that all children and youth be offered the opportunity and be encouraged to develop competency and power in science and mathematics to the extent of the individual's ability, (2) that reemphasis be placed on basic skills, meanings, and understandings, and (3) that specific preinduction courses in mathematics and science *not* be introduced until their value has been determined by the cooperating organizations of MOE and the Federal Agencies working through the U. S. Office of Education."

There is no call for schools to set aside any part of a sound curricula or school program; but it is urgent that present programs and courses be re-examined to check on permanent values. At present, the military people are not asking for specialized or pre-induction courses in the high school. They do advise, however, that children and youth get a sound and substantial training in basic understandings and fundamental skills.

H. W. CHARLESWORTH,
President

Organization Patterns of Affiliated Groups

JOHN R. MAYOR,
Chairman, Committee on Affiliated Groups

LEADERS of organizations of mathematics teachers considering affiliation with the National Council ask many questions about the nature of the organization of Groups already affiliated. The Committee has now on file constitutions of the various Affiliated Groups and these are made available to any Groups, affiliated or unaffiliated, which are writing their first constitutions or considering revisions. The constitutions are the best source of information on the nature of the various organizations and are the source of most of the information in this report.

REGIONAL ORGANIZATION

The regional bases of organization of the 43 Groups which had completed affiliation by January 1, 1951, are:*

City—11
[Chicago (Men, Women, Elementary), Cleveland, Dallas, Detroit, New York, Oklahoma City, Richmond, Tulsa, Wichita]
County—4
[Dade and Hillsborough, Florida; Nassau and Suffolk, New York]
Sectional within a state—4
[Eastern Colorado, Southern Oregon, Western Pennsylvania, Eastern Tennessee]
State and Province—22
[Alabama, Arizona, Arkansas, California, Colorado, Florida, Georgia, Illinois, Indiana, Iowa, Kansas, Kentucky, Maryland, Minnesota, Nebraska, New Jersey, Oklahoma, Ontario, Texas, Virginia, West Virginia, Wisconsin]
Sectional including more than one state—2
[Louisiana-Mississippi, New England]

Of the eleven city organizations eight are in states which also have a state-wide affiliated organization. The remaining three are in Cleveland, Detroit, and New York. These cities are in states where a state-wide organization, not yet affiliated, has either recently been formed or is in process of formation.

Two of the county groups are in Florida which has a state-wide Affiliated Group while the other two are in New York, where much progress is being made on a state-wide organization. It should also be pointed out that a third Florida county has previously been affiliated and probably will have renewed its affiliation by the time this report is published.

The four sectional organizations within a state are in Eastern Colorado, Southern Oregon, Western Pennsylvania, and Eastern Tennessee. Colorado also has a state-wide affiliated organization and Pennsylvania probably will have by the time of the Pittsburgh meeting. A new organization in Western Tennessee has taken the first steps to complete affiliation.

States with more than one Affiliated Group are:

	No. of Groups
Colorado	2
Florida	3
Illinois	4
Kansas	2
New York	3
Oklahoma	3
Texas	2
Virginia	2

Of the 22 state-wide Affiliated Groups, including Ontario, 14 are in states which do not also have another Affiliated Group within the state. In the March issue of THE MATHEMATICS TEACHER, a request was made for assistance in interesting the remaining states in affiliation. These states, 13 in number, are Delaware, Idaho, Missouri, Montana, Nevada, New Mexico, North Carolina, North Dakota, South Carolina, South Dakota, Utah, Washington, and Wyoming. Interest in affiliation on the part of mathematics teachers in two of these states has already been reported in this section in recent

* NOTE: The official list for the Second Delegate Assembly included 50 Groups. Seven Groups completed affiliation between January 1 and March 29, 1951. These Groups are:
City—Philadelphia, Washington, D. C. (Benjamin Banneker Club)
County—Pinellas, Florida
State—Pennsylvania, South Carolina, South Dakota, Utah.

issues and this spring correspondence with two others has been encouraging.

Two of the most successful of all Affiliated Groups have been the Association of Teachers of Mathematics in New England and the Louisiana-Mississippi Branch of the National Council. In these areas teachers have found advantages in a larger organization which crosses state lines.

A question often discussed is whether it might be better for the National Council to recognize only state-wide groups or groups made up of several states as Affiliated Groups and then to encourage these organizations to recognize in a similar manner city, county, and regional groups. It seems doubtful that a required pattern of this type will ever be desirable. There are such great variations in distribution of population among the states that what works most efficiently in one area cannot be expected necessarily to be best for another. Those responsible for the organization of any new group should give careful consideration to the various possibilities, and should also, if possible, study constitutions of and communicate directly with Affiliated Groups with organizations of all of the types under consideration.

This study has revealed even more clearly than was previously realized that strong city and county organizations have had a great deal to do with the establishment of state organizations and are continuing to contribute much to their support. These city and county organizations are likewise continuing to contribute directly to and to gain directly from the National Council through their affiliation.

Relationships With Other Associations

Many of the Affiliated Groups are associated with their state education association in a relationship similar to that they bear to the National Council. Among these are the state groups in:

Alabama	New Jersey
Arkansas	Pennsylvania
Colorado	Tennessee
Florida	Texas
Georgia	Virginia
Iowa	West Virginia
Maryland	Wisconsin

In most instances these Affiliated Groups are either the mathematics sections of the state education association, or organizations which have grown from these sections. An opportunity for important and fruitful endeavor is certainly provided by such a relationship. This was well brought out in the panel discussion on Affiliated Groups at the Florida meeting of the National Council.

In a number of instances in which Affiliated Groups are also associated with their state education association this relationship has provided the basis for the state organization to maintain special relationships with the mathematics sections of the divisional education associations within the state.

Two of the regular meetings of the Association of Mathematics Teachers of New Jersey are held in co-operation with larger state educational associations. In November the Association meets at Atlantic City as an Affiliated Group of the New Jersey Education Association and in May, at Rutgers University, New Brunswick, in connection with the New Jersey Secondary School Teachers Association. New Jersey is one of the first of the state Groups to include chairmen of sectional groups on its Executive Board.

In the very comprehensive annual report of the New Jersey Association, prepared by Mary C. Rogers, one of the factors listed as contributing to the excellent growth of the New Jersey Group was the co-operation of the State Department of Education with the Association. This co-operation has been shown both in special studies and problems of educational research and in matters pertaining to membership growth, such as supplying periodically lists of teachers of mathematics within the state.

The Indiana Council of Teachers of Mathematics recognizes five regional groups and local affiliated mathematics clubs as important components of the state-wide organization. A diagram showing the relationships within the Indiana organization was included in the second issue of the Newsletter on Affiliated Groups.

The California Council, in adopting a new constitution two years ago, was particularly interested in developing a "unified state organization" and in determining that "local initiative and activity shall be stimulated." The Council realized the importance of local interests and the need for the discovery and development of leadership ability. In the new organization, functioning in close harmony with the president, vice-president-secretary, and treasurer of the State Council are six additional State Executive Board members, who are the officers of the Northern and Southern Sections of the state. The directors of the Northern and Southern Sections are aided by a secretary and treasurer for each Section and the chairmen of various "sub-groups" that are organized in "naturally related areas." These chairmen, with three sectional officers, form the board of directors for each section. It follows quite naturally that one of the more important functions of this board is to exchange ideas on programs and activities for all sections and sub-groups.

The constitution of the Wisconsin Mathematics Council provides for an executive committee to be composed of the officers of the Council, the presidents of various state divisional education associations, presidents of any city or area mathematics clubs with at least 35 members, the past president of the Wisconsin Mathematics Council, and the state representative of The National Council of Teachers of Mathematics.

Many other groups have planned programs intended to promote more and better cooperation with local organizations. The Dade County (Florida) Council of Teachers of Mathematics sponsors a curriculum committee which coordinates its work with the various curriculum agencies of Dade County and with the Florida State Department of Education in discussing and formulating curriculum policy.

The Association of Teachers of Mathematics in New England has provided in its constitution for the "organization of sections on the basis of geographical location of members of the association," these sections being authorized to hold meetings at their own discretion.

The Kentucky Council of Mathematics Teachers keeps in contact with other interested persons by sending out notices of meetings to districts where mathematics teachers are not organized.

The Iowa Association of Mathematics Teachers maintains an advisory board which considers policies proposed for adoption by other groups interested in mathematics and assists and advises chairmen of the district mathematics sections in planning programs for district meetings. The annual meeting of the Iowa Association is held in connection with the Iowa State Education Association.

While the general pattern of organization is for Affiliated Groups to include among their members teachers of mathematics from all levels of instruction, there are two Groups which are concerned primarily with instruction in mathematics at the elementary school level. These are the Chicago Elementary Teachers Mathematics Club and the Dallas Elementary Mathematics Association. Because we often fail in mathematics organizations to offer as much to teachers at the elementary level as we are able to give at the secondary school level, these Groups should have a special welcome. The state-wide Affiliated Groups in Illinois and Texas are able to operate more effectively at the elementary school level because of strong, active Groups for elementary teachers within their area of responsibility.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Membership Record

MARY C. ROGERS, *Roosevelt Junior High School, Westfield, N. J.*

100% Schools—as of March 20, 1951

1. Tampa, Florida ..... Brandon High School
2. South Bend, Indiana ..... James Madison Junior High School
3. Elizabethtown, Pennsylvania ..... Elizabethtown College
4. Gray Court, South Carolina ..... Hickory Tavern High School
5. Richmond, Virginia ..... John Marshall High School
6. Washington, D. C. ..... McKinley High School

"All but One" Schools—as of March 20, 1951

1. Penns Grove, New Jersey ..... Regional High School
2. Washington, D. C. ..... Phelps Vocational High School
3. Washington, D. C. ..... Taft Junior High School

We have prepared for you a few interesting membership data in addition to the listing of the Membership Honor Schools. Much more information of this sort is available and may be released to you from time to time. Our present *membership total* is *8479*.

TEN STATES LEADING IN MEMBERSHIP TOTALS

1. Illinois	710	6. California	348
2. New York	671	7. Indiana	324
3. Pennsylvania	550	8. New Jersey	321
4. Texas	387	9. Michigan	275
5. Ohio	364	10. Massachusetts	272

STATES SHOWING CONTINUOUS GROWTH

1947–1951

Delaware	Indiana	Tennessee
Florida	Maryland	Texas
Georgia	Pennsylvania	Virginia

STATES WHOSE MEMBERSHIP HAS MORE THAN DOUBLED

1947–1951

Arizona	Idaho	Nevada
Delaware	Indiana	New Mexico
Florida	Louisiana	Utah
Georgia	Mississippi	Virginia

STATES WITH 50%-100% INCREASE IN MEMBERSHIP

Maryland	Oklahoma	Texas
New Hampshire	Tennessee	West Virginia

Our congratulations to each of you for this excellent record. Please accept our sincere thanks for your outstanding support. We are deeply grateful for the enthusiastic cooperation which you continuously give us in the many professional services which we are undertaking together.

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KNOW YOUR NATIONAL COUNCIL REPRESENTATIVE

By KENNETH E. BROWN
University of Tennessee, Knoxville, Tennessee

MISS MAUDE C. HOLDEN

NEBRASKA REPRESENTATIVE

MISS MAUDE C. HOLDEN, a high school teacher at Ord, Nebraska, has recently assumed the responsibilities of National Council Representative for Nebraska. Miss Holden is a Past President of the Nebraska Section of the National Council Council of Teachers of Mathematics and she gave favorable reports of the activities of the Nebraska Section at the National Council meetings at Denver, Colorado, and Madison, Wisconsin. She is looking for energetic key members to help plan an intensive campaign in Nebraska for new National Council members.

SOUTHERN CALIFORNIA REPRESENTATIVE

DALE CARPENTER, the National Council Representative in Southern California, is also the Mathematics Supervisor of the junior and senior high schools in Los Angeles. His experience in teaching for twelve years in the junior high school in addition to a similar period as a supervisor of high school mathematics has given him an invaluable background for his present position. Mr. Carpenter has had a special interest in improving the mathematics for the non-college preparatory student. This interest has resulted in the basic mathematics courses in the Los Angeles high schools for those students not preparing for college. His enthusiasm for better mathematics teaching has contributed greatly to the steady increase in the National Council membership in California.

DALE CARPENTER

Texas Representative

Miss Mary Ruth Cook, as a member of the faculty of North Texas State College, conducts the courses in the teaching of mathematics and supervises the student teachers in the Laboratory School of the College. Her interest in professional improvement is indicated by the fact that she is a member of the National Education Association, National Association of Student Teaching, College Classroom Teachers Association, Kappa Delta Pi, Delta Kappa Gamma, Delta Psi Kappa, a life member of the Texas State Teachers Association and the Representative of the National Council since 1945.

Since Miss Cook has been the National Council Representative, the membership has increased from 200 to approximately 400. For this increase in membership she gives the credit to her twelve Regional Representatives. However it has required considerable effort to secure the help of these energetic key teachers and personally send out hundreds of invitations to join the National Council. According to Miss Cook, a sample copy of The Mathematics Teacher is one of the best means of publicity for the National Council of Teachers of Mathematics. In a recent letter she states, "I am particularly pleased with the issues of The Mathematics Teacher to date this year."

Miss Mary Ruth Cook

Miss Eva Burkhalter

Oregon Representative

Miss Eva Burkhalter is not only a native of Oregon but, with the exception of advanced graduate work at the University of Washington, her academic education was received in Oregon. Miss Burkhalter has devoted her professional career to the improvement of the teaching of mathematics on the junior and senior high school level. During her ten years at Klamath Union High School, she has done much to realize this goal.

Miss Burkhalter's interest in the National Council began when her teacher gave her a copy of The Mathematics Teacher. Since that day, she has read all the copies of The Mathematics Teacher from Volume 1, Number 1, to the present issue. Also she states that the Yearbooks have been most helpful. Miss Burkhalter has been the Oregon Representative for two years and she is seeking key teachers who will help her in the National Council.

MATHEMATICS FOR NATIONAL SECURITY

THE events of the past few months have reminded us of similar ones which took place in the late thirties and the early forties. During that period it was necessary to enlist the cooperation of mathematics teachers in every part of the country. Today, the need for such assistance is even greater.

Special emphasis has been placed by those concerned with our country's semi-mobilization needs on securing personnel with an amount of training far exceeding that ever required before in our history. Since much of this training parallels that which was found necessary in the period referred to above, we wish to call attention to a few of the important recommendations made by committees and members of the National Council in earlier issues of THE MATHEMATICS TEACHER. These statements and many others are among those which call for our most careful consideration today.

From "Pre-Induction Courses in Mathematics" on page 115 of the March 1943 issue:

The emergency need for boys and girls trained in mathematics has focused attention on the highly technical features of our mechanized civilization. The armed services and the supporting war industries need boys and girls trained in the proficient use of mathematics ranging from a real mastery of arithmetic fundamentals and such practical uses as are found in courses in general mathematics to the uses of higher mathematics in meteorology, ballistics, and other branches of science. Girls trained in mathematics are needed to replace men in industrial and other civilian positions which require the same range of uses of mathematics.

From "Progress Report of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America" on page 301 of the November 1941 issue:

In the junior and senior high schools, each boy and girl *of sufficient mathematical aptitude* should be urged by his advisers to observe that the study of mathematics through the stage of trigonometry and some solid geometry may serve as a distinctly patriotic action.

From "The First Report of the Commission on Post-War Plans" on page 231 of the May 1944 issue:

The simplicity of mathematical skills needed by the masses in the armed forces is even greater than was generally assumed, but is, as has been pointed out elsewhere, very difficult for many persons to achieve. Provision for growth in the mastery of arithmetic should be continuous throughout the elementary and secondary schools. Achievement of ability in mathematical reasoning and in sensible use of mathematical concepts requires much time and continuous practice. Courses of study of the elementary school should be reconsidered and adequate time be given to the teaching of arithmetic. Moreover, we have not given enough attention to arithmetic in teaching the senior high school courses. In fact, we have too often allowed the fundamental skills to deteriorate. Hereafter, we should keep an eye on arithmetic as we teach the advanced courses. Finally, the administrator can't eat his cake and have it. If we are to be held for results, we need to have the students for whatever time it takes to do the job.

From "Influence of the War on the Teaching of Secondary Mathematics" by E. R. Breslich on pages 293 and 294 of the November 1944 issue:

Proficiency in arithmetic should be a requirement for graduation from the high school.

The teaching of all high school mathematics should stress understanding of concepts and principles.

From "Mathematical Education in War-Time" by Virgil S. Mallory and Howard Fehr on page 298 of the November 1942 issue:

The army, navy, industry, and scientific organizations indicate that the mathematics teachers can be of immediate aid in the war emergency of securing the following:

1. Every boy in high school should study mathematics according to his abilities, the more capable taking four years of mathematics including trigonometry and solid geometry.
2. Advisers must realize the acute need for mathematics and direct all capable boys into such courses.
3. It is not necessary to create new courses or to entirely reorganize the present courses in the wartime effort. If our objectives are valid, and our methods of teaching will obtain these objectives, all that is needed is an added emphasis on the practical. . . .
5. Good teaching of straight mathematics

by fully qualified teachers is what is needed. Such teaching is a far superior procedure than to attempt courses in aeronautics or navigation in the high school given either by teachers not fully qualified to teach or taken by pupils who have not the necessary foundations for their successful study.

6. Modify your subject matter and your rate of teaching to attain complete understanding and you will best fit boys and girls for both peace-time and war-time uses of mathematics . . . .

From "The Second Report of the Commission on Post-War Plans" in the May 1945 issue:

The school should guarantee functional competence in mathematics to all who can possibly achieve it.

We must discard once for all the conception of arithmetic as a mere tool subject.

We must conceive of arithmetic as having both a mathematical aim and a social aim.

We must give more emphasis and much more careful attention to the development of meanings.

We must abandon the idea that arithmetic can be taught incidentally or informally.

We must realize that readiness for learning arithmetical ideas and skills is primarily the product of relevant experience, not the effect of merely becoming older.

We must evaluate learning in arithmetic more comprehensively than is common practice.

The mathematical program of grades 7 and 8 should be essentially the same for all normal pupils. (a) It should provide an adequate, organic continuation of the work of grades 1–6. (b) It should provide a substantial beginning in achieving functional competence. (c) It should provide a dependable foundation for subsequent courses in mathematics. (d) It should be so organized as to enable the pupils to achieve mathematical maturity and power.

The large high school should provide in grade 9 a double track in mathematics, algebra for some and general mathematics for the rest.

The main objective of the sequential courses should be to develop mathematical power.

Nominations for Officers and Members of the Board of Directors

The By-Laws of the National Council of Teachers of Mathematics require that "the Nominations and Elections Committee shall cause an announcement to be published in the official journal at least five months before the annual meeting inviting members of the Council to suggest nominees for elective offices."

The 1952 ballot will present candidates for the following offices:

A President (to serve for two years)

A Vice-President representing elementary school mathematics (to serve for two years)

A Vice-President representing senior high school mathematics (to serve for two years)

A Vice-President representing junior high school mathematics (to serve for one year only. This is a new office created by the revised By-Laws, for which election will thereafter be held at two-year intervals.)

Three members of the Board of Directors (to serve for three years)

The candidates for Directors may represent any level of instruction. The present geographic plan requires that only one of the nine Directors may come from one state. This limitation does not apply to officers. Hence, the following states will not be represented on the ballot for the three Directors: Louisiana, Minnesota, New Jersey, Florida, Kansas, and Iowa.

The Nominating Committee invites members of the Council to suggest persons to be nominated and to submit brief statements of their qualifications and their participation in local or national affairs pertaining to mathematics. Suggestions should reach the committee as soon as possible, and no later than September 1, 1951. Suggestions may be sent to any member of the Nominating Committee: E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana; Ona Kraft, Collingwood High School, Cleveland, Ohio; Veryl Schult, Wilson Teachers College, Washington, D. C.; Lenore John (Chairman), Laboratory School, University of Chicago, Chicago, Illinois.

The Twelfth Christmas Meeting of the National Council of Teachers of Mathematics will be held December 27, 28, 29, 1951, at the new Student Union Building on the campus of The Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma. The Program Chairman is Miss Lenore John, Laboratory Schools, University of Chicago, Chicago, Illinois and the Local Chairman is James H. Zant, Department of Mathematics, Oklahoma A. and M. College, Stillwater, Oklahoma. Plans for the tentative program include a general session on Friday and on Saturday morning at 9:00 A.M. as well as sectional meetings at 10:30 and in the afternoon on both days.

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

The Correlation of Mathematics and Science

FIFTY YEARS ago the Englishman John Perry advocated greater emphasis upon the practical uses of mathematics arising in mechanical drawing, physics, chemistry and engineering. A quarter of a century later, the National Committee report (1923) in this country recommended (p. 28) that there should be a conscious effort, through the selection of problems, to correlate the work in mathematics with other courses in the curriculum, especially in science. In 1942 the National Council of Teachers of Mathematics published as its 17th Yearbook, *A Source Book of Mathematical Applications*. Despite the wealth of excellent material assembled in that yearbook, to what extent are mathematics and science being correlated *systematically* and *effectively* in typical mathematics courses throughout the country today?

In view of the crucial role played by science and technology in contemporary civilization, and because of the intimate relationship between science and mathematics, both pure and applied, it would seem that a more adequate correlation can and should be effected between the two disciplines. It has been possible to integrate them so thoroughly that the casual visitor could not decide whether the lesson he was observing was one in mathematics or in science. Such complete correlation probably need not be our goal, but present practice certainly leaves much to be desired. To facilitate improvement in this connection, the following source material has been collected in the hope that it may prove helpful to teachers, curriculum committees, authors of textbooks, and mathematics clubs.

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MATHEMATICAL RECREATIONS

Edited by AARON BAKST
School of Education, New York University, New York 3, N. Y.

THE GAME OF TICK-TACK-TOE

By HARRY D. RUDERMAN
High School of Aviation Trades, New York, N. Y.

THE Game of *Tick-Tack-Toe* may be the nemesis of a teacher when two or more pupils prefer it to the performance of any sort of drill or decide to divert themselves away from a dull lesson. However, should this occur in a classroom, a teacher may seize the opportunity to utilize this game for the purpose of teaching some elementary topics in probability, combinations, and simple arithmetic.

The simplest *Tick-Tack-Toe* is played by two pupils with a three-by-three celled square. A move consists in placing a selected mark in each of the cells. One pupil agrees to use the mark X, and the opponent agrees to use the mark O. Only one mark is placed in any one cell. The object of the game is to obtain a straight-line sequence of three marks of the same *kind*. These sequences may be horizontal, vertical, or diagonal.

As soon as such a sequence is obtained by one of the two players the game is considered over, and he who obtains the sequence first is declare a winner.

As soon as one mark is placed in a cell (see Figure 1*b*) there is a definite number of ways a winning sequence can be obtained by the player as well as by his opponent. The O-player has four possible ways of winning (see Figure 1*b*) when placing his mark in the center cell, provided the winning sequence contains this mark. The strategy of his opponent, the X-player, is to prevent him from winning, while creating a favorable situation for himself. In Figure 1*c* the X-player blocked one way for the O-player and opened one possible way for himself. In Figure 1*d* the O-player blocked his opponent and threat-

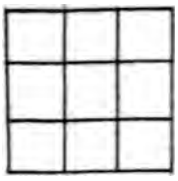


FIG. 1a

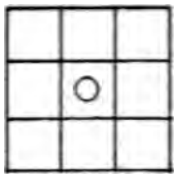


FIG. 1b

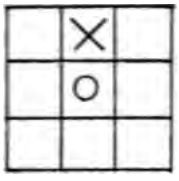


FIG. 1c

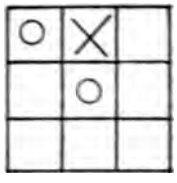


FIG. 1d

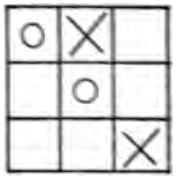


FIG. 1e

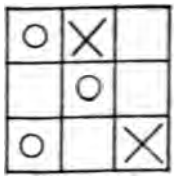


FIG. 1f

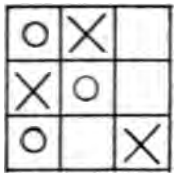


FIG. 1g

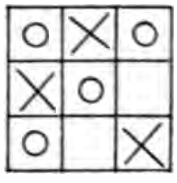


FIG. 1h

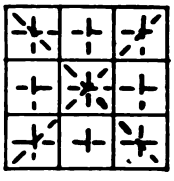


FIG. 2

3	2	3
2	4	2
3	2	3

FIG. 3

ens to win in the next move. The remainder of the game is forced.

The further analysis of the game shows that there are exactly eight possible winning sequences (see Figure 2). We shall indicate these sequences by eight straight lines. Within every cell of Figure 2 a certain number of straight lines intersect. Each straight line represents a winning pattern. Counting the number of these intersecting straight lines within each and every cell we obtain the *weight-value* of the respective cell as shown below (see Figure 3).

The strategy of playing this game must take into consideration the *weight-value* of the vacant cell which is utilized in any given move. Thus, at the beginning of the game the most preferable cell is the central one whose *weight-value* is 4. And the player who is allowed the first move should place his mark there. This gives him four different possible ways of winning. His opponent cannot block him in all the four ways during the next move. Whatever move he makes, he cannot secure for himself more than 2 possible ways to win, while he reduces his opponent's advantage to 3.

All further moves should be examined in a similar manner.

In other words, the simple *Tick-Tack-Toe* game may be very useful in the processes of analysis. Furthermore, this game offers opportunities for variation if the game is extended to squares with a greater number of cells.

The above contribution by Mr. Harry D. Ruderman, President of the Association of the Teachers of Mathematics of New York City, on the *Tick-Tack-Toe* game may appear to be too elementary for some of the teachers of mathematics. However, this department urged Mr. Ruderman to present a discussion on this topic because almost every game which is played by young or adults has a mathematical theory. The knowledge of this theory will generally improve the mastery of a game. On the other hand, many a classroom situation which begins to border on deterioration may be turned into a useful activity . . . "if only we knew how!" It is the hope of this department that the readers of THE MATHEMATICS TEACHER who know of some other games will send their contributions for publication. Mr. Ruderman informed this department that his interest in the game of *Tick-Tack-Toe* originated in a class while in service with the Armed Forces. The instructor in the class was weak on the motivation side. Thus, Mr. Ruderman's neighbor offered to teach him the three-dimensional *Tick-Tack-Toe* game. After a few minutes of instruction, Mr. Ruderman noted the mathematical properties of the game. From that moment on his friend never had a chance to beat him. This department hopes to publish the theory of the three-dimensional *Tick-Tack-Toe* game in one of the subsequent issues of THE MATHEMATICS TEACHER.

Let us consider a five-by-five *Tick-Tack-Toe* game with the rules slightly modified. In this game *all* the 25 cells must be filled with X's and O's. While the game is played a score is kept. Three like marks in a straight line and not separated by the opposing mark is given a score of 1. Four like marks in a straight line and not separated by the opposing mark is given a score of 2. Five like marks in a straight line and not separated by the opposing mark is given a score of 3. Thus, if three consecutive O's appear, the O-score is 1. If during the play this sequence of marks is increased by another O-mark, then the O-score is increased by 1, and so on.

The *weight-values* of the various cells in the five-by-five square are obtained in the same manner as in the case of the three-by-three square. These *weight-values* are shown in Figure 4 below.

9	10	11	10	9
10	15	17	15	10
11	17	24	17	11
10	15	17	15	10
9	10	11	10	9

FIG. 4

For example, the central cell has a *weight-value* 24. The reader will note that horizontally we can obtain three sequences with three consecutive marks, two sequences with four consecutive marks, and one sequence with five consecutive marks. Thus, horizontally we can obtain six scoring sequences. Vertically we can obtain six scoring sequences. Diagonally we obtain two six scoring sequences. Thus, the total number of the scoring sequences for the central cell is 24. The *weight-values* of all the other cells of the square are obtained in a similar manner.

A completed game (won by the X's) is shown in Figure 5.

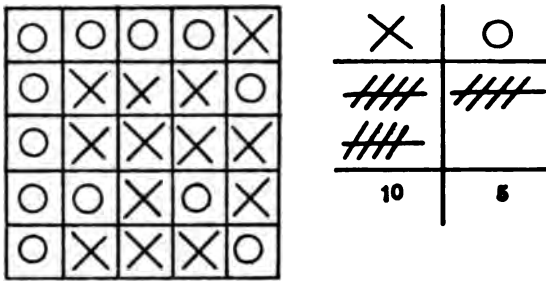
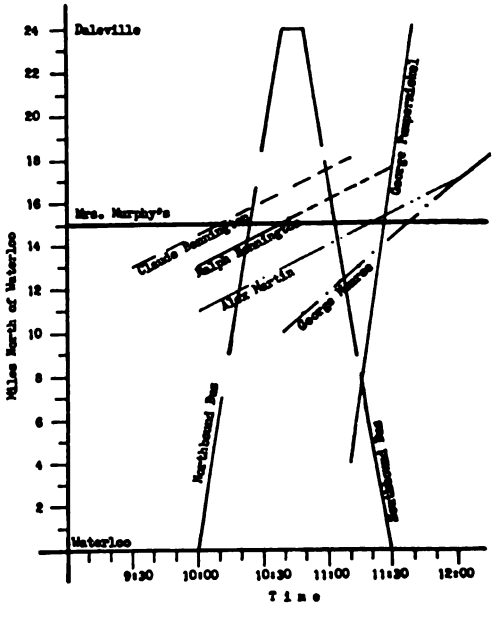


FIG. 5

Editorial Comment. In the case of the five-by-five square *weight-values* may be also obtained in terms of the scores which are possible for any given cell. Thus, for example, in the case of the central cell, horizontally, we have three possible sequences whose scores are 1 each, two possible sequences (of four consecutive marks) whose scores are 2 each, and one possible sequence (of five consecutive marks) whose score is 3. Thus, horizontally, the *score weight-value* is 10. The vertical *score weight-value* is also 10. Diagonally we obtain two *score weight-values* of 10 each. Thus, the *score weight-value* of the central cell is 40. The computation of the *score weight-values* follows the same pattern as the computation of statistical scores with corresponding frequencies.

The above variation of the game need not be necessarily confined to squares with



the same number of cells horizontally and vertically. We may vary the game so that rectangles, or any other geometric figures may be employed.

A. B.

Here is the solution of the theft of MRS. MURPHY'S laundry. The information, which was gathered by Detective Foams (see the March 1951 issue of THE MATHEMATICS TEACHER), is represented in the graph on p. 346. The only possible culprit was Ralph Bennington.

Correction: In the March issue of THE MATHEMATICS TEACHER (page 203) problem 2 should read

$$\left(\frac{5}{8}\right)^2 + \frac{3}{8} \quad \text{as} \left(\frac{3}{8}\right)^2 + \frac{5}{8}$$

BOOK SECTION

Edited by JOSEPH STIPANOWICH

Western Illinois State College, Macomb, Illinois

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is available whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

JUNIOR HIGH SCHOOL

Everyday Arithmetic (Junior Book 1), by Harl R. Douglass, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Principal, Ridge Road School, Parma, Ohio. Cloth, ix+488 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.08.

Everyday Arithmetic, (Junior Book 2) by Harl R. Douglass, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Principal, Ridge Road School, Parma, Ohio. Cloth, ix+502 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.08.

HIGH SCHOOL

1. Algebra

Algebra by Visual Aids (4 Volumes and Answer Book), by G. Patrick Meredith and edited by Lancelot Hogben. Cloth, 550 pages, 1948. George Allen and Unwin Ltd., Ruskin House, Museum Street, London. 10 Shillings; 8 Shillings, 6 Pence; 7 Shillings, 6 Pence; 9 Shillings, 6 Pence; 6 Shillings.

2. Plane Geometry

Plane Geometry, Revised Edition, by F. M. Morgan, Clark School, Hanover, N. H.; and W. E. Breckenridge, Teachers College, Columbia University. Cloth, viii+520 pages, 1951. Houghton Mifflin Company, 2 Park Street, Boston. $2.32.

3. Second Year Algebra

Algebra—Meaning and Mastery, Book Two, by Daniel W. Snader, University of Illinois. Cloth, v+500 pages, 1950. John C. Winston Company, 1010 Arch Street, Philadelphia 7, Pa. $2.60.

4. Special Courses in Mathematics

General Mathematics for the Shop, by Gilbert D. Nelson, Lincoln High School, Cleveland, Ohio; Frank C. Moore, Cleveland Public Schools; Carl Hamburger, Cleveland Public Schools; and Philip Becker, Editor, William E. Grady Vocational High School, Brooklyn, N. Y. Cloth, viii+440 pages, 1951. Houghton Mifflin Company, 2 Park Street, Boston. $2.64.

COLLEGE

1. Trigonometry

Trigonometry, by Cecil Thomas Holmes, Bowdoin College. Cloth, ix+246 pages, 1951. McGraw-Hill Book Co., 330 West 42nd Street, New York 18, N. Y. $3.00.

2. College Algebra

Essentials of College Algebra, by Joseph B. Rosenbach and Edwin A. Whitman, Carnegie Institute of Technology. Cloth, x+322 pages, 1951. Ginn and Company, Statler Building, Boston. $3.00.

3. Advanced Mathematics

Theory of Probability, by M. E. Munroe, University of Illinois. Cloth, viii+213 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $4.50.

The Kernel Function and Conformal Mapping, by Stefan Bergman. Mathematical Surveys No. 5. Cloth, vii+161 pages, 1950. American Mathe-

matical Society, 531 West 116th Street, New York 27, N. Y. $4.00.

The Elements of Mathematical Logic, by Paul C. Rosenbloom, Syracuse University. Cloth, iv+214 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $2.95.

Jacobian Elliptic Function Tables by L. M. Milne-Thomson, Royal Naval College, Greenwich, England. Cloth, xi+123 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $2.45.

The Fourier Integral and Certain of its Applications, by Norbert Wiener, Massachusetts Institute of Technology. Cloth, xi+201 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $3.95.

Teaching of Mathematics

Secondary Mathematics, A Functional Approach for Teachers, by Howard F. Fehr, Teachers College, Columbia University. Cloth, xi+431 pages, 1951. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Massachusetts. $4.25.

The Teaching of Secondary Mathematics, by Charles H. Butler, Western Michigan College of Education; and F. Lynwood Wren, George Peabody College for Teachers. Second Edition. Cloth, xiv+550 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $4.75.

The Teaching of Mathematics, by David R. Davis, State Teachers College, Montclair, N. J. Cloth, 384 pages, 1951. Addison-Wesley Press, Inc., Cambridge 42, Massachusetts. $4.50.

The Teaching of Calculus in Schools, A Report prepared for the Mathematical Association. Cloth, iv+76 pages, 1951. G. Bell and Sons, Ltd., London.

Miscellaneous

The Main Stream of Mathematics, by Edna E. Kramer. Cloth, xii+321 pages, 1951. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. $5.00.

REVIEWS

Arithmetic 6—The World of Numbers, Dale Carpenter and Dorothy Leavitt Pepper. New York, The Macmillan Co., 1950. iv+316 pp., $1.68.

This book is the sixth in the series called *The World of Numbers*, covering grades three through eight. The text contains some unusual features for books at this level. The topics are developed gradually and with meaning.

The material included for the purpose of improving the pupils' problem solving abilities is varied and interesting. The use of colorful drawings and pictorial material has made this one of the strongest features of the text. The suggested steps for solving problems are sound.

The text contains some valuable testing material. In the reviewer's opinion, the diagnostic testing program, if carried out by the teacher, should yield valuable information for organized review and re-teaching. The series of tests entitled "progress tests" contain some valuable items for measuring understanding. Chapter tests are also included. These contain computational exercises plus challenging "word problem" situations.

The authors have included some very interesting enrichment material that should prove of value in making mathematics more challenging to the bright pupil. The enrichment material also lends help to the authors' purpose of stressing meaning in arithmetic.—William L. Carter, Western Illinois State College (Laboratory School), Macomb, Illinois.

Arithmetic 7—The World of Numbers, Dale Carpenter and Elizabeth Cuthbertson. New York, The Macmillan Company, 1950. 332 pp., $1.68.

This textbook represents an arithmetic course for the seventh grade which has been well thought out, well graded, and well prepared. Emphasis is placed on understanding number concepts, principles, and generalizations. The word problems are well suited to the age and experience range of the average seventh grade child. They are presented in interesting social settings, illustrating the everyday usefulness of arithmetic. The orderly progressive organization of materials makes arithmetic in this book meaningful to the pupil, and provides effective guidance for the teacher.

An abundant variety of practice materials is provided to help increase his understanding and develop speed and skill. The vocabulary of numbers is carefully developed and maintained. Numerous attractive functional illustrations are found throughout the text; a special help to the eye minded pupil. All in all this book is on the right road to recapture the important traditional role that the textbook once held as a tool in teaching.—Louis M. Stern, Garrison Junior High School, Baltimore, Maryland.

Making Arithmetic Plain, Rose and Ruth Weber. Books I and II (workbooks). Wichita, Kansas, The McCormick-Mathers Publishing Company, 1949. Paper, 144 pp., $0.52 (each).

The books integrate the content of the textbook, the practice material, and the testing program in an attempt to satisfy the needs of the seventh and eighth grade children. The four functions of arithmetic; the informational, sociological, psychological, and computational are given consideration. Achievement tests are included at the end of each unit. Additional achievement tests and problems for practice in the fundamental operations are provided in separate booklets. A Teacher's Answer Book to facilitate scoring the exercises and tests is available for each grade.

The content of the books is good and the treatment of it, for the most part, is likewise

good. The traditional method of treating approximate measurement, however, is disappointing. The examples used to illustrate the process of cancellation in the multiplication and the division of decimals are objectionable from a teacher's view-point since they fail to show the complete division process.—L. H. WHITCRAFT, Ball State Teachers College, Muncie, Indiana.

Experimenting With Numbers (Teachers Manual), Catherine Stern. New York, Houghton Mifflin Company, 1950. 108 pp., $0.60.

This Teachers Manual is written to accompany newly designed types of number blocks, wherein the author describes what she calls *Structural Arithmetic* that employs the principle of measuring number—where comparisons are more readily understood and number relations more easily facilitated. Numerous activities and games are described in detail and can be understood by observing the diagrams and pictures in the book, but they can only be carried out with the special blocks described—and these are expensive. It is quite possible that the activities described will achieve desirable results because the approach makes use of education through the three senses—auditory, visual, and kinaesthetic.—JOSEPH J. URBANCEK, Chicago Teachers College, Chicago, Illinois.

Children Discover Arithmetic, Catherine Stern. New York, Harper and Brothers, 1949. xxiv +295 pp., $4.50.

Children Discover Arithmetic is the result of twenty years of experimentation by the author. This book is unique in its contribution to the teaching of arithmetic. The emphasis is on understanding the structure of numbers. It shows how to build the number concepts from 1–10 by using manipulative devices. The structure of whole numbers, common fractions and decimal fractions is presented in unforgettable pictures. The laboratory approach is used to gain understanding of numbers before attempting any sort of computation with the numbers. Special techniques for teaching bridging, and a variety of procedures that lead to mastery of the so-called fundamentals are included.

Teachers of kindergarten and of the first six grades will find that this book offers a new approach to the study of numbers. All facts are discovered through measurement rather than counting.

One of the most delightful parts of the book is the anecdotal records of the children's experiences at the Castle School in New York City where Dr. Stern is founder and director. The reader gains added insight on how children think about numbers by "listening in" on their conversations.—IDA MAE HEARD, Southwestern Louisiana Institute, Lafayette, Louisiana.

Algebra, Book 1, A. M. Welchons and W. R. Krickenberger. Boston, Ginn and Company, 1949. xi+580 pp., $2.12.

This first-year algebra text is one of the most carefully prepared, and therefore one of the most complete books that has come to the attention of this reviewer. Much attention has been given to the language of algebra and to formulas. Explanations are unusually complete, with emphasis on meanings and understanding, especially on such troublesome points as transposition and the square root process. There is a continuing and successful attempt made to show algebra as generalized arithmetic, and to integrate the subject matter with geometry, trigonometry, and analytics. The historical data inclosed is excellent as are the frequent references to the use of mathematics in the engineering and scientific world. Most teachers will particularly appreciate the unusually large number of problems of all types carefully graded into A, B, and C levels of difficulty. There are adequate tests and review materials provided throughout the text, a chapter reviewing the subject matter by topics, and a last chapter on arithmetic drill for those students particularly weak in that field. The text is well indexed and provides all the mathematical tables necessary for the work of the course. An answer book is not inclosed, but is available separately.—ROBERT V. BELDING, Thomas Carr Howe High School, Indianapolis, Indiana.

Algebra, Its Big Ideas and Basic Skills, Daymond J. Aiken and Kenneth B. Henderson. New York, Harper and Brothers, Publishers, 1950. xv+409 pp., $2.48.

Materials within this text are based upon recommendations of The National Council of Teachers of Mathematics and recent research pertaining to important concepts and processes needed for the study of advanced mathematics and college science. Creating interests that will encourage students to acquire greater understanding is of primary importance in this book. Many illustrations and photographs are included as incentives toward greater efforts. Novel caricatures are inserted for purposes of explaining mathematical processes stressing important facts and advancing thought provoking questions. An introduction addressed to the student helps initiate the program of interest creation. The reader is continually informed about algebraic applications in science and advanced mathematics.

Seven chapters are devoted to a presentation of "Big Ideas" and include discussions concerning general numbers, equations, signed numbers, dependence and mathematical operations, graphical representations of algebraic quantities, exponents at work, and indirect measurement. The other six chapters envolve further study and integration of these ideas. A breakdown of the individual chapters discloses the following presentations: more exercises than are needed permit application and review of basic principles; a section devoted to a practice of valuable skills; unit tests and a summary of things to remember. The summary will be especially valuable for teachers desiring a quick resume of a

chapter for purposes of pretesting, teaching and achievement testing. A unit entitled supplementary topics enables additional instruction in quadratics, imaginary roots and irrational numbers.—RODERICK MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

General Mathematics, Virgil S. Mallory and Kenneth C. Skeen. Chicago, Benjamin H. Sanborn and Company, 1951. v+478 pp.

This text, according to the authors, is designed for the large majority of high school students who need that socially useful mathematics which will give them a preparation for competent and happy citizenship.

Various methods are used to motivate the work and most of the problems are within the sphere of interest of an average ninth-grade student. Each chapter ends with a test covering all work up-to-date; in other words, the test after Chapter 8 is called a Test on Chapter 1–8.

A chapter known as Practice in Computation is included toward the end of the book. This is written so that it may either be used by a student individually or as a class group. The answers to this chapter are included in the book.

The format is good, type is large, and the many illustrations and diagrams are extremely well done.—MADELINE MESSNER, Abraham Clark High School, Roselle, New Jersey.

Self-Help General Mathematics Workbook, G. E. Hawkins and L. S. Walker. Chicago, Scott, Foresman and Company, 1949. 80 pp., $0.72.

This workbook, which can be used to supplement any standard text in general mathematics, is designed, as the title indicates, to provide "self-help" to the individual pupil in a diagnostic, remedial and continuous program.

The workbook contains thirty separate cumulative drills or tests, composed of twenty examples each, based on a number of different topics. Some examples are computational, some multiple-choice, and others true or false. Each drill contains helpful illustrations and diagrams. Space is provided on the page for working the computational exercises. A progress chart provides the self-competing motivation from drill to drill.

Perhaps the most outstanding feature of this workbook is the self-help chart accompanying each of the thirty drills which enables the pupil to determine just what he needs to study in his text and workbook to correct and overcome his own individual weaknesses. There are, also, five "self-help" study units, one after every six drills, which cover the hardest examples in the preceding drills as found by the authors through the standardization of these tests.

This workbook is an excellent addition to "self-help" type of material which not only aids the pupil to locate his specific difficulties in computations, ideas and concepts, but it, also, provides a remedy and an opportunity for him to overcome his weaknesses.—HARRY L. MCCULLOUGH, Woodrow Wilson Junior High School, Terre Haute, Indiana.

Algebra, Book II, A. M. Welchons and W. R. Krickenberger. Boston, Ginn and Company, 1949. x+516 pp., $2.20.

This book, designed for the second year's study of algebra is, like the first year book, unusually complete. In fact it contains most of the topics normally given in a first year college course lacking, however, a study of least squares, partial fractions, infinite series and mathematics of finance. Treatment of other topics is most excellent. Explanations are clear and well illustrated. Chapter and cumulative tests are included. There is adequate review of first year fundamentals, most of which is well integrated with the advanced work. Individual differences are cared for through grading of exercises into A and B levels of difficulty. Techniques have been emphasized, but only after concepts are thoroughly developed. Without labeling it as such, an excellent chapter on the calculus has been included. Necessary tables are included. Answers are available under separate cover. Applications of mathematics to various fields of work, historical data and other motivation materials are well done. They are appropriate, and should appeal to both student and teacher. —ROBERT V. BELDING, Thomas Carr Howe High School, Indianapolis, Indiana.

Essentials of Plane Trigonometry. Joseph B. Rosenbach, Edwin A. Whitman, and David Moskovitz. Boston, Ginn and Company, 1950. vii+158 pp., $2.70.

This book was written for early college or engineering school level, but because of its many outstanding features may be used by high school students.

The material is presented in a clear but concise manner. Each new principle or process is carefully and completely presented with all the necessary definitions, theorems, and proofs. This is followed by many illustrations, illustrative examples, and graded problems.

Other outstanding points about the book are:

(1) Historical notes are woven into the discussion. More than twenty are found in the book.

(2) The authors have tried to analyze pupil difficulties by placing *notes* throughout the book.

(3) Where misunderstanding might occur the authors have placed *warning* paragraphs.

(4) Problems are chosen from a wide subject matter field.

(5) Tables are thorough and complete.

(6) Authors suggest need for certain review before some topics are undertaken.

(7) Complete grouping of all the necessary formulas.

(8) General exercises listed by chapters and not as a composite group.—MRS. CECIL CRUM, Rossville High School, Rossville, Indiana.

A Course in the Slide Rule and Logarithms (Revised Edition), E. Justin Hills, Boston, Ginn and Company, 1950. iv+108 pp., $1.40.

This book is intended to introduce the student to the slide rule, to logarithms, to show the relationship between slide rule and logarithm procedures, and to give basic and general applications.

There are cuts of eighteen standard and special-use slide rules. Hence, the student is made aware not only of the demonstration slide rule in the class room, or the one he may possess, but also of many additional rules in existence.

Presented in twenty-two short units, each is suitable for a class period of study or for study by pupils on an individual basis. Explanations are clear and concise. Sample problems permit the student to follow through step-by-step settings on the slide rule. Adequate additional problems are given for practice, and sufficient answers are available to permit self-measurement.

Additional features include the trigonometric solution of triangles by both slide rule and five-place logarithms and the introduction to the student of slide rule scales based on natural logarithms.

The application to business, finance, and statistics which the text presents as well as basic computation problems can serve as a motivating device in appropriate secondary mathematics courses.—RUSSELL L. SCHNEIDER, Eastern High School, Lansing, Michigan.

Business Mathematics (Third Edition), Cleon C. Richtmeyer and Judson W. Foust. New York, McGraw-Hill Book Company, Inc., 1950. xviii+441 pp., $3.50.

A successful college textbook designed primarily for students preparing to teach commercial arithmetic and students planning to enter business. Special attention has been given to development of formulae used in solving more advanced problems. After considering the usual elementary material the authors have included chapters on logarithms and statistics.

Changes in the third edition include revised exercise and problem lists, an additional chapter on statistics, and the set of miscellaneous problems has been expanded and placed in a separate chapter.—CAROLINE A. LESTER, New York State College for Teachers, Albany, New York.

Elements of Analytic Geometry (Third Edition), Clyde E. Love. New York, The Macmillan Company, 1950. xii+218 pp., $2.75.

This is the third edition of a well-known text. It includes all the topics usually found in a course in plane analytic geometry plus chapters on coordinates in space, surfaces and curves, the plane, the straight line (in space), and quadric surfaces.

Topics that are treated more completely than in most texts include algebraic curves, asymptotes, bending of beams, rate of change of a function, trigonometric functions, exponential functions, logarithmic functions, families of curves, and the drawing of figures in three-dimensional space.

There is an abundance of illustrative examples and figures, and a wide range of exercises (about 1700), providing sufficient material so that the instructor may vary his assignments from semester to semester.

The figures are well drawn, and the type is easy to read, although the reviewer would have preferred a "glossier" grade of paper.

As a whole the book is a worthy successor to those which have preceded it.—CLEON C. RICHTMEYER, Central Michigan College, Mount Pleasant, Michigan.

Strategy in Poker, Business and War, John McDonald. New York, W. W. Norton and Company, Inc., 1950. 128 pp., $2.50.

As indicated by the title, the theme of this little book is *strategy*—not just military strategy, but strategy in its multifarious applications to the affairs of mankind.

Uncertainty arises from imperfect information. Strategy is used either to clarify, or to further obscure this information. A certain kind of strategy is used to resolve conflicting desires in such a way that an optimum condition is achieved.

Part One deals with the game Poker, including an interesting account of the history of the game since its inception in America over one hundred years ago.

Part Two takes up the Theory of Games, following the general treatment in von Neumann and Morgenstern's treatise. Here we are introduced to the distinction between games of chance and games of strategy. Solitaire is strictly a game of chance, for example, since all voluntary moves are made by the single player. In games involving strategy the players have a choice of action, and endeavor to base that choice on the actions of the other players. Hence there is an interdependence, but not cooperation involved. The important principle of "minimax" is discussed at some length.

Part Three has to do with the Game of Business. The gist of this chapter is that free competition among many individuals has been replaced by strategical combinations of individuals into corporations, trade unions, and other associations. The buyer-seller relation in an open market is visualized as a two-man game wherein we again see the minimax principle at work.

The entrance of a third party into the game makes for considerable alternation, since now the possibility of coalition emerges. Viewing economic movement as a game, the author says: "The complications that one extra player adds to game solutions are both good and bad for the theory of games. For although the extra player reveals the law of coalitions, by the same token he shows the fantastic complications involved in extending actual mathematical computations to situations involving large

numbers." But the author then explains that other factors tend to resolve the situation back into a few-party game: "The trade union movement shows how large numbers of economic individuals can group themselves back into small numbers of strategy-minded units."

In Part Four the author applies the three principal features of game theory—randomized strategy, minimax strategy, and coalitions—to political and military situations. The author's discussion of Stalin's rise to power, as a lesson in coalition strategy, is extremely interesting, as is also his brief description of some of the activities of the Operations Evaluation Group in our Navy.

The material contained in this book is a development of articles originally written for *Fortune* magazine. The author's very entertaining style, supplemented by Robert Osborn's piquant illustrations, make for an enjoyable reading experience.—E. W. BANHAGEL, Northwestern University, Evanston, Illinois.

The Problem of Problem-Solving in Mathematical Instruction. Aaron Bakst. New York, New York University Bookstore, 1950. ii+53 pp., $0.40.

In this essay, prepared for teachers of mathematics and teachers of teachers, the author has undertaken a detailed analysis of the problem of problem-solving in the light of modern mathematical theories as well as the theories of learning. Not only does he examine the question in view of the purely mathematical aspects, but also in light of the psychological implications, the nature, and the inner structure of mathematical problem-situations. Attention is given to the definition of a problem; the criteria for a real problem; and the kind of mathematical problems. The author stresses the fact that unless the pupil can and does identify himself with a problem-situation, it is not real to him.

The resolution of a problem-situation, in general, is considered under the following fundamental steps:

1. Anatomy of a problem-situation
2. Transition from a problem-situation to a problem
3. Anatomy of a problem
4. Analysis of the facts and of the conditions of the problem
5. Critical evaluation of the problem
6. Plan of the solution procedure
7. Evaluation of the plan of the solution procedure
8. Analysis of the solution.

The author clarifies and shows how these steps may be applied in the resolution of a problem-situation by solving a verbal (work) problem in algebra, and an original proposition in geometry.

Unique in this field, the work provides the basis for further study and experimentation. However, the technical nature of the discussion and the author's involved mode of expression make it rather difficult reading for the average teacher of secondary mathematics.—ELIZABETH ZEIGEL WINTER, Delta State Teachers College, Cleveland, Mississippi.

Mathematics, Queen and Servant of Science, by E. T. Bell. New York, McGraw-Hill Book Co., 1951. xx+437 pages, $5.00.

This book is an integration and amplification of two earlier volumes by the same author: *The Queen of the Sciences* (1931) and *The Handmaiden of the Sciences* (1937). All the material of the two previous volumes has been retained and skillfully blended into one presentation. New material has been added to the extent of about a third of the present volume. The newer portions retain all the flavor which permeated the two originals. For those well acquainted with the earlier volumes further recommendations are hardly necessary.

The avowed purpose of this book is twofold: to give those non-mathematicians who remember enough of their elementary mathematics a better understanding of what modern mathematics is all about, and, secondly, to enable young students to catch a glimpse of modern mathematics. Certainly there is nothing more disquieting to a mathematician than to find some person holding the opinion that a mathematician is a sort of super accountant who is a whizz at calculating. This book could do much to dispel such a misconception. It would be especially suitable for collateral reading in the last year of high school or the first years of college for all students who hope to have some understanding of what mathematics is about and how it serves science.

The theme of the book is indicated by its title. It is shown how pure mathematics, the Queen, has been of great significance to science, and how, in turn, science, in using applied mathematics, the Servant, has given impetus to pure mathematics. In short, it is shown that the two have been and still continue to be inseparable. The ideas and the spirit of mathematics as it has developed from ancient times to the present day are clearly presented by Dr. Bell in a fascinating story. Interesting bits of information on the history of mathematics and mathematicians are skillfully interspersed. And many priceless gems in the form of anecdotes and witticisms are thrown in for good measure.

Chapter I is concerned with the object of mathematics and is followed by a discussion of definitions of mathematics and of the postulational method. The discussion leads up to Chapter 5, entitled "The art of abstraction," in which the basic concepts of modern algebra are presented. This chapter is completely new. In Chapter 7 geometry takes over and the author traces its development from the ancient Greeks on to relativity theory in Chapter 10. Included in Chapter 8 is a brief presentation of the basic ideas of topology. This is also new. Following Chapter 11, which is on number theory, the author turns to mathematics, the Servant of science. Following a stimulating

philosophical discussion the reader is led through the story of Kepler and Newton on to the calculus and a brief (but adequate for the purposes of the book) mention of chief topics of modern analysis. Chapter 18 is devoted to statistics and probability. The volume concludes with two chapters on 'infinity' as a mathematical term and the related questions concerning the foundations of mathematics.

This book belongs in the library of every teacher of mathematics and every serious student of science. To both it could not help but be a source of inspiration and enlightenment, for Dr. Bell has the gift of presenting the profound concepts of the subject with a minimum of technical difficulty. It has been remarked that Dr. Bell is "perhaps mathematics' greatest interpreter." There is no perhaps about it. —D. H. Potts, Northwestern University, Evanston, Illinois.

ATTENDANCE RECORD OF THE TENTH SUMMER MEETING UNIVERSITY OF WISCONSIN, MADISON, WISCONSIN AUGUST 20-24, 1950

Compiled by Edwin W. Schreiber,
Western Illinois State College, Macomb, Illinois

State	Members	Guests	Total	Mississippi	2	0	2
Alabama	2	0	2	Missouri	5	0	5
Arkansas	2	0	2	Montana	1	0	1
California	3	2	5	Nebraska	5	3	8
Colorado	3	3	6	New Hampshire	1	0	1
Delaware	2	0	2	New Jersey	6	14	20
District of Columbia	2	0	2	New York	5	12	17
Georgia	1	1	2	North Dakota	0	1	1
Illinois	59	30	89	Ohio	9	1	10
Indiana	18	12	30	Oklahoma	6	5	11
Iowa	17	19	36	Tennessee	2	1	3
Kansas	5	8	13	Texas	5	10	15
Louisiana	1	0	1	Vermont	1	1	2
Maine	0	1	1	Virginia	1	0	1
Maryland	1	1	2	West Virginia	1	1	2
Massachusetts	1	0	1	Wisconsin	77	105	182
Michigan	13	6	19				
Minnesota	12	12	24	Totals	269	249	518

NEWS NOTES

An Exhibit of Instructional Aids and Demonstration Materials will be one of the features of the 1951 Summer Meeting of the National Council of Teachers of Mathematics to be held at St. Olaf College, Northfield, Minnesota. If you have materials which you would like to display, please write to Emil J. Berger, Monroe High School, St. Paul, Minnesota and let him know how much space to reserve for you. Materials may be shipped any time after May 15 to Mr. Arthur K. Solum, St. Olaf College, Northfield, Minnesota and will be stored until the time of the meeting in August.

Dates of Fall Meetings of Affiliated Groups are as follows:

Illinois Council of Teachers of Mathematics, October 13, 1951, at Northern Illinois State Teachers College at De Kalb

Iowa Association of Mathematics Teachers: Joint Meeting with Iowa State Educational Association, November, Des Moines

Oklahoma Council of Teachers of Mathematics: October 12, 1951, Oklahoma City

Mathematics Section, Eastern Tennessee Education Association: October 26, University of Tennessee, Knoxville.

A Joint Meeting of the **Kansas Association of Teachers of Mathematics** and the Kansas Section of the Mathematical Association of America was held at the University of Kansas at Lawrence on April 7. The program included the following: "Some Novel Methods for Estimating the Eigenvalues of Linear Operators" by Dr. John H. Curtiss, Chief of the National Applied Mathematics Laboratories of the National Bureau of Standards; "The Mathematical Needs of College Freshmen" by Kathleen O'Donnell of the University of Kansas; and "Motivation Devices" by W. V. Unruh of the Shawnee Mission High School.

The Mathematics Division of the Alabama Education Association held its spring meeting on March 15, 1951. **Mary E. Potter**, Supervisor of Mathematics in the Racine, Wisconsin Public Schools was the principal speaker.

(Continued on page 364)

Program

Eleventh Summer Meeting
The National Council of Teachers of Mathematics

St. Olaf College, Northfield, Minnesota
August 19, 20, 21, 22, 23, 1951

SUNDAY, AUGUST 19

3:00 P.M.–8:00 P.M. Registration and Room Assignment—Library Lobby

MONDAY, AUGUST 20

General Session

9:30 A.M.–10:30 A.M.—Gymnasium

Presiding: H. W. CHARLESWORTH, President, National Council of Teachers of Mathematics

Meeting the Needs of the Present Emergency, WILLARD E. GIVENS, Executive Secretary, National Education Association, Washington, D. C.

Study Groups

10:45 A.M.–12:00 NOON

Each study group will meet at this time each day of the conference: MONDAY, TUESDAY and WEDNESDAY. Register for the group of your choice in advance since enrollment is limited.

Group S-1—Library 201

Theme: *Mobilizing Secondary and College Mathematics Education for the National Emergency*

Leader: VERYL SCHULT, Public Schools, Washington, D. C.

Recorder: VIRGINIA LEE PRATT, Public Schools, Omaha, Nebraska

Monday's Topic: *Mobilization and the Mathematics Curriculum*

Group S-2—Holland 501

Theme: *Applications of Mathematics*

Leader: AARON BAKST, New York University, New York, N. Y.

Recorder: MAUDE HOLDEN, Public Schools, Ord, Nebraska

Monday's Topic: *Graphical Methods*

Group S-3—Library 302

Theme: *Training Teachers of College Mathematics*

Leader: JAMES H. ZANT, Oklahoma A. and M. College, Stillwater, Oklahoma

Recorder: EDWIN W. SCHREIBER, Western Illinois State College, Macomb, Illinois

Monday's Topic: *The Kind of Mathematics Preparation Needed*

Group S-4—Holland 412

Theme: *Psychological and Semantical Foundations of Meaningful Arithmetic Instruction*

Leader: HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa

Recorder: GLENADINE GIBB, Iowa State Teachers College, Cedar Falls, Iowa

Monday's Topic: *What Do We Know about How Children Learn Arithmetic?*

Group S-5—Holland 415

Theme: *Problem Solving*

Leader: PHILIP PEAK, Indiana University, Bloomington, Indiana

Recorder: HENRIETTE L. BRUDOS, State Teachers College, Valley City, North Dakota

Monday's Theme: *Objectives of Problem Solving*

Participants: JOHN RIDENOUR, Rockford, Illinois, and NAOMI MARGARET HICKS, Boone, Iowa

Group S-6—Holland 414

Theme: *Problems in Teaching Geometry*

Leader: F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

Recorder: AGNES HERBERT, Public Schools, Baltimore, Maryland

Monday's Topic: *The Purpose of Geometry in the School Program*

Group S-7—Library 202

Theme: *Problems in Teaching General Mathematics*

Leader: MYRON F. ROSSKOPF, Syracuse University, Syracuse, New York
Recorder: ALICE M. HACH, Slauson Junior High School, Ann Arbor, Michigan
Monday's Topic: *The Use of Models in the Teaching of General Mathematics*

Group S-8—Main 4
Theme: *Teaching Selected Topics of Algebra*
Leader: DANIEL W. SNADER, University of Illinois, Urbana, Illinois
Recorder: MARGARET JOSEPH, Shorewood High School, Milwaukee, Wisconsin
Monday's Topic: *Teaching Solutions of Equations and Formulas by Inverse Operations*

Group S-9—Main 5
Theme: *Problems in Teaching Non-College Preparatory Mathematics in the Senior High School*
Leader: HOWARD F. FEHR, Columbia University, Teachers College, New York, N. Y.
Recorder: To be appointed from the group
Monday's Topic: *Who Are Our Non-College Preparatory Students of Mathematics?*

Group S-10—Library 304
Theme: *Materials of Instruction*
Leader: HENRY W. SYER, Boston University, Boston, Massachusetts
Recorder: RUTH KNAUS, High School, Waseca, Minnesota
Monday's Topic: *The Relationships between Objectives, Materials and Activities*

Group S-11—Main 2
Theme: *Providing for Individual Differences*
Leader: MARY A. POTTER, Supervisor of Mathematics, Racine, Wisconsin
Recorder: MARY LEE FOSTER, Henderson State Teachers College, Arkadelphia, Arkansas
Monday's Topic: *Some Basic Differences in Slow and Fast Learners*

Discussion Groups

10:45 A.M.–12:00 NOON (Monday only)

Register for the group of your choice in advance since enrollment is limited.

Group D-11—Annex 5
Topic: *Ways of Solving Algebra Problems*
Leader: EDITH WOOLSEY, Sanford Junior High School, Minneapolis, Minnesota

Group D-12—Annex 13
Topic: *Student-Teacher Planning*
Leader: GLADYS W. JUNKER, University of Chicago Laboratory School, Chicago, Illinois

Group D-13—Main 13
Topic: *Remedial Mathematics at the Twelfth Grade Level*
Leader: JOHN T. LADD, Highland Park High School, Highland Park, Michigan

Group D-14—Annex 6
Topic: *Family Trees and Similar Devices in the Teaching of Mathematics*
Leader: HENRY SWAIN, New Trier Township High School, Winnetka, Illinois

Group D-15—Annex 1
Topic: *Some Trends in College General Mathematics*
Leader: LYLE J. DIXON, University of Kansas, Lawrence, Kansas

Group D-16—Main 8
Topic: *How to Organize and Run a Mathematics Clinic*
Leader: NATHAN LAZAR, Ohio State University, Columbus, Ohio

Films and Filmstrips

10:45 A.M.–12:00 NOON—Holland 211
Arithmetic Films and Filmstrips

Sectional Meetings

1:30 P.M.–2:45 P.M. *General Section*—Library 304
Sponsored by Iowa Association of Mathematics Teachers
Presiding: ORVILLE A. GEORGE, Mason City, President, Iowa Association of Mathematics Teachers
A Career as an Actuary, FLOYD

S. HARPER, Drake University, Des Moines, Iowa

Mathematical Values of High School Freshman Exploration Days and Senior Career Days, ORLANDO C. KRIEDER, Iowa State Teachers College, Cedar Falls, Iowa

Mathematical Forms Found in Nature, KATHRYN B. HINSENBROCK, High School, Charles City, Iowa

1:30 P.M.–2:45 P.M. *General Section*—Library 302

Sponsored by Nebraska Council of Mathematics Teachers

Presiding: MILTON W. BECKMAN, President, Nebraska Section, National Council of Teachers of Mathematics

Consumer Mathematics in the Twelfth Grade, RICHARD R. SHORT, Public Schools, Lincoln, Nebraska

Creating and Maintaining Interest in Mathematics Classes. DUANE M. PERRY, Public Schools, Omaha, Nebraska

The Relative Merits of Algebra and General Mathematics in the Development of Mathematical Literacy. MILTON W. BECKMAN, University of Nebraska, Lincoln, Nebraska

1:30 P.M.–2:45 P.M. *General Section*—Holland 415

Presiding: LOUISE KINN, Junior High School, Brainerd, Minnesota

Some Implications of the Formula $K=f(d)$, where $d=$ discovery and $K=$ knowledge, for Teachers of Mathematics. LEONARD S. LAWS, University of Minnesota, Minneapolis, Minnesota

General Educational Values of Mathematics and the Attempt of a Faculty to Teach Them. JOHN ABERNATHY, Arkansas Polytechnic College, Russelville, Arkansas

1:30 P.M.–2:45 P.M. *Arithmetic Section*—Steensland Auditorium

Presiding: HARRY C. JOHNSON, University of Minnesota, Duluth, Minnesota

Some Fun With Arithmetic. HAROLD D. LARSEN, Albion College, Albion, Michigan

Problem Solving, The Backbone of Arithmetic. HOLMES BOYNTON, Northern Michigan College of Education, Marquette, Michigan

1:30 P.M.–2:45 P.M. *General Mathematics*—Holland 501

Presiding: PAUL JORGENSON, High School, Northfield, Minnesota

The Possibility and Desirability of Teaching Arithmetical Concepts and Operations without the Use of Numerals and Algorisms. NATHAN LAZAR, Ohio State University, Columbus, Ohio

Teaching General Mathematics Effectively. GILBERT D. NELSON, Lincoln High School, Cleveland, Ohio

1:30 P.M.–2:45 P.M. *Algebra Section*—Library 303

Presiding: WORTH J. OSBURN, University of Washington, Seattle, Washington

Getting to the Heart of Algebra. DANIEL W. SNADER, University of Illinois, Urbana, Illinois

The Function Concept in Secondary School Mathematics. J. H. BANKS, George Peabody College for Teachers, Nashville, Tennessee

1:30 P.M.–2:45 P.M. *Geometry Section*—Library 201

Presiding: THEODORE KELLOGG, President, Mathematics Section, Minnesota Education Association, Minneapolis, Minnesota

The Nature of Evidence. HOWARD F. FEHR, Teachers College, Columbia University, New York, N. Y.

The Mult-Converse Concept in Geometry. FRANK B. ALLEN, High School, LaGrange, Illinois

1:30 P.M.–2:45 P.M. *College Section*—Library 202

Presiding: CLARENCE S. CARLSON, St. Olaf College, Northfield, Minnesota

A Combined Course in College Algebra and Trigonometry. JAMES H. ZANT, Oklahoma A. and M. College, Stillwater, Oklahoma

Courses Desirable for Training Teachers of High School Mathematics. CECIL B.

READ, University of Wichita, Wichita, Kansas

Films and Filmstrips

1:30 P.M.–2:45 P.M.—Holland 211
Arithmetic Films and Filmstrips

Coffee Hour

3:00 P.M.–4:00 P.M.—College Center

Laboratory Sessions

3:30 P.M.–4:30 P.M.

Each laboratory group will meet at this time each day of the conference: MONDAY, TUESDAY, and WEDNESDAY. Register for the laboratory of your choice since enrollment is limited to 50. There may be a small charge for the materials which are used in the making of teaching devices by the members of each group.

Arithmetic Laboratory Section—Library 304
Leader: ELDA MERTON, Chicago, Illinois
Junior High School Laboratory Section—Library 303
Leader: MARVIN JOHNSON, Junior High School, Rochester, Minnesota
Senior High School Laboratory Section—Library 301
Leader: EMIL J. BERGER, Monroe High School, St. Paul, Minnesota
Participant: FRANKLYN BLUME, Principal, Monroe High School, St. Paul, Minnesota
Topic: *The Role of the Administration in Developing a Mathematics Laboratory*

Film and Filmstrip Exhibit

3:30–4:30 P.M.—Holland 211
Arithmetic Films and Filmstrips

Exhibits

9:00 A.M.–5:00 P.M.—Library Reference Room
Commercial Exhibits
Mathematical Models and Teaching Devices
Charts, Pamphlets and other Materials

TUESDAY, AUGUST 21

General Session

9:30 A.M.–10:30 A.M.—Gymnasium
Presiding: EDITH WOOLSEY, President, Minnesota Council of Teachers of Mathematics, Minneapolis, Minnesota
What Mathematics Should We Be Teaching? JOHN R. MAYOR, University of Wisconsin, Madison, Wisconsin

Study Groups

(See Monday's Sechedule)

10:45 A.M.–12:00 NOON

Group S-1—Library 201
Topic: *The Role of Mathematics in National Defense*
Leader: KENNETH HENDERSON, University of Illinois, Urbana, Illinois
Group S-2—Holland 501
Topic: *The Mathematics of Atomic Energy*
Group S-3—Library 302
Topic: *The Professional Courses Needed*
Group S-4—Holland 412
Topic: *Fundamental Arithmetic Situations*
Group S-5—Holland 415
Topic: *Characteristics of Good Problem Material*
Group S-6—Holland 414
Topic: *Functional Dependence in Geometry*
Group S-7—Library 202
Topic: *The Use of Films and Filmstrips in the Teaching of General Mathematics*
Group S-8—Main 4
Topic: *Teaching Fundamental Operations with Directed Numbers*
Group S-9—Main 5
Topic: *What Are the Best Teaching Devices for This Group?*
Group S-10—Library 304
Topic: *The Location and Evaluation of Materials*
Group S-11—Main 2
Topic: *Helpful Topics for Slow Learners*

Discussion Groups

10:45 A.M.–12:00 NOON (Tuesday only)

Group D-21—Annex 1
Topic: *What Shall We Teach in First Year Algebra*

Leader: A. M. WELCHONS, Arsenal Technical Schools, Indianapolis, Indiana

Group D-22—Annex 5

Topic: *The Emerging Extension of General Mathematics in Senior High School*

Leader: F. G. LANKFORD, JR., University of Virginia, Charlottesville, Virginia

Group D-23—Annex 13

Topic: *How Can We Best Achieve Soundness in Our Procedure, Reasoning, and Terminology*

Leader: HENRY A. MEYER, Central High School, Evansville, Indiana

Group D-24—Main 13

Topic: *Applications of Teacher-Pupil Planning in the Mathematics Classroom*

Leader: LOTTCHEN LIPP HUNTER, Curriculum Division, Wichita, Kansas

Group D-25—Main 8

Topic: *Trends in the Teaching of Solid Geometry*

Leader: WILLIAM D. POPEJOY, High School, Savanna, Illinois

Group D-26—Annex 6

Topic: *Ways and Means to Implement Thinking in Arithmetic*

Leader: ANN PETERS, Teachers College, Keene, New Hampshire

Film and Filmstrip Exhibit

10:45 A.M.–12:00 NOON—Holland 211

Junior High School Films and Filmstrips

Sectional Meetings

1:30 P.M.–2:45 P.M. *General Section*—Drama Studio

Report of Representative of Affiliated Groups

Presiding: MARY C. ROGERS, High School, Westfield, New Jersey

Significant Services of Mathematics Education

1:30 P.M.–2:45 P.M. *General Section*—Library 304

Presiding: LAWRENCE WAHLSTROM, State Teachers College, Eau Claire, Wisconsin

The Preparation of Teachers. MYRON ROSSKOPF, Syracuse University, Syracuse, New York

The Evaluation of Successful Teaching of Mathematics. PHILIP PEAK, Indiana University, Bloomington, Indiana

1:30 P.M.–2:45 P.M. *General Section*—Library 201

Presiding: ONA KRAFT, Collinwood High School, Cleveland, Ohio

Reasoning Patterns as Proof Aids. VELMA OAKES, Siloam Springs, Arkansas

Adjusting First Year College Mathematics to Meet the Needs of Special Interest Students. ADRIEN L. HESS, Montana State College, Bozeman, Montana

Mathematics in Michigan's Outdoor Education Program for High School Students. DONALD F. MARSHALL, High School, Dearborn, Michigan

1:30 P.M.–2:45 P.M. *Arithmetic Section*—Steensland Auditorium

Presiding: MARIE S. WILCOX, Washington High School, Indianapolis, Indiana

Developing Number Concepts in the Primary Grades. AGNES G. GUNDERSON, University of Wyoming, Laramie, Wyoming

Basic Meanings in Arithmetic. F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

1:30 P.M.–2:45 P.M. *General Mathematics Section*—Holland 501

Presiding: MILTON W. BECKMAN, University of Nebraska, Lincoln, Nebraska

Examples of Laboratory Teaching in Mathematics. F. G. LANKFORD, University of Virginia, Charlottesville, Virginia

Motivation as the Key. RALPH HUFFER, Beloit College, Beloit, Wisconsin

1:30 P.M.–2:45 P.M. *Algebra Section*—Library 303

Presiding: FLORENCE SHOTTLER, High School, Austin, Minnesota

The Semantics of Algebra. WORTH J. OSBURN, University of Washington, Seattle, Washington

Four Fundamental Processes in Mathematics. JOHN RECKZEH, New Jersey State Teachers College, Jersey City, New Jersey

1:30 P.M.–2:45 P.M. *Geometry Section*—Library 201

Presiding: CLARICE KAASA, High School, Red Wing, Minnesota

Honor Work in Demonstrative Geometry. ELOISE BAKST, Jamaica High School, New York, N. Y.

Successive Levels of Generalization in Geometry. CLARENCE H. HEINKE, Capital University, Columbus, Ohio

1:30 P.M.–2:45 P.M. *Advanced General Mathematics Section*—Holland 415

Presiding: JACK WELLS, Board of Education, Minneapolis, Minnesota

What Should Be Taught at the Twelfth Grade Level in Consumer Mathematics. MARGARET JOSEPH, Shorewood High School, Milwaukee, Wisconsin

A Course of Study for the Second Year of General Mathematics. WILLIAM POSORSKE, High School, Rochelle, Illinois

1:30 P.M.–2:45 P.M. *College Section*—Library 202

Panel Discussion: *The Improvement of Instruction in College Mathematics*

Participants: MAX KRAMER, New Mexico College of Agriculture and Mechanic Arts, Las Cruces, New Mexico; HAROLD D. LARSEN, Albion College, Albion, Michigan; and KENNETH WEGNER, Carleton College, Northfield, Minnesota

Film and Filmstrip Exhibit

1:30 P.M.–2:45 P.M.—Holland 211

Junior High School Films and Filmstrips

Coffee Hour

3:00 P.M.–4:00 P.M.—College Center

Recreational Activities

3:00 P.M.–5:00 P.M.

Laboratory Sessions

3:30 P.M.–5:30 P.M. (See Monday Schedule)

Film and Filmstrip Exhibit

3:30 P.M.–4:30 P.M.

Junior High School Films and Filmstrips

Exhibits

9:00 A.M.–5:00 P.M.—Library Reference Room

Commercial Exhibits

Mathematical Models and Teaching Devices

Charts, Pamphlets and other Materials

Dinner

6:30 P.M.

Norwegian Dinner

Entertainment

8:30 P.M.

Folk Dancing, Square Dancing, Bridge

WEDNESDAY, AUGUST 22

General Session

9:30 A.M.–10:30 A.M.—Gymnasium

Presiding: HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa

Making Mathematics Meaningful. LEO J. BRUECKNER, University of Minnesota, Minneapolis, Minnesota

Study Groups

(See Monday Schedule)

10:45 A.M.–12:00 NOON

Group S-1—Library 201

Topic: *Successful Features of the Training Program During World War II*

Leader: WILLIAM L. HART, University of Minnesota, Minneapolis, Minnesota

Group S-2—Holland 501

Topic: *The Banker's Number—e*

Group S-3—Library 302

Topic: *The Psychology of Learning Applied to Mathematics*

Group S-4—Holland 412

Topic: *Problem Solving*

Group S-5—Holland 415
Topic: *Problem Solving Applied to Life*
Group S-6—Holland 414
Topic: *What About Solid Geometry*
Group S-7—Library 202
Topic: *Sharing our Experiences in Teaching General Mathematics*
Group S-8—Main 4
Topic: *Teaching Problem Solving, or Teaching for Integration of Algebra and Geometry, or Teaching of Logarithms and the Slide Rule*
Group S-9—Main 5
Topic: *The Subject Matter and the Training of the Teachers*
Group S-10—Library 304
Topic: *The Proper Use of Materials*
Group S-11—Main 2
Topic: *Adapting Teaching to Slow Learners*

Discussion Groups

10:45 A.M.–12:00 Noon (Wednesday only)

Group D-31—Annex 5
Topic: *What Elements of Statistics Should We Teach in High School*
Leader: MAX BEBERMAN, University of Illinois High School, Urbana, Illinois
Group D-32—Annex 13
Topic: *What Belongs in Fourth Semester High School Algebra*
Leader: ONA KRAFT, Collinwood High School, Cleveland, Ohio
Group D-33—Main 13
Topic: *Junior High School Mathematics in the New Education*
Leader: MARY C. ROGERS, High School, Westfield, New Jersey
Group D-34—Main 8
Topic: *How May Plane Geometry Be Revised to Meet the Life Adjustment Education Program*
Leader: CELIA H. CANINE, North High School, Wichita, Kansas
Group D-35—Annex 1
Topic: *Ideas for Adding Interest to the Mathematics Club or Classroom*
Leader: EDNA NORSKOG, Illinois State Normal University, Normal, Illinois
Group D-36—Annex 6
Topic: *The Measurement of Understanding in Demonstrative Geometry*
Leader: JACK SILVERMAN, St. John's High School, Winnipeg, Canada

Film and Filmstrip Exhibit

10:45 A.M.–12:00 Noon—Holland 211
Geometry Films and Filmstrips

Sectional Meetings

1:30 P.M.–2:45 P.M. *General Section*—Gymnasium
Presiding: HOLMES BOYNTON, Northern Michigan College of Education, Marquette, Michigan
The Educated of the Gifted. WILBUR MURRA, Educational Policies Commission, Washington, D. C.
Human Relations in the Mathematics Classroom. H. E. BENZ, Ohio University, Athens, Ohio
1:30 P.M.–2:45 P.M. *Applications Section*—Drama Studio
Presiding: MARVIN JOHNSON, Rochester, Minnesota
Mathematics and Music. AARON BAKST, New York University, New York, N. Y.
1:30 P.M.–2:45 P.M. *General Section*—Library 304
Sponsored by the Wisconsin Council of Mathematics Teachers
Panel Discussion: *Improving the Testing Program*
Presiding: KENNETH R. FISH, President, Wisconsin Council of Mathematics Teachers, Beloit, Wisconsin
Participants: JOHN R. MAYOR, University of Wisconsin, Madison; RALPH E. HUFFER, Beloit College, Beloit; JOHN BROWN, Wisconsin High School, Madison; MARGARET STRIEGL, Senior High School, Wauwatosa; DOROTHY SWARD, Roosevelt Junior High School, Beloit; ELLI OTTESON, Senior High School, Eau Claire; IRENE LARSON, Elementary Supervisor, Green Bay; and SISTER MARY FELICE, Mount Mary College, Milwaukee

1:30 P.M.–2:45 P.M. *Arithmetic Section*—Steensland Auditorium

Presiding: LENORE JOHN, University of Chicago Laboratory School, Chicago, Illinois

Dr. Stern's Structural Arithmetic Materials. ELIZABETH BALDWIN, Kamehameha Schools, Honolulu, Hawaii

Teaching Mathematical Generalizations in the Elementary School. LUCY ROSENQUIST, Colorado State College, Greeley, Colorado

How to Make Individualized Instruction in Arithmetic Meaningful. ALICE FUNFAR, University of Iowa Graduate Student, Iowa City, Iowa

1:30 P.M.–2:45 P.M. *General Mathematics Section*—Holland 501

Presiding: GEORGE MCCUTCHEON, Minnesota State Department of Education, St. Paul, Minnesota

Teaching the Concepts of Area and Volume to Avoid Certain Pitfalls. ALICE M. HACH, Slauson Junior High School, Ann Arbor, Michigan

Dimensionality—A Proposal of a New and Important Thread of Emphasis in Secondary Mathematics and Science. SHELDON S. MYERS, Ohio State University School, Columbus, Ohio

1:30 P.M.–2:45 P.M. *Algebra Section*—Library 303

Presiding: NICK LOVDJIEFF, University of Minnesota High School, Minneapolis, Minnesota

Oral Practice Work in Mathematics. P. A. PETRIE, University of Toronto, Toronto, Ontario

Let's Return to the Fundamental Axioms. KENNETH R. CONKLING, Frankton, Indiana

1:30 P.M.–2:45 P.M. *College Section*—Library 202

Presiding: KENNETH MAY, Carleton College, Northfield, Minnesota

The Mathematics of Meteorology, MAX KRAMER, New Mexico College of Agriculture and Mechanic Arts, Las Cruces, New Mexico

The University Recreational Mathematics Lecture for High School Students. CHARLES HATFIELD, University of Minnesota, Minneapolis, Minnesota

Films and Filmstrip Exhibit

1:30 P.M.–2:45 P.M.
Geometry Films and Filmstrips

Coffee Hour

3:00 P.M.–4:00 P.M.—College Center

Laboratory Sessions

3:30 P.M.–5:30 P.M.
(See Monday Schedule)

Films and Filmstrip Exhibit

3:30 P.M.–4:30 P.M.
Geometry Films and Filmstrips

Recreational Activities

3:00 P.M.–5:00 P.M.

Exhibits

9:00 A.M.–5:00 P.M.—Library Reference Room

Commercial Exhibits

Mathematical Models and Teaching Devices

Charts, Pamphlets, and other Materials

Banquet

7:00 P.M.

Toastmaster: WILLIAM L. HART, University of Minnesota, Minneapolis, Minnesota

Mathematics, Logic and Values, DR. ROBERT H. BECK, University of Minnesota, Minneapolis, Minnesota

THURSDAY, AUGUST 23

Sight-Seeing Trip

9:00 A.M.–4:00 P.M.
Twin Cities

ANNOUNCEMENTS

Registration: The registration fee is fifty cents for members of the National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools.

The fee for non-members and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press and commercial exhibitors are not charged a registration fee, but should register. You are urged to register in advance using the advance registration and reservation form.

Room Reservations: Applications for room accommodations in St. Olaf dormitories should be sent to David Johnson, Director of Public Relations, St. Olaf College, Northfield, Minnesota, by August 10. Rates are as follows: $1.50 per person per day. All rooms are double or triple rooms, with single beds and bunks, and must be used as such. The dormitories will provide linens and towels, however no maid service is available. Rooms will be available for the period, Sunday, August 19, through Friday, August 24. Reservations for residence accommodations may be made by mailing the Advance Reservation and Registration Form, *not* later than August 10. Room charges are payable in advance at time of registration.

Food Service: Meals and coffee hours will be provided by college facilities, Monday morning through Friday morning, August 20–24, and breakfast and luncheon will be cafeteria style.

Evening Meals: The evening meals will consist of a picnic on Monday ($1.00); Norwegian dinner on Tuesday ($2.00); and the banquet on Wednesday ($2.50). Reservations for evening meals should be made on the Advance Reservation and Registration Form.

Study Groups: In order to permit more intensive study of classroom problems, study groups have been organized, to meet from 10:45 to 12:00 Noon each day of the conference. If you wish to meet with a study group, registration should be made in advance on the Advance Registration and Reservation Form, since enrollment in most of the groups is limited. If you wish to be a member of a study group, you will register for the group of your choice, and attend the meetings of the group on each day of the conference. Attendance to that group will be by admission card which will be issued at the time of registration.

Discussion Groups: Discussion groups will be small to permit the audience to participate in the topic of discussion. Since enrollment is limited, registration should be made in advance with first, second, and third choices indicated. Attendance to that group will be by admission card which will be issued at the time of registration.

Mathematics Laboratories: There will be three mathematics laboratories sections, each section meeting on each day of the conference. These groups will demonstrate and discuss the construction and use of models in the classroom. Then each member of the group will be given the opportunity to make models. A minimum cost charge for materials used will be made. You are urged to register in advance since enrollment in each section is limited.

Recreational Facilities: The recreational facilities of St. Olaf College will be available to everyone attending the conference and members of their families. These facilities include swimming pool, tennis court, golf course. Special activities for wives and children in attendance will be provided each afternoon of the conference.

Sight Seeing Trip: A sight seeing trip by bus will be provided on Thursday, August 23, to points of interest in the Twin Cities. The bus will leave St. Olaf College at 9 A.M. Thursday, August 23, and return at approximately 4 P.M. The trip will include an excursion through the Ford Assembly Plant, luncheon at the Automobile Club, a tour of the Minnesota River valley, Minnehaha Falls, Minneapolis lakes, the University of Minnesota campus and the Museum of Art. The cost of bus fare for this excursion will be $1.75. The cost of the luncheon will depend upon individual orders. Make reservations for this trip on the registration form before August 10.

Exhibits: There will be exhibits of

mathematical models, instruments, teaching aids and other classroom materials. Teachers are invited to bring materials for exhibit. Those who are willing to bring materials from their schools, should communicate in advance with Emil Berger, Monroe High School, St. Paul, Minnesota.

Many free and inexpensive materials will also be on exhibit. Suggestions of recent materials which have become available and have classroom value should be sent to Florence Schottler, High School, Albert Lea, Minnesota.

Commercial Exhibits: Textbooks and commercial teaching aids will be on exhibit. Inquiries for exhibit space should be addressed to Lawrence Goodrich, High School, Owatonna, Minnesota.

Supplies and Equipment: Speakers and other participants in the program who need projection equipment or other materials should communicate not later than August 10 with Florence Collins, 1406 Osceolo Street, St. Paul, Minnesota.

Films and Film Strips: Persons wishing to view films while in attendance at the meeting should communicate with Oscar A. Wilcox, High School, Anoka, Minnesota, not later than August 1.

Refunds: No ticket refunds will be made later than three hours preceding functions for which reservations were made, e.g., picnic, Norwegian dinner, banquet and sight seeing trip.

Information: The Hospitality Committee will furnish you with information on vacation facilities, points of interest, etc. A daily bulletin of special activities will be published. When at the conference ask for help at the Information Desk in the library lobby.

Luncheons: Luncheons will be served cafeteria style from 12 noon through 1 P.M. Special programs of a social nature will be provided.

Coffee Hour: Coffee hours will be provided each afternoon in a college center. This will provide an opportunity to meet friends and relax between sessions.

ADVANCED REGISTRATION AND RESERVATION FORM

Fill out completely and mail to Mr. David Johnson, Public Relations Office, St. Olaf College, Northfield, Minnesota, as soon as possible.

Mr, Miss, Mrs _____
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Registering as: Member of NCTM _____; MAA _____; Student _____ Elementary Teacher _____ Exhibitor _____; Non-Member _____

Members of families and friends of those attending the meetings are welcome at all social and recreational events.

Registration Fee: $.50 or $1.50 _____

Room Registration:

_____ Myself only. I wish to room with _____

_____ Myself and wife or husband _____

_____ Myself and the following members of my family. Give ages of children _____

_____

_____

Encircle days room is desired. August 20, 21, 22, 23

See over

News Notes

(Continued from page 353)

Members of the **Association of Mathematics Teachers in New England** at their March 10 meeting were provided with interesting teaching materials as a result of visiting twenty exhibits consisting of a game or puzzle based on a principle of mathematics. The procedure for visiting the exhibits was governed by mathematical formulas governing the selection of different partners at each table. Typical exhibits involved playing games of three-dimensional tick-tack-toe, piecing together various Geometric Dissections and Mystic Pyramids, construction of geometric figures using the 7-pieces of a tangram set, making and solving magic squares, working with Moebius strips, and making geometric "faces." The program was supervised by quiz-masters William R. Ransom of Tufts College, Henry W. Syer of Boston University, and Mrs. Isabel Savides of the Newton Public Schools.

Charles H. Butler, Professor of Mathematics at Western Michigan College at Kalamazoo, will be one of the members of the guest faculty for the 1951 Summer Session at the **Colorado State College of Education at Greeley.** Dr. Butler will participate in a workshop for mathematics teachers during the pre-session period, June 11–22 and will teach classes during the regular eight-weeks session, June 22–August 17.

New York University's Department of Mathematics Education of the School of Education is offering the following courses during its session from July 2 through August 10: The Teaching of Arithmetic, by Amanda Loughren; The Teaching of Junior High School Mathematics, by Harry Ruderman; The Teaching of Mathematics in the Senior High School, by John J. Kinsella; Teaching and Curricular Problems of College Mathematics, by A. Day Bradley; Research Investigations in Mathematics Education, by J. J. Kinsella; and Applications of Mathematics, by A. Day Bradley.

Films for Secondary Mathematics is the title of a 90 page mimeographed booklet which may be obtained for seventy-five cents from Professor Henry W. Syer, School of Education, Boston University, Boston, Massachusetts. It contains descriptions of most of the films now available as well as a summary of teachers' opinions which were obtained in a special research project at Boston University.

Mathematical Pie is the name of a new publication of the Mathematics Staff of the Gateway School, Leicester, England, consisting of puzzles and problems of special interest to students at secondary school level. The first issue which appeared in October, 1950, consists of 4 pages and is available at 1d. while the second issue of 12 pages which was published in February 1951 may be secured for 2d. from the editor R. H. Collins at the Gateway School, Leicester, England. Arrangements are being made for distribution of future issues through an agent in the United States.

Mr. A. H. Wheeler, a teacher of mathematics at Worcester High School and professor of mathematics at Clark University, died on December 19, 1950. He served as a member of the first Board of Directors of the National Council in 1920–21 and as a member of its Multi-Sensory Aids Committee in 1943–45.

President Truman has appointed 24 persons as members of the Board of Directors of the **National Science Foundation.** In addition to **Dr. Harold Marston Morse,** professor of mathematics at Princeton University, there are three physicists, four chemists, four biologists, three medical scientists, four educators, two engineers, one geologist and two industrialists on the Board.

(Continued from reverse side)

Evening Registration: Indicate the number of reservations desired for each

Picnic ($1.00) _____ Norwegian Dinner ($2.00) _____ Banquet ($2.50) _____

Group Registration: Be sure to state complete Group Number and leader. Remember, each study group meets *every* day of the conference.

Choice:	*Monday*	*Tuesday*	*Wednesday*
First	_____	_____	_____
Second	_____	_____	_____
Third	_____	_____	_____

Laboratory Section Registration: (Register for only one section)
Each group meets Monday, Tuesday, Wednesday

Arithmetic Section _____ Junior High School _____ Senior High School _____

Enrollments in study groups, discussion groups and laboratory sections will be made on the basis of the time of registration and the number enrolled will be limited.

Join the National Council of Teachers of Mathematics

Any person who is interested in the field of mathematics is invited to apply for membership in the Council which has for its object the advancement of mathematics teaching at all levels of instruction. Its activities include:

1. The publication of *The Mathematics Teacher* which appears monthly except in *June, July, August, and September.* The annual membership dues of $3.00 for individuals include a one year's subscription to *The Mathematics Teacher* beginning with the issue you may designate. The subscription price to others (libraries, schools, colleges, etc.) is $5.00 per year. Canada, 25¢ additional; foreign, 50¢ additional.

2. The publication of yearbooks. The only available yearbooks at present are
   The Third: *Selected Topics in Teaching Mathematics*
   The Fourth: *Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910*
   The Sixth: *Mathematics in Modern Life*
   The Eighth: *The Teaching of Mathematics in Secondary Schools*
   The Fourteenth: *The Training of Mathematics Teachers of Secondary Schools*
   The Fifteenth: *The Place of Mathematics in Secondary Education*
   The Sixteenth: *Arithmetic in General Education*
   The Eighteenth: *Multi-Sensory Aids in Teaching Mathematics*
   The Nineteenth: *Surveying Instruments—Their History and Classroom Use*
   The Twentieth: *The Metric System of Weights and Measures*

These may be obtained for $3 each, postpaid. Send orders and checks directly to Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York 27, New York.

3. The conducting of research through committees on problems at all levels of instruction. One recent report is entitled "Guidance Pamphlet in Mathematics." Copies may be obtained for 25¢ each or 10¢ each if ordered in lots of ten or more. Send orders and remittance to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C.

4. Cooperation with over forty affiliated groups which average three or four meetings annually. If there is one in your area, be sure to join it.

5. Holding at least four conventions annually. Watch *The Mathematics Teacher* for dates and locations.

If you are not a member, fill out the application blank below (or a copy of it) today and join with thousands of other mathematics teachers who are members of the only national organization devoted solely to the interests of mathematics teachers in Elementary and Secondary Schools, Junior Colleges and Teacher Education.

If you are a member, will you tell others, who are not members, of the work of the National Council and make the application blank below (or a copy of it) available to them for their use?

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I hereby apply for membership in The National Council of Teachers of Mathematics. I enclose $3.00 for my dues and would like to have my subscription to *The Mathematics Teacher* begin with the ..... issue. (If this is a renewal, please indicate.)
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K, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, others ..... Arith, Gen Math I, Gen Math II, Alg I, Alg II, Pl Geom, Sol Geom, Trig, Col Alg, Others ..... This application and your remittance should be sent to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C.

MATHEMATICS . . . ITS MAGIC AND MASTERY

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School of Education, New York University, New York
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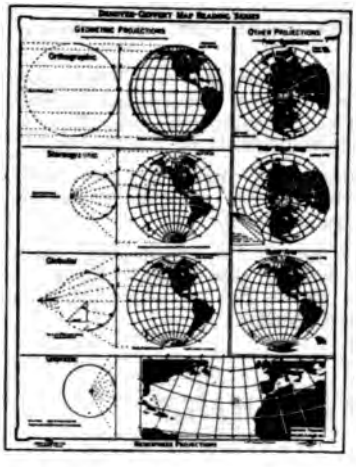

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-GFa Under glass, 3" frame . 12.50

MW2—Galileo

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THE MATHEMATICS TEACHER

Volume XLIV Number 6

A Mathematics Program for the Able*

By G. BALEY PRICE
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1. *Introduction.* In 1946 Herbert Hoover [10, pp. 436–437] said, "It is dinned into us that this is the century of the common man. . . . But if we are to have leadership in government, in science, in education, in the professions and in the home, we must find and train some *uncommon* men and women." The able and the gifted—once the primary concern of our schools, colleges, and universities—are now largely a forgotten group as a result of the growth of mass education. There are encouraging signs of improvement, however. Last year the Educational Policies Commission issued a report [6] entitled *Education of the Gifted* which considers the gifted student and his importance to society, and which recommends ways and means of identifying and educating the gifted. The present paper may be considered an effort to examine and interpret, in the special field of mathematics, the general conclusions and recommendations of this report. The paper begins with a background of facts; recommendations follow.

2. *A need for mathematicians.* The nation is desperately in need of mathematicians for its defense. Dr. Curtiss of the National Bureau of Standards recently said [3]: "You will recall that it has been said that World War I was a chemists' war and that World War II was a physicists' war. There are those who say that the next world war, if one should occur, will be a mathematicians' war." The instruments of modern warfare—the atomic bomb, radar, guided missiles, aeroplanes, and so on—are so complex and expensive that mathematical analysis must be used to the utmost in their design, development, and production. Mathematicians are required also for waging war even after the equipment has been built. *Operational research* or *operations analysis*, which has been defined [1, p. 4] by Dr. Kittell as "a scientific method for providing executives with a quantitative basis for decisions," and which came into being during World War II, employed many mathematicians and is seeking more at the present time. In order to secure sufficient mathematical talent to carry on World War II, most of the universities were stripped even of their pure mathematicians, and fundamental research and the training of young mathematicians were stopped.

But mathematics is needed also in time of peace. Dr. Fry [7] in 1940 traced the applications of mathematics in the communications industry, in the petroleum industry, and in several others; and the demand for mathematicians in industry

* A paper read at the 29th Annual Meeting of The National Council of Teachers of Mathematics in Pittsburgh, Pennsylvania on March 31, 1951.

Numbers in square brackets refer to the bibliography at the end.

is much greater now than it was then. The Industrial Mathematics Society was founded in May, 1949, and had attained a membership, largely in the Detroit area, of 200 by October, 1950. There are important applications of statistics to quality control, sampling procedures, and to other problems in the industrial field. Statisticians are needed for important work in the social and biological sciences, including economics, psychology, medicine, agriculture, and so on. Important mathematical work is being done in the field of biology already [16, 24], and there are need and prospect for much more. Rafferty, a biometrician, recently said, [15, p. 549] "Biologists should become more theoretical; mathematicians should become more biological."

3. *The present situation: a shortage of mathematicians.* Although there is a great need for mathematicians and statisticians, there is also a great shortage of both. An article in *Fortune Magazine* stated [18, p. 107]:

"There is at present an acute shortage of scientists. The shortage is sharper in some branches (physics, mathematics) than in others (geology), greater in some areas (basic research) than in others (applied or industrial research). But the shortage is widespread and unique in U. S. history, a product of wartime draft policies that for the duration practically halted the creation of young scientists, and a postwar expansion in scientific research beyond all expected bounds."

Although the United States occupies a leading position in science at the present time, it is well to remember that Europe held the lead up to World War II. The truth of this statement is well indicated by the record of the awards of Nobel Prizes. A recent summary [8] shows that, of the 124 prizes (to 158 persons) awarded in physics, chemistry, and physiology (there is no Nobel Prize in mathematics) up to the end of 1949, 20½ (29 persons) went to the United States while 33½ (38 persons) went to Germany, 20 (28 persons) went to Great Britain, 11½ (16 persons) went to France, and the remainder were scattered among numerous smaller countries. It is well known that the United States was almost wholly dependent on England, France, and Germany for fundamental research in applied mathematics up to the beginning of World War II.

Another aspect of our present position should be noted carefully. In spite of our present shortage of mathematicians and statisticians, it is generally agreed that the United States ranks first in mathematics in the world. But it must be remembered that the United States reached its present position of leadership only with the help of a large number of mathematicians who were imported from abroad. Professor Dresden [4] has listed the names of 130 mathematicians and mathematical physicists who came to the United States during the years 1933–1942. Many others came before 1933 or after World War II. There is no stockpile of mathematicians, and access to our foreign sources of supply might be cut off completely at any time.

4. *Highest ability required.* Perhaps you are puzzled. Why should there be a shortage of mathematicians when our colleges and universities are crowded, and when instruction in mathematics is given to hundreds and thousands? The answer is simple: in mathematics, as in other fields of science, it is only the exceptional and highly gifted individual who can be a creative scholar.

Conant said [2; 5, p. 413]: "Advance in science depends upon the number of first-class men engaged in scientific work, and no possible number of second-rate men will provide a substitute. . . . We must see to it that, as far as humanly possible, all the potential talent of the country . . . is recognized at an early age and given adequate opportunity."

An article in *Fortune Magazine* states [18, p. 108]: "The significance of these figures lies in the fact that there are all together only some 140,000 people engaged in scientific research, development,

and teaching in the U. S., and of these only about 25,000 hold doctors' degrees in the physical, biological, and agricultural sciences. The whole scientific and technological progress of the country, therefore rests upon a small fraction of 1 per cent of the population. And the doctorate group, representing the vanguard of science, could easily be assembled in one large hall. Strictly speaking, it is mainly the doctors who are the scientists, men who advance the frontiers of knowledge into the unknown."

Professor André Weil, a leading pure mathematician, has written [22, p. 304]: "Moreover, mathematical talent usually shows itself at an early age; and the workers of the second rank play a smaller role in it than elsewhere, the role of a sounding board for sounds in whose production they had no part."

Dr. T. C. Fry, writing on industrial mathematics [7, p. 270] said: "In other words, the mathematician in industry, to the extent to which he functions as a mathematician, is a consultant, not a project man. . . . He must be a man of outstanding ability. No one wants the advice of mediocrity. Among industrial mathematicians there is no place for the average man."

Finally, the universities receive many calls for mathematicians of the highest ability and extensive training (to the Ph.D. degree and beyond), but relatively few calls for holders of A.B. degrees with majors in mathematics.

5. *The young create mathematics.* It is well known that mathematics is created largely by the young. Weil's statement concerning the early appearance of mathematical talent has been quoted already. As examples, many will recall Abel and Galois, two of the really great mathematicians of the early part of the nineteenth century. Abel died at age 26 and Galois at age 20.

G. H. Hardy, one of the leading pure mathematicians of the first half of the twentieth century, has had much to say on mathematics and youth [9, pp. 10–13]. He asserted that, although he still had many good ideas at age sixty, he no longer had the strength necessary to develop them and carry them through to completion.

An article in *Fortune Magazine* states [18, p. 111]: "If the most creative years of a large number of scientists are charted on a graph, the peak comes about age thirty-two. But this, too, varies with the sciences. Mathematicians generally bud early, often before twenty-five; physicists come later, between thirty and forty; and biologists, whose science depends on a mass of patient observation, usually are most productive late in life."

Finally, it must be pointed out that, in a large majority of cases at the present time, the training of a mathematician in the United States begins in his senior year in college or in his first year as a graduate student, that is, at age twenty-one or twenty-two. In most cases the student takes no course before this time which is an adequate test of either his interest or his ability in mathematics. A reading knowledge of French and German is required for successful work in mathematics, and a reading knowledge of Russian is now highly desirable. In a large majority of cases, the beginning graduate student in mathematics in the United States is unable to read any foreign language.

6. *What is the mathematicians' apology?* In the effort to teach mathematics to the gifted, and especially to develop creative mathematicians, it will be helpful to understand the reasons which lead people to undertake a serious study of mathematics and to devote their lives to mathematical research. Some information is available, but it is surprisingly difficult to give an answer that satisfies even the mathematicians.

There are at least a few who become mathematicians because, they say, they find that mathematics offers an easier means of earning a living than any other field they know. But the meager salary of

the mathematician [18, pp. 109–110] cannot be considered an inducement to devote one's life to mathematical research.

Professor Henry D. Smyth, author of the report on the atomic bomb and member of the Atomic Energy Commission, has said [17, p. 427]: "The primary motive that leads men to study the behavior of nature is intellectual curiosity, a very personal desire to understand the 'how' of things."

In discussing the question "What makes a scientist?", the article in *Fortune Magazine* [18, p. 111] says: "One explanation is that science is one of the true vocations—a calling—and the best scientists become scientists because they cannot help themselves. By far the leading motive is the freedom offered by the sciences for the constructive exercise of the mind upon the material world, a motive that received first mention by 53 per cent of scientists surveyed for the Steelman report. Science is probably the only profession that not only encourages radical thought, in the sense of an experimental search for new principles below the surface of the accepted, but reserves its highest honors for radical ideas. Nearly 50 per cent of all scientists surveyed, regardless of where employed, picked university work as ideal because of its greater freedom."

But to these statements it is necessary to add another. G. H. Hardy, himself a mathematician of outstanding accomplishments, wrote a small book [9] to explain why he devoted his life to mathematical research. His explanation has not satisfied many people. Recently Sir Edmund Whittaker, also an important mathematician, has written [23, p. 42] as follows:

> A small book published in 1940 by Hardy under the title *A Mathematician's Apology* attracted a great deal of attention. "Why," he asked, "is it really worth while to make a serious study of mathematics? What is the proper justification of a mathematician's life?" Hardy's own answer was that "if a man has any genuine talent, he should be ready to make almost any sacrifice in order to cultivate it to the full," and he justified his choice of a career by his consciousness of possessing great mathematical ability. But the critics asked: "If Hardy had been conscious of great agility in climbing drainpipes, ought he to have become a cat-burglar? If he had found that he had a great attraction for the other sex, ought he to have set up as a Don Juan?" The fact is that the problem he proposed really belongs to moral philosophy, and Hardy, who was a stranger to moral philosophy, had no principles available to deal with it. Curiously enough, I do not remember ever having seen a sustained argument by any author which, starting from philosophical or theological premises likely to meet with general acceptance, reached the conclusion that a praiseworthy ordering of one's life is to devote it to research in mathematics. Whatever the philosophical situation may be, it must be admitted that the 20th century would not have been the same without its mathematicians.

7. *A mathematics program for the gifted.* The above facts will now serve as a background for certain specific recommendations concerning the education of the gifted in mathematics. The "highly gifted" are understood to be [6, p. 43] the top one per cent with respect to intellectual capacity (that is, roughly, individuals with an I.Q. above 137); similarly, the "moderately gifted" are understood to be the individuals who fall within the top 10 per cent but below the top one per cent (that is, individuals with an I.Q. between 120 and 137). The recommendations may be grouped under five headings as follows:

(a) Identify the gifted student and, in particular, those with special ability in mathematics.

The student who is highly gifted in mathematics is usually "highly gifted" in the sense defined above. Thus the first instrument to be used in searching for the mathematically gifted is the intelligence test. The second instrument to be used is a competitive mathematics examination which has been designed to locate mathematical ability. The first of these instruments is well known, but the second probably is not.

There is convincing evidence by this time that students of high intellectual capacity—students who will far outstrip their fellows in their later intellectual and professional accomplishments—can be suc-

cessfully identified early [20]. There is evidence [5] that the efforts to select those superior in scientific aptitude have been successful. Finally there is evidence that the creative mathematician can be identified with considerable success at least by the time he graduates from high school. Professor Tibor Radó, reporting [14] on the mathematical competitions held each year for determining the winner of the Eötvös-prize founded by the Hungarian Mathematical Society in 1894, has written as follows:

> Is it possible to test by problems creative mathematical ability? Certainly it would be possible to prove the impossibility of such an attempt on the basis of psychological theories. First of all, a boy can be really able, and still be unable to solve anything in four hours, in strange surroundings. There is the great danger of discouraging this type. There are many other objections; it might be interesting, but certainly futile, to discuss them, simply because the experience of almost forty years shows conclusively that this mathematical competition *could* be made a success.

It is in order to report that three competitive mathematical examinations are held each year in the United States. The oldest of these is [21] the William Lowell Putnam Mathematical Competition, for college students, conducted by the Mathematical Association of America, and held each year since 1938 except for an interruption of three years during World War II. The next is [19] the Stanford University Competitive Examination in Mathematics, for high school seniors, which has been held each year since 1946. The newest addition to the list is the Mathematics Contest of the Metropolitan Section (of the Mathematical Association of America), for high school students, which was held for the first time in 1950. These competitive mathematical examinations are serving a useful purpose, and more high school and college students should be given an opportunity to compete each year.

(b) Educate the mathematically gifted to the limit of their abilities—in spite of the draft or war—for the sake of the safety and welfare of the nation.

Mathematical ability is a scarce item, and an adequate supply is essential for the defense of the nation. During World War II the training of scientists was essentially stopped because all students were drafted, and the nation is suffering the consequences now. The countries of Europe, where military conscription has been practiced for many years, have always been more foresighted. In Russia, in China, in Finland, in England, and in other countries of Europe the gifted student was educated even during World War II—not drafted.

Our policies concerning the drafting of students and universal military service are again the subject of much debate. There has been support by President Conant of Harvard, the American Association of Universities, the Department of Defense, General Eisenhower, and the Senate for a plan which would include college training for perhaps 75,000 selected students annually. Because of the present scarcity of mathematical talent and the long time required for its development, because mathematics is an activity of youth, and because our only hope of overcoming the superior numbers of our enemies is by means of superior skill and ability, a provision to continue the training of mathematicians and other scientists seems absolutely necessary.

Dr. Henry D. Smyth, speaking to the American Association for the Advancement of Science at its meeting in Cleveland on December 28, 1950, recommended the formation of a Scientific Service Corps for scientists of military age and of a Student Science Corps for the scientifically promising student. The Student Science Corps would enroll science students of high intelligence and promise in their freshman year. They would stay in through graduate school training if they were intelligent, imaginative, original, and good performers. Others who were only technically competent would be released to the military and civilian life after finishing college, and students would be "continually

weeded out." This Corps would be under the general jurisdiction of the National Science Foundation.

Dr. Smyth's recommendation recognizes the obligation of all to serve their country in time of need, but adds the important principle that the gifted, like all others, should serve in the position in which they can make their greatest contribution.

The National Science Foundation is now in existence. Through scholarships and other aids, it will undoubtedly lend valuable assistance to carrying out the recommendation to educate the mathematically gifted to the limit of their abilities.

(c) Counsel all students, including the mathematically gifted, to study the mathematics they will need in their later work.

This recommendation may appear to be trivial, but there is reason to believe that it is one of the most important of the five. Many students, upon entering college, are surprised to learn that their high school preparation in mathematics is inadequate. Some report that their schools did not give the necessary courses. Others say, "But nobody told me that I would need these courses."

No better specific recommendations (even for the gifted in mathematics) can be given than those in *Education of the Gifted* [6, pp. 63–64]:

> For these reasons we recommend that there be included in the secondary-school program of nearly every *highly* gifted student the following:
>
> (1) A *foreign language;* studied for a long enough time and with sufficient intensity to achieve, at least, reading mastery.
>
> (2) Advanced *mathematics;* certainly through advanced algebra, probably through trigonometry, possibly through calculus.
>
> (3) Additional study of the *social studies,* with emphasis on history, beyond the amount taken by the typical high-school student.
>
> It has been said that "nearly every highly gifted student" should follow the program outlined above. There are two exceptions. (1) The youth who is highly gifted in music or art as well as in general intelligence may safely omit advanced mathematics but not social studies or foreign language. (2) The highly gifted youth who suffers from physical disability or from social or emotional maladjustment in adolescence to the extent that intensive academic work for him in high school may be either impossible or undesirable should have his high-school program modified accordingly.
>
> With these exceptions, we recommend the above program for *all* highly gifted youth. We say this with a full realization that some of them may have relatively little use for some parts of the program in their later study and life work. Thus, the young man or woman who studies law or public administration may make little use of his mathematical training and the research chemist may have little use for a broad background in history (as far as his professional work is concerned). But, because all avenues for later specialization should be kept open for the gifted youth until he is in his upper years of college, he should not be limited in his choice by lack of previous preparation. It is for this reason that we recommend the full program for all highly gifted youth. The discredited claim that study of foreign language and mathematics are valuable because they "discipline the mind" has no bearing on the matter.
>
> There are two further and related considerations. In the first place, the highly gifted student who has completed a substantial amount of advanced academic work in high school, along the lines here recommended, has a better basis for choosing his area of specialization in college and he can make a better decision on this matter at an earlier stage in his college work. In the second place, he is spared the necessity of taking valuable time from his more mature student years to equip himself with the tools and basic information that are essential to advanced study and to creative achievement in scholarship and the intellectual professions.
>
> Many *moderately* gifted students could also profit from more social studies, advanced mathematics, and foreign language in high school; but in their case the need is neither so clear nor so compelling as it is in the case of the highly gifted.

(d) Stimulate the interest of the mathematically gifted by means of good teaching, competitive mathematical examinations, and recognition of high accomplishment.

An important function of the teacher of the mathematically gifted is the stimulation of interest. Even the most gifted student will be repelled if he encounters only trivial courses and dull teachers. The teacher must have, above all else, a knowledge of, and a genuine interest in, mathematics.

The stimulation of interest is probably more difficult in mathematics than in the other sciences. One of the reasons is that

modern mathematics, including the modern applications, is almost unknown to high school teachers of mathematics. The high school student can build radios and model airplanes, operate television, and so on, but he is fortunate if his progress in mathematics carries him as far as the calculus—a subject now almost three hundred years old! Even the college textbooks on calculus would look quite familiar to the student of a hundred years ago. How different from the situation in physics, chemistry, engineering, medicine, and so on! The same differences exist in the publicity concerning current developments in the various fields. The newspapers and magazines usually find mathematics not understandable and hence gently ridicule it. The treatment given the other sciences is very different. It is worth calling attention to one exception in the treatment of mathematics: recently the *Scientific American* has published a number of significant articles on mathematics. The lack of publicity and popular knowledge of mathematics certainly places an additional obligation on the teacher. It is recorded [18, p. 112] that "many [chemists] were originally lured into the science by two books—*The Romance of Modern Chemistry* and Stokley's *Science Remakes Our World*—which for many years were about the only scientific books available in small-town libraries."

Lacking some of the aids available in other fields, the teacher of mathematics can nevertheless employ the stimulation of competition. Large numbers of students [11, 19, 21] enter the competitive mathematical examinations that are held each year in the United States, and it seems certain that they help to increase the number of students who make a serious study of mathematics.

Finally, recognition should be given for outstanding achievement in all fields, including mathematics. Every community in the nation honors its athletes, and considerable recognition is given to students in music and art. Students in mathematics are usually ignored. Recognition may be based on performance in a competitive mathematical examination (as it usually is in athletics and music), and at least one examination [11] appears to have been designed so that it gives recognition to a large number of participants.

(e) Instruct, using the best teachers, the most approved courses, and all the care and solicitude possible in each individual case.

Special provision should be made for the mathematically gifted whenever possible. Remembering that mathematics is an activity of the young, and that some investigators [20] find *acceleration* to be quite desirable in the case of the gifted generally, careful consideration should be given to this provision in the case of a student who shows promise in mathematics. At least in the later years of high school, *grouping* on the basis of ability seems highly desirable; the instruction that is required for the gifted is not appropriate for the average student. The grouping will probably be brought about to a considerable extent by means of the *elective courses* which have been recommended for the gifted, and which will not be taken by the average student. *Enrichment* should be employed whenever possible. The full development of the gifted requires a great deal of attention to the individual.

The mathematics teacher of the mathematically gifted should know mathematics; a knowledge of teaching methods is not sufficient. This fact places an obligation on the colleges and universities to train good teachers. The university training of a teacher should awaken an appreciation of the beauty of mathematics, include courses which are not trivial and dull, and involve contact with some phases of modern mathematics. The usual teacher preparing to teach in high school takes the standard sequence through calculus and differential equations and then subjects such as modern synthetic geometry, theory of equations, and so on, because these

courses concern the subject matter of high school courses. Modern mathematics remains beyond the senior year, and the prospective teacher makes no contact with it unless he takes graduate work. It is a surprising fact that the high school and college courses in mathematics have remained essentially unchanged for a hundred years. It is possible that better and more modern courses could be devised. The University of Chicago has tried [12, 13], but it is too early to appraise the results.

Finally, the teacher of the mathematically gifted will fail if he stresses only the *usefulness* of his subject. The young are full of curiosity and easily attracted by something that is interesting and elegant. The high school student is much less attracted by an opportunity to take up the world's burdens. Hardy has said [9, pp. 59–60]: "It is not possible to justify the life of any genuine professional mathematician on the ground of the 'utility' of his work." The teacher of the mathematically gifted must not forget to present the aesthetic side of the subject.

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Misconceptions about Rationalization in Arithmetic

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WITHIN recent years the doctrine of meaning in arithmetic has received wide recognition. Much professional writing has been done in an attempt to translate sound theory into effective practice. Unfortunately, however, certain aspects of the basic doctrine have been misinterpreted. The rationalization of computational processes or skills is an important case in point.

All who subscribe to the "meaning theory" of instruction place considerable emphasis upon rationalization, and rightfully so. But all persons do not agree on its specific role as part of a broad instructional program. For example, disagreement has been observed in regard to the temporal relation between the *how* of a computational process or skill and its *why*. With apology one may ask: "*How-why*, or *Why-how*?" That is the question.

THE *How-why* POINT OF VIEW

There are numerous persons who advocate exclusive adherence to a *how-why* sequence: the *how* of a computational process or skill must precede the *why*. One of the most emphatic statements of this viewpoint was given a short time ago by Johnson[1] in THE MATHEMATICS TEACHER. Although it may be dangerous to quote out of exact context, the writer feels that no injustice is done in citing the following statements from the Johnson article.

> The *how* of any process should be fully understood before rationalization.
>
> The *why* cannot be given before the *how* a thing is done is known because the step which we are trying to tell the reason for in a process must be known before the reason for it can be given.
>
> . . . a rationalization of a process in arithmetic is meaningless unless the *how* to do that process is understood first.
>
> Teaching the rationalization of a process before the *how* of the process is fully understood may confuse rather than clarify the process.

These statements, then, typify the basic position of persons who would answer the question, How-why or Why-how?, with an emphatic "Teach the *how*, then the *why*." This becomes an instructional policy to be followed without exception.

The present writer is not at all certain that the *how* of a process or skill must necessarily precede the *why*. In fact, a *why-how* sequence is both possible and desirable in many instances. An acceptance of the *how-why* sequence as a policy to be followed without exception is dangerous—not because of what it does, but because of what it fails to do. Unless a change is made from *how-why* to *why-how* in connection with numerous skills, an important aid to learning will be lost.

THE *Why-how* POINT OF VIEW

The skill of carrying in addition may be used to illustrate the feasibility and desirability of a *why-how* temporal sequence. Let us assume that this new skill first is encountered when 27 must be combined with 38.[2] *With a background of previous meaningful instruction* an attack upon this quantitative situation will, rather naturally, follow lines similar to the following:

$$\begin{array}{rl} 38 = & 3 \text{ tens} + 8 \text{ ones} \\ +27 = & 2 \text{ tens} + 7 \text{ ones} \\ \hline 65 = & 5 \text{ tens} + 15 \text{ ones, or} \\ & 6 \text{ tens} + 5 \text{ ones, or } 65 \end{array}$$

[1] J. T. Johnson, "What Do We Mean by Meaning in Arithmetic?" THE MATHEMATICS TEACHER, XLI (1948), 362–367.

[2] Some textbooks and courses of study unfortunately introduce carrying with an example having a 2-digit addend and a 1-digit addend, which truly is a higher-decade combination with bridging and does not necessitate carrying.

Undoubtedly, representative materials will be used by teacher and children to dramatize this and similar examples as previous experiences and understandings are applied to the new situation. In other examples United States money may be used to advantage. The accompanying verbalization will be in terms of "dimes" and "pennies" rather than "tens" and "ones."

At first the written algorism serves only as a symbolic record of a particular quantitative experience. Then, as a result of a variety of meaningful experiences of this nature, many children very frequently discover and formulate for themselves the more mature and efficient way to find the sum from the algorism itself—i.e., by carrying. The common crutch, indicated below, may or may not be suggested by the children:

$$\begin{array}{r} 1 \\ 38 \\ +27 \\ \hline 65 \end{array}$$

Regardless, the *how* of carrying has been discovered as a consequence of experiences associated directly with the *why* of that skill.

A comparable type of rationalizing activity is useful in connection with numerous other skills such as borrowing in subtraction or carrying in multiplication. In each instance the child learns *how* to formulate the preferred algorism through meaningful experiences which are related to the *why* of that algorism. In each instance *why* leads to a *discovery* of *how*. Thus for many skills, but not necessarily all, the *why-how* temporal sequence is definitely feasible. And not only is it feasible; it is desirable as well.

The Value of the *Why-how* Sequence

The important distinction between the products of learning and the process of learning is pertinent in this connection. Brownell[3] described this contrast and its implications very clearly some years ago. More recently, Anderson and Gates[4] have written as follows:

> By *process* is meant the way in which the learner operates in order to attain certain learning products. Process refers to the *way* in which one learns. But it also has a larger significance because process, a way of learning, itself becomes established, and a particular way of learning with its consequent meaning for a way of attacking new problems is often as important to an individual as a particular product. . . . It seems sound that the school should attempt to stimulate learning through processes which are meaningful, insightful, or problem-solving in character, because this kind of learning behavior is especially valuable for children and adults in our society.

Now consider the child who first encounters a subtraction such as 40−27. At this initial stage for the skill in question, *process of learning* should be of primary concern. Confronted with a new situation for which he has no immediately satisfactory response, the child may make a variety of "provisional trys." He may utilize the skill of counting; he may work with the actual objects described in the basic quantitative situation; he may resort to the use of some semi-concrete or representative materials; he may work with the number symbols themselves. In any event, these types of response will differ in relevancy, maturity, and efficiency, depending largely upon the background of related experience which the child may bring to this new situation. Less relevant, mature, and efficient responses will be eliminated as they lead to more acceptable ones. If a background of meaningful experience is brought to the new situation, it is not at all uncommon to find children discovering the mechanics of the skill of borrowing in a rational way with a minimum of irrelevant and imma-

[3] William A. Brownell, "Psychological Considerations in the Learning and the Teaching of Arithmetic," *The Teaching of Arithmetic*, Chapter I. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935.

[4] G. Lester Anderson and Arthur I. Gates, "The General Nature of Learning," *Learning and Instruction*, pp. 27–28. Forty-ninth Yearbook of the National Society for the Study of Education, Part I. (Chicago: University of Chicago Press, 1950.)

ture "provisional trys." Thus it is seen that, at this initial learning stage, *why* may lead to *how*. In so doing, children gain invaluable experience in a process, or "way of learning," which will serve them well in their attack upon new quantitative situations now and later.

If the sequence is reversed, if why *follows* how (sometimes by days or months or years), it becomes increasingly difficult for initial learning experiences to be most effective in the manner described. If first we tell a child *how* to borrow in subtraction, but defer the *why* until some future time, there is grave danger that initial learning may be largely mechanistic or authoritative. The advantages to be gained from the standpoint of learning *process* in the *why-how* temporal sequence frequently may be lost when the order is reversed. This loss to the child is one for which there is no compensation. Whenever it is possible to do so, let us gain the advantage of the *why-how* sequence.

A Point of View Concerning *How-why*

Thus far, this writer has objected to the adoption of a *how-why* sequence as an instructional policy to be followed without exception. Further, an attempt has been made to show that, whenever possible, definite psychological advantage is to be gained by following a *why-how* temporal sequence. No contention has been made or implied that it is always feasible for *why* to lead to *how*. In some instructional situations it may seem virtually necessary to present the algorismic form of certain computational skills on the basis of a *how-why* sequence. In such instances, when *how-why* is selected as the course to be taken, let us be certain that the *why* is coupled with the *how* just as soon as possible or feasible. There is grave danger that *why* may follow *how* at such temporal distance that ultimate rationalization is minimized in effectiveness.

Anderson and Gates[5] make the following statement in their discussion of Transfer of Learning.

> It seems clear that learning which is cognitive in process and control is more likely to have varied utility than learning which is mechanically associative. The most brilliant illustration of this concept is found in the development of the "meaning theory" for teaching arithmetic.

If learning is to transfer most effectively and have most varied utility, if learning is not to be "mechanically associative," it is urgent that *why* be coupled as closely as possible with *how*. In the event that the *how* of a skill does precede the *why*, let us see that rationalizations follow without unnecessary loss of time.

In a recently published research monograph Brownell and Moser[6] submit clear-cut evidence regarding the relative transfer values when the skill of borrowing in subtraction was taught meaningfully and mechanically. When subtraction examples such as $43-18$ were clearly understood and rationalized by children, there was significantly greater transfer to untaught examples such as $428-263$ than when the initial instruction had been "mechanically associative." Thus, subsequent sub-skills in borrowing were learned and taught more easily and effectively when the initial instructional experiences were rational and meaningful. Similar conditions may well exist with other skills and sub-skills of other processes. The more we defer rationalization, the less effectively may we utilize transfer of learning and the greater becomes the danger of learning and instruction which is "mechanically associative."

It has become almost axiomatic to aver that drill should *follow* understanding. So long as rationalizations are withheld, understanding will be incomplete or lacking. If we present the *how* of a skill and withhold the *why* for an unnecessary length of time, there is every possibility that drill will *precede* understanding. The dangers of premature drill are familiar to all. Let us minimize these dangers by linking *why* with *how* as soon as possible.

[5] *Ibid.*, p. 29.

[6] William A. Brownell and Harold E. Moser, *Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction*. Duke University Research Studies in Education, No. 8. (Durham: Duke University Press, 1949.)

An Unfortunate Criterion

Frequently in the preceding section of this discussion the writer has stated and implied that rationalization should follow *how* "as soon as possible." There is an indefiniteness about such an emphasis that is not appealing to some persons. Rather, they would prefer a more specific criterion for determining and stating the proper time for rationalization. It is most unfortunate that in their search for such a criterion some persons have turned to the mental age concept.

Undoubtedly the reader is familiar with the work of the "Committee of Seven"[7,8] relative to the placement of arithmetic topics in terms of mental age levels. More recently, Brueckner and Grossnickle[9] have advocated a similar basis for the gradation of arithmetic processes. Finally, the following statement by Johnson[10] extends the mental age criterion to cover the matter of rationalization:

> Research would have to lead the way showing at what mental age the various arithmetic processes could be rationalized.

The reader certainly is familiar with the significant critiques of the "Committee of Seven's" work, such as those by Brownell.[11,12] These comments are equally apropos to the matter at hand: the use of mental age as the prime factor in "readiness" for rationalization. A more recent statement by Brownell and Moser[13] epitomizes the sane position to take:

> If children have the necessary ability, knowledge, and understanding, they are 'ready.' And their 'readiness' is determined, not by their age or by their grade, but by the kind of arithmetic they have had. By manipulating their arithmetic backgrounds we may shift the placement of any arithmetical topic over a range of several years and of several grades.

"Readiness" for rationalization is primarily a function of the type of quantitative experiences which children have had, and cannot be judged primarily in terms of the criterion of mental age.

The present writer feels it advantageous to call attention to further evidence, both experimental and theoretical, which shows the inadequacy and danger of the mental age criterion. First, consider Curr's[14] study of grade-placement. It is unfortunate that this scholarly piece of research has received so little publicity. In essence the study was analogous to that of the "Committee of Seven," and was concerned with the process of division by one- and two-digit divisors. The Table* on page 381 is adapted from Curr's material and shows minimum mental ages, as deduced by regression formulae, necessary to attain designated mastery standards in seven different schools.

Consider the data for divisors of 2 through 12. Notice that the minimum mental age ranged from 8−0 in one class to 10−4 in another class. For divisors 13 through 99, the range was from 10−9 to 17−7. In either case, which mental age shall be used as the minimum? The lowest? The highest? The average? In this connection Curr has stated:

> It is immediately evident . . . (from the Table) . . . that the ability to derive any specified amount of profit from instruction in an arithmetic topic is not a universal phenomenon which occurs at the same mental age in every child's development, irrespective of the condi-

[7] *Report of the Society's Committee on Arithmetic.* Twenty-ninth Yearbook of the National Society for the Study of Education, chap. XIII. 1935.

[8] *Child Development and the Curriculum.* Thirty-eighth Yearbook of the National Society for the Study of Education, Part I, chap. XVI. 1939.

[9] Leo J. Brueckner and Foster E. Grossnickle, *How to Make Arithmetic Meaningful,* chap. III. (Philadelphia: The John C. Winston Co., 1947.)

[10] *Op. cit.,* p. 366.

[11] William A. Brownell, "Readiness and the Arithmetic Curriculum." *Elementary School Journal,* Vol. 38, pp. 344–354, 1938.

[12] ———, "A Critique of the Committee of Seven's Investigations on Grade Placement of Arithmetic Topics," *Elementary School Journal,* Vol. 38, pp. 495–508, 1938.

[13] *Op. cit., pp.* 156–157.

[14] William Curr, "Placement of Topics in Arithmetic," *Studies in Arithmetic,* Vol. II. Publications of the Scottish Council for Research in Education: XVIII, pp. 183–218. (London: University of London Press, 1941.)

* Reprinted by permission from the Scottish Council for Research in Education and the University of London Press.

Divisors	Mastery Standards	Minimum MA's by School Classes						
		#1	#2	#3	#4	#5	#6	#7
thru 12	83%	9–5	9–5	9–8	8–0	9–1	10–4	9–0
thru 99	56%	11–3	10–9	17–7	13–10	11–6	12–10	12–7

under which he is taught. . . . it would ar that to derive a minimum age for a from the results of a large number of s, taught, no doubt, by methods and by ers ranging from the supremely efficient e hopelessly inefficient, and then to impose minimum mental age on all teachers, ir- ctive of their own methods and efficiency, uivalent to . . . (having) . . . the superior er reduce his work to the mediocre level ed from the combined results of all meth- nd types of teaching, from the best to the .

strong statement by Curr points ly to the unsoundness of any attempt et minimum or optimum points of ement for factors such as "rational- on readiness" in terms of *mental age.* final objection, more theoretical in re, may be raised to the use of mental o determine the point of rationaliza- for any process or skill. Within recent s men such as Wechsler have re- ced completely the concept of mental as a truly satisfactory basis for the aisal of intellectual capacity.[15,16] The wing statement from the WISC Man- is most appropriate.

al age is considered to represent an abso- evel of mental capacity, so that an MA of ans the same irrespective of whether it is ned by a child of 6, 10, or 16. If all that is ed by this statement is that subjects of ent ages can obtain the same test scores, rue but so obvious. But if one also under- s it to mean that their intelligence levels lentical, the statement at best is an assump- which has to be proved. In point of fact, clinical and statistical evidence is clearly st such an assumption. Instructors in psy- etrics have devoted many hours to teach- heir pupils first that the MA is a *measure ntal level* (an MA of 7 is a mental level of ll children attaining this rating) and then teaching them that this is not true after all (a 5-year-old with an MA of 7 has not the same kind of mind as a 10-year-old with an MA of 7).

This is but one of several reasons cited by Wechsler for the rejection of mental age as an intermediate step in the determination of IQ. This same reason throws serious doubt upon the use of mental age as a major determiner of placement for a given rationalization. For example, assume that an MA of 9—0 has been forwarded as the optimum point for rationalization of an arithmetic skill. Two children having MA's of 9—0 may have respective CA's of 8—0 and 10—0, for example. Yet, as Wechsler has stated, these two children are not similar in the sense of having the "same kind of mind." And it is highly questionable whether they are similar from the standpoint of "readiness."

The MA concept is far from being as absolute as we may unconsciously assume it to be. As a consequence, serious doubt is thrown upon the use of mental age as the primary determiner in connection with arithmetic readiness and with placement problems in general.

Recapitulation

In this discussion the writer has attempted to deal with several misconceptions concerning rationalization in arithmetic. Exception was taken to the exclusive adherence to a *how-why* temporal sequence. Advantages of the reverse sequency, *why-how*, were presented. Further, in those instances in which the *how-why* sequence may be the more feasible one to use, attention was directed to the importance of coupling *why* with *how* just as soon as possible. Finally, discussion centered around the unsoundness of using *mental age* as the primary criterion for determining readiness for rationalization.

David Wechsler, *The Measurement of Intelligence*, chap. 3. (Baltimore: Williams Wilkins, 1944.)

———, *Manual: Wechsler Intelligence for Children.* (New York: The Psychologi- orporation, 1949.)

Ibid., p. 2.

The Numeral Frame—A Device for Use in the Teaching of Counting

By DONNA NORTON
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THE numeral frame is a development from the old-fashioned bead counter, or abacus, which for a long time was considered essential for teaching beginners in arithmetic. The bead counter was a device small enough for a child to hold in his hand, while the numeral frame, as shown by Figure 1, is large enough for demonstrations before a class. In the case of the bead counter, the movable beads were permanently attached to the device, while in the numeral frame, the blocks may be removed. The beads were not numbered, but the blocks used with the numeral frame are numbered from one to one hundred. The number symbols on the blocks are large enough to be read by every child in a class of ordinary size, and it is this feature which largely determines the practical size of the numeral frame.

As shown by Figure 1, the numeral frame consists of a structure about 5 feet in height, and 2½ feet in width, in which there are ten horizontal rails. The ten rails each have a projection, or tongue, on the upper side. Ten blocks, which have grooves to fit the tongue, are placed upon each rail. A practical size for the blocks is 2 inches long, 1 inch thick, and 1¼ inch high. The entire frame is mounted upon a base to hold it upright. The device is easily moved, and it may be placed before a class, or stood in a convenient place for use by the pupils.

The word counting, as commonly used, has two different meanings which should be carefully distinguished. When we are counting to place objects into an ordered sequence, as first, second, third, and so on, we are using number in its *ordinal* sense. When we count to determine the number of objects in a collection without reference to their order, we are using number in its *cardinal* sense.

In teaching counting, we should recognize that it is essential for the child to understand first the meaning of number in its ordinal sense, since the child must use the serial order of the number names in determining how many objects there are in a given group.

It must be recognized that mathematics is a conceptual structure in which the learning of ideas must take place in a definite order if they are to be understood. For example, the child can not understand multiplication, which is repeated addition, before he has grasped the meaning of addition.

The Numeral Frame

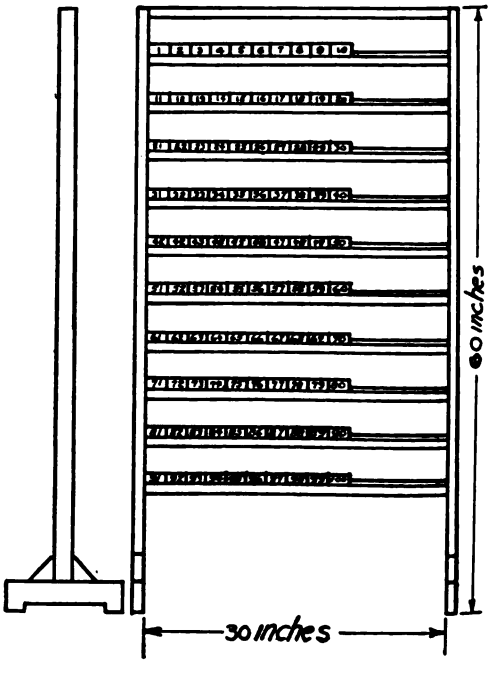

FIG. 1

Similarly, counting as a logical mathematical procedure must be mastered in a series of steps which may be designated as rote counting, enumeration, identification, reproduction, and comparison. In teaching, it is essential to preserve this order. Identification should precede reproduction because it is more easy to go from the concrete to the abstract, than from the abstract to the concrete.

The step of rote counting* is merely the learning of the number names either in a repetitive manner, or through rhymes. The object of this rote step is to give the child the number names in their correct serial order so that he may use them in making further steps. The names themselves are purely a matter of language, while the conception of serial order is mathematical. It is a matter of logical necessity for the child to develop this mathematical conception before he can use it for the next step, that of enumeration.

By enumeration, we mean using the number names to designate each successive object of a group which has been arranged in a serial order. In doing this, each object is recognized as an individual entity which should have a name of its own and a definite place in the series. Until rote ccunting is mastered, the child can not perform enumeration correctly.

When we have a collection of objects arranged in a series, we recognize that we can destroy the particular serial order by rearranging the objects. However, in doing this, we recognize that the group of objects has an aspect which is unchanged by the rearranging. The unchanged, or invariant, aspect of the rearranged series is the cardinal number of the objects. By identification in counting, we mean the determination of the ordinal number which serves to identify the cardinal number of the objects in the group.

In the above steps in counting, the child is first given groups of objects to study and enumerate. In the step of reproduction, the child is asked to perform the inverse and more difficult operation of forming the group of objects which corresponds to a designated cardinal number. The child does this by placing objects into a one to one correspondence with the ordered series of number names until he reaches the designated number.

In the step of comparison, two groups of objects are studied at once from both the ordinal and the cardinal points of view. For example, the child learns that a group of eight objects contains two more than a group of six objects. He also learns that one number stands between six and eight in the series of numbers.

By learning to count in the above senses, the child learns the names for numbers, and gains a clear understanding of the decimal system of notation, and of the meaning of number in both its ordinal and cardinal senses. Since both of these meanings are much used socially, he makes an important start toward understanding how adults use the number concept.

Counting also forms the essential foundation for learning the meanings of addition, subtraction, multiplication and division, all of which in the final analysis can be reduced to counting. A knowledge of counting enables the child to develop for himself the elementary facts used in our rapid methods for performing these operations.

The numeral frame was developed as a teaching device to use in every step in counting for demonstrating to the child, and in having the child discover the essential ideas in counting, and the meanings of addition, subtraction, multiplication, and division as developments from counting.

Rote counting, that is, learning the names for numbers in their serial order, may be taught by having the child move the blocks along the rails one at a time, and say the number names without placing any emphasis upon the serial position of each number. In this activity, the kinesthetic sense helps in establishing a cor-

* Herbert F. Spitzer, *The Teaching of Arithmetic*, Chapter III. (Chicago, Houghton Mifflin Co., 1948.)

respondence of the number names of the blocks.

Teaching enumeration or ordered counting, the next stage in the counting process, is accomplished with the numeral frame by having the child move the blocks along the rails and designating each block with its particular name. The arrangement of the blocks along the rails serves to emphasize their serial order. Having the number symbols on the blocks assists the child in forming the association of the numerals with the serial concept of number and with the number names.

The meaning of cardinal number, and its relation to ordinal number, which is termed identification, is taught by having the child move blocks one at a time to form a group. In doing this, the child says the name of each number as he moves the block into the group, and the last number named is the cardinal number of the group. Since number symbols are placed upon only one side of the blocks, and since the blocks can be turned around on the rails, the symbols may be, or may not be visible to the child. Both arrangements have advantages.

Reproduction of a designated number is accomplished by moving the desired number of blocks into a group, and the symbol upon the final block serves the purpose of verifying the child's counting.

In teaching the comparison of groups, the teacher shows the child how to form two different groups by placing the blocks into two groups upon the rails. Then the teacher shows how to compare the groups by adding blocks to a group and by removing blocks from a group. The teacher also shows the child how to compare the position of a number in the series of numbers. For example, if the group contains five blocks, the child recognizes that its number is two less than seven and three more than two.

The nature of our decimal system of notation is indicated in the numeral frame by having ten rails, and ten blocks for each rail. In this arrangement, as shown by Figure 1, numerals having the same final symbol form a column. The object of this arrangement is to make it easy for the child to see and understand the systematic character of our system of notation.

The numeral frame is also useful in exercises in which counting is used to obtain the sum, difference, product, and quotient of two numbers. The child, having mastered the step of reproduction, can produce groups of any designated number. Then he can put two groups together, and by counting determine the sum of the two. Similarly, he may form an arbitrarily designated group, and then remove from it a designated number, and in this manner determine the difference between two numbers. It is evident that since equal groups can be put together, the meaning of multiplication can be demonstrated easily. Starting with a number such as twenty-five, the teacher may show the child how to divide the blocks into five equal groups, and in this manner show the meaning of division.

From the above discussion, it will be seen that the numeral frame serves as a powerful means used by the teacher for communicating ideas about number to the mind of the child so that these ideas become perfectly clear and firmly fixed. The numeral frame also enables the child, through his handling of the blocks, to communicate his thoughts to his teacher, who can then judge what he has learned and what he has failed to learn. Therefore there is a closer meeting of minds than we can obtain without such a device.

The teacher who starts to use this device will quickly find that one of its important contributions to teaching is that it makes it easy to obtain and hold the close attention of an entire class. Moreover, children greatly enjoy using this device, and their joy motivates and gives purpose to their learning.

Secondary School Mathematics in Great Britain and the United States*

By WILLIAM S. BRACE
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1. PRIVILEGE OF SPEAKING

I am very conscious of the honor done to me and to the teachers of Britain by your president in asking me to address you. My qualifications for talking about American schools and educational methods are very slight indeed compared with those of many of you here today, so I am quite sure that it is not on those grounds that I am being asked to speak to you. Perhaps it is that someone is a little suspicious of the report that I am going to carry back to England, and is contemplating "bumping me off" if I haven't formed satisfactory impressions of this beautiful state of yours! For my own safety, then, I will say at once that I think that Colorado is one of the most beautiful places in the world, that the people of Colorado are among the most hospitable in the world, and that the schools of Denver, which are the only Colorado schools that I know much about, are among the best in the world.

2. GREETINGS FROM SHEFFIELD

Shortly before I left England I attended a meeting of the Sheffield Branch of the Mathematical Association of Great Britain, an organization corresponding to the National Council of Teachers of Mathematics in the United States, and I was commissioned to bear their greetings to the mathematics teachers of the part of America that I was to visit. I have very great pleasure at this time in delivering that message of greeting.

* Presented at a meeting of the Colorado Council of Teachers of Mathematics, April 20, 1951.

3. PERSONAL ATTITUDE IN THIS ADDRESS

I feel that this address to some extent puts me "on the spot," for one of the duties of an exchange teacher is to try to further friendly relations between the two countries, and it has been well said that "comparisions are odious." It seems that there are three courses open to me: first, to say that I find everything in the United States is wonderful, and be convicted before all of you here who know better as being an ignoramus or a sycophant; second, of allowing my critical propensities full reign, and finding everything which is not British absolutely horrid; or, third, to try to steer a middle course and give an objective analysis and assessment of the differing arrangements of the two countries. It is obvious that one of these courses would serve no useful purpose. For some audiences the first approach might be desirable, but I am sure that this audience is of stronger fiber, so I shall adopt the third method, and endeavor to give an honest account of my experiences.

4. ANALYSIS OF THE SECONDARY SCHOOL SYSTEMS OF BRITAIN AND THE UNITED STATES

The differences in organization between the schools of Britain and those of the United States are great enough to affect very appreciably the courses and methods of teaching in the two countries, so I had better begin by outlining the major differences in this area. At the secondary level in the United States you have a system of common schools. The goals of these schools are, according to "Planning for

American Youth,"[1] largely social, but there are also aims which seem to have a bearing in the field of mathematics. The imperative needs of youth which can be at least partially satisfied by teachers of mathematics in their mathematics classes include the development of salable skills and those understandings and attitudes that make the worker an intelligent and productive participant in economic life: the development of knowledge of how to purchase and use goods and services, understanding both the value received by the consumer and the economic consequences of their acts; the development of an understanding of the methods of science and the influence of science on human life; the development of the ability to think rationally, to express thoughts clearly, and to read and listen with understanding. No one pretends that all these needs are satisfied exclusively, or even primarily through the mathematics classes, but it does seem that these needs provide the justification for the inclusion of a considerable amount of mathematical work in the programs of all pupils.

In Britain general aims are less well formulated. Our broad aim is to give each child an education suited to his age, aptitude and ability, without enquiring too closely just what fulfills these conditions. We break our secondary schools up into three parallel streams, each covering the whole secondary period, instead of into two consecutive phases, as is common here. Our three streams cater, broadly, for those who require a college preparation, in rather a narrower sense than is usual here, for those who require a thorough preparatory training so that they may become technicians or craftsmen, and for the general run of workers. Access to the three streams is controlled solely by ability, and social or economic status has no direct bearing. In addition we have a rather well developed system of private schools, known for historical reasons as public schools. These have, I judge, rather more impact on the community at large than comparable American private schools.

[1] National Association for Secondary-School Principals, *Planning for American Youth.* (Washington, D. C.: National Association for Secondary-School Principals, 1944.)

5. My Experience

I feel that it is only fair that I should make it clear at this stage that my experience in Britain is almost exclusively with the college preparatory stream, and quite largely with the best students of mathematics in that stream. This makes comparison with the general run of pupils in the American high school rather difficult for me. Out of consideration for my limited experience, my colleagues on the faculty at South High School have seen to it that the greater part of my work here has been with those pupils showing some evidence of mathematical ability. The most elementary classes I have had to teach have been an Algebra 1 and an Algebra 2 class. I have been into one or two junior high schools and observed a few classes there. But by some mischance when I visit a school, instead of being given a chance to learn, I am looked upon as some peculiar performing specimen, and called upon to give lectures on all sorts of remote and recondite subjects! I trust that I have made it clear that I cannot speak with any sort of authority about any type of school, English or American, except the English grammar school.

6. Preconceptions

I arrived in America with certain preconceptions about what I should find in American high schools. Those preconceptions, in so far as the teaching of mathematics was concerned, were based largely on The Mathematics Teacher, which I have read for some years, and also on the wonderful year-books put out by the National Council. I had been greatly impressed by the tremendous amount of research, painstaking and detailed, which is being done into the technique of teaching mathematics, and which I had seen re-

ported in those documents. I had also read a good deal about the work being done to humanize mathematics, so that the pupils might have a more adequate motivation, so that they might see the relevance of the work they were doing to the experiences they met in their daily lives, and so that they might understand the basic mathematical concepts. Education in England is in a state of abnormal fluidity at the present time, and one of the reasons why I applied for an exchange with an American teacher was so that I might see how these new ideas and some of this research was being applied. My reactions to what I have seen here are naturally colored by my background of experience and by the success I have had in looking for the things I hoped to find.

7. Mathematics Teaching in the United States

As I now begin to comment on what I have found in the American high schools I must again emphasize that what I say is based on what I have seen in the high schools of Denver and in a few of the school districts of Colorado. I am well aware that this is not an adequate sample of the schools of the United States. The mathematics classes which I have found to be common to all the schools with which I have come into contact seem to me to fall into three groups. First there are the so-called remedial mathematics classes, going under a variety of different names, but all having as their objective the removal of certain defects in basic mathematical competence or understanding. It would seem that there are two possible ways of dealing with these difficulties: one is to work on the assumption that the previous teachers of the pupils were incompetent or negligent and did not give the pupils sufficient practice; the other is to conclude that the pupils were presented with certain material when they had not sufficient maturity for that material to be meaningful to them. The former attitude might be called the classical attitude; the second appears to be the one taken in the research articles in The Mathematics Teacher. I have been impressed by the excellent results being achieved in many of these classes, by the care taken to identify the exact skills lacking, and the trouble taken to remedy defects identified. But I have not yet observed a class being conducted on other than classical lines, and I have spoken to only a very few teachers who made it clear to me that they were endeavouring to use the results of recent research.

The second type of class which I have met is the general mathematics class. These classes, it is said, mark an attempt to get away from the sub-division of mathematics into watertight compartments, at the same time making the work relevant to modern life and meaningful to the pupils. In practice the classes frequently turn out to be arithmetic, with a very slight admixture of algebra—substitution in easy formulas and a few simple graphs, perhaps with the intention of introducing the idea of functionality—and a little geometry—some mensuration. It is stated that these classes are for all pupils, but in practice it seems that the weaker pupils are the ones who find their way into them. Again I would say that an excellent job is being done within the limits accepted, and the teachers strive manfully to overcome the limitations of the situation in which they find themselves. Nevertheless, the classes in practice seem to me to be rather different from what was envisaged by their proponents.

The third type of class is the traditional algebra, geometry, solid geometry, trigonometry, or mathematical analysis class. These classes move on solid traditional lines, and concentrate on giving a thorough training in the basic techniques of the branches of mathematics being studied. In the algebra classes particularly, the subdivision of the subject matter is not quite so complete as the titles of the classes seem to indicate, and some work of a geometrical type is often included. Standards

attained seem to be reasonably high, having due regard to the amount of time spent and the age at which the studies are started. The most interesting classes with which I have come into contact were a complete surprise to me—classes ostensibly in plane geometry, but actually having as their main object the exploration of the theory and practice of clear thinking. These classes are obviously an attempt to apply the psychological discoveries of some thirty years ago to the announced aims of formal geometric teaching. The work I have seen being done is of a very high standard indeed. The pupils really seem to gain not only a good understanding of logical processes, but also considerable powers of critical thought, so that they are much less easily fooled by plausible but spurious arguments. For some curious reason, although harder critical and constructive thinking is called for than in the traditional geometry courses, the weaker pupils are the ones generally counselled into these courses. Even so, the results achieved are more than satisfactory. This is most certainly an idea which I propose to take back to England with me.

8. Mathematics Teaching in Britain

Mathematics teaching in England is organized rather differently. In the first place most of our pupils enter a secondary school when they are between eleven and twelve years old—say as they enter the seventh grade—and remain in that school until their school days are over. There is a wide variation in the way mathematics courses are arranged within a common framework which is quite generally adopted. The division of mathematics into separate subjects so that for one term a pupil will do algebra exclusively, for another term geometry exclusively, and so on is not favored. I would like to say that this is because we regard mathematics as a single subject, and that we adopt any of its tools as they seem to be appropriate for the subject in hand. That idea is preached by the leaders of thought in mathematics teaching. The real reason, I fear, is the tradition that all pupils shall start some geometry and some algebra in the seventh grade. In practice this means that if five periods a week are allocated to mathematics—we work on a weekly timetable, rather than a daily timetable—then two periods will be allocated by the teacher to arithmetic, two to algebra, and one to geometry, and the subdivision is as complete as it is here, except that the different branches are taught concurrently, and usually by the same teacher. Some teachers really do strive for integration of the different branches, and I think that the number doing so is increasing very steadily. Such teachers look upon the study of mathematics primarily as the study of number, space, and time, particularly of the ways in which these enter modern life.

Our courses in mathematics are planned as four or five year courses in which the basic concepts are encountered. We try to help our pupils to acquire those basic skills which they need while at school, and which they will use as young adults. We also endeavour, by simple and vivid examples, to bring them into contact with the great ideas of mathematics and their applications to the life of our civilization. These aims are high aims, and are certainly not often satisfactorily achieved, but I do think that during the last twenty or thirty years, in pursuit of these aims, we have gone a long way in cutting out dead wood that has obstructed mathematical teaching and made the name "mathematician" synonymous with "unworldly crank." We have, I think, gone a good deal further than you have in removing manipulation for manipulation's sake and juggling for juggling's sake from our elementary mathematics. In consequence, we have more time to stress the really important ideas. The pupils who need the manipulative powers have the necessary background of ideas on which to build when the time comes, and then, because they have a real reason to do the

manipulation, they are sufficiently alert to get it right. Prompted by a personal sense of urgency they quickly develop the necessary skills.

The five year course includes some of the arithmetic of finance, algebra of simultaneous quadratic equations and finite geometric series, geometry to the properties of circles and of similar polygons, a little trigonometry, say the use of the six ratios, and some very elementary analysis. This course is started by almost all pupils, though not all pupils go the whole way. We go in rather thoroughly for homogeneous grouping in our mathematics classes, which helps us to temper the wind to the shorn lambs, or to cut our coat according to our pupil's cloth.

The greatest difference in teaching methods I have found has been in the field of plane geometry. Here the ideas of geometry and the use of mathematical instruments are introduced on a limited scale in the seventh and eighth grades. Then there is a gap while algebra is taken up, so that not until the tenth or eleventh grades is formal geometry begun. The few pupils who remember anything about their geometrical instruments after this interval are then made aware of the sin of using those instruments. I have found in my own classes pupils saying that if triangles are congruent, corresponding angles are equal—and not knowing how to measure an angle to test the accuracy of a congruence construction. Faith in reasoning is good, but a realization of the frailty of human reasoning is also scientific.

In the early twenties the Mathematical Association of Great Britain set up a committee to examine and report on the teaching of geometry. This committee suggested a three stage approach: Stage A—gaining experience of the basic spatial ideas by drawing and measurement; Stage B—ad hoc proofs of properties discovered in Stage A, making such assumptions as appeared reasonable; Stage C—establishment of a logical geometric structure. These stages were visualized as consecutive but overlapping, with gradually increasing emphasis on stages B and C. In practice, Stage A is the only type of geometry considered for about a year (one or two periods a week), and much emphasis is placed on accurate and correct use of the standard geometrical instruments—ruler, compasses, dividers, protractor, triangles. After the first year the five year course will be a mixture of the first two stages, with stage B progressively displacing stage A. For the most able pupils some attention may be paid to stage C in the last year. This approach gained rapid acceptance, and is still being developed. It is clear, of course, that the subject matter is primarily geometry, and only secondarily, reasoning processes.

When pupils have completed this five year course successfully—success being determined by performance in an examination—they have fulfilled the minimum requirements for college entrance. They will not proceed to college for a further two or three years, so those who propose to study mathematics at the university will begin a further course extending over those two or three years. The normal time allowance will be eight to twelve periods weekly, and in this course a considerable amount of work in analysis, trigonometry, solid and analytical geometry, and theoretical mechanics is done. This special group of able pupils do work which is considerably more advanced than anything which is done in high schools here.

9. Textbooks

The teaching of mathematics, and indeed of most subjects, is for the average teacher, closely bound up with the textbooks he has to use, or which are available to him. The work of gathering suitable illustrative material and of compiling a sufficiently lengthy and closely graded list of practice exercises is so great that the teacher with a full program can hardly contemplate it. He certainly cannot do it

and at the same time keep up with the necessary correction and grading of papers. Naturally he relies on the textbook for much of his illustrative material and almost all of the drill examples. This means that the teacher's methods and approach are very much affected by the attitude of the textbook writers. Now it is exceedingly unfortunate that the large amount of research which goes into the production of a new textbook, and the capital investment which it represents imply that every textbook must be sufficiently conservative in outlook to command a wide sale, and radical experiments with textbooks which from their nature are likely to need considerable revision as experience accumulates are almost prohibited. American textbooks on conservative lines and which concentrate on carefully arranged and graduated drill are very good indeed. The arrangements made for a pupil to assess his own progress and identify for himself the work he needs to do excite my intense admiration, for I have not met books of this type before. Many English teachers would be quite shocked at the idea that a child could possibly discover his own weaknesses, and that, having discovered them, he would have the fortitude to work at them.

On the other hand, the more advanced textbooks have been rather disappointing to me. Quite frankly, most of the mathematics textbooks now in use in the Denver public schools would be regarded in England as at least twenty years out of date, and that by those enemies of change, English teachers. The features which lead me to this conclusion are, in algebra, the emphasis on drill and formal manipulation, often of a very complex type, at the expense of reasonably attractive story problems and other applications of the algebraic methods considered, and, in geometry, the tendency to start very quickly with formal axioms and postulates which are presented as such, and which often relate to entities with which the pupils have had little or no first-hand contact. I am bound to say, however, that some of the most recent textbooks appear to me to show a very considerable advance.

In Britain we seem to have a much wider variety of textbooks available, including many with a far less formal approach. I may have this impression because my knowledge of English textbooks has been built up over a number of years, and I now know almost exactly where to look for a text of any particular type. One rather exciting book which has been introduced lately and is intended for the less mathematically able pupils has the title *Designing and Making.** It is a collection of about one thousand experiments of a type likely to appeal to boys and girls who can use their hands, and illustrating or involving the basic mathematical ideas which we are all trying to get across. This is, admittedly, a radical departure, but it is based on the experience of the author, who is a practicing teacher, until recently at the secondary level. It has certainly aroused a great deal of interest in Britain. On more orthodox lines we have many excellent textbooks giving a wide variety of applications of the various mathematical ideas and concepts with which they deal. There are also more formal books, some of which have been in use for forty years and are being replaced by the more modern books as rapidly as school finances will permit.

As an example of the sort of differences which I am referring to, most American algebras I have seen start with a short chapter on the use of formulas, and then get down to a lengthy study of the use of letters for numbers and drill on the basic operations with letters. The English method would be to make the chapter on formulas more substantial, and include in it much of the formal work on the four rules as incidental to the manipulation of the formulas; alternatively, many English algebras start with the solution of simple

* W. W. Sawyer and L. G. Srawley, *Designing and Making.* (Oxford: Basil Blackwell, 1950.)

concrete problems, introducing algebra in the historical way through the syncopation of verbal statements. With either method there is an understandable reason for the drill work involved. Another example which struck me forcibly: a solid geometry text which I was given to use has some beautiful pictures of airplanes, buildings, machinery, etc. and the student is informed that solid geometry is used in the design and making of these things, but I have searched the book in vain for some reference hinting what or how. An English text would probably be short on the pictures, but would have some real information in the text.

It seems to me that the function of the textbook is not well understood either in Britain or America by the writers of these necessary instruments. Some textbooks are apparently designed to make a teacher unnecessary; in these books there is much waste of space on detailed instructions better given by the teacher. Other books are just collections of examples, leaving all explanations and applications to the teacher. I consider that any satisfactory textbook should perform three functions: first, it should provide a lengthy list of practical applications of the matter under consideration—the average teacher does not have the knowledge, the facilities, or the time to compile this information each day; second, the textbook should provide a comprehensive set of well graded examples and drill material—again the average teacher has neither the technical skill nor the time to prepare such a list; third, the textbook should provide for the pupils' reference a simple explanation of the common procedures. In my experience the first function is grossly neglected in favor of the second. The teacher is thus left without the assistance which he could reasonably expect.

10. Organization and Uniformity

Before I conclude I feel I ought to say something about the way in which a measure of uniformity between the schools is achieved in the two countries. In both countries the schools are free from central control of curriculum. In England each school, in theory, can make its own syllabuses and curriculum, according to its own ideas; in America each school system has the same liberty. Yet no one could possibly mistake an English scheme of studies for an American one, or vice versa. Why? Whence the uniformity in view of the liberty to vary? In America there is great uniformity within a school system, and for good reasons; a measure of uniformity between school systems is secured partly by the agreement there is between college requirements, partly by the popularity of the various attainment tests. For better or for worse many teachers at the high school level seem to have at least part of an eye on one or other of these standardizing influences. In Britain uniformity at the elementary level within a school system is brought about by the Selective Examinations for Entrance to Secondary Schools. Each school system organizes its own examinations, and it is essential that the pupils shall be adequately prepared for the examination they will take. Uniformity within the secondary schools is brought about by the General Certificate of Education Examinations, which are effectively college entrance examinations. At each level there is a systematic arrangement for consultations between teachers and examiners.

In both countries, therefore, the colleges have an important unifying influence and the differences between the courses in the schools of the two countries are to a considerable extent due to the differences in the outlooks of the universities. In Britain, universities are regarded as places of academic learning, selecting their pupils accordingly. In the United States every person who wishes to lay claim to an education must have attended a university, and demands the right to attend, so the universities modify their courses accordingly. Of course in each country the

(*Continued on page 394*)

The Slide Rule Made Meaningful

By GEORGE L. KEPPERS *and* PERRY A. CHAPDELAINE
Iowa State Teachers College, Cedar Falls, Iowa

WHAT constitutes "meaning" in slide rule instruction? The most frequent answer, probably, is that the student should know the main principle underlying slide rule construction. That principle, of course, is the representation of the logarithm of a number by a distance along a line as the decimal part of a given constant.

It has been within the experience of the writers to note the results of faulty slide rule instruction in their classes. On questioning the pupils, it would seem that teachers follow one general procedure to effect slide rule competence. The teacher may have a large wall model or, most generally, just a ten inch slide rule and a manual "to accompany the slide rule." The students are equipped with a slide rule. The various scales are explained and there may have been some preliminary work such as a thorough study of logarithms (a thorough study is not necessary) and a study of scientific notation. Next the teacher suggests that a simple problem be worked such as $2\times3=6$. The procedure is as follows: line up the left hand index of the "C" scale with the two on the "D" scale, then move the hair line to the three on the "C" scale and read the answer on the "D" scale. This is continued for many other examples including problems in division, finding square roots and raising to a power. By this method the student can, in many cases, become proficient at manipulating the slide rule. However, in most cases the student does not know what he is doing or why he is doing it.

The writers would propose an exercise for the students to perform which has worked successfully in their trigonometry classes. Remarks such as these: "How interesting!," "Well! I never realized it was this simple!," "Why wasn't I taught this before?," and so on, are evidence that the procedure recommended is meaningful and simple to perform.

In teaching the slide rule in a meaningful way, a certain amount of preliminary work is necessary. The student should know and understand the four fundamental operations of logarithms—multiplication, division, raising to a power, and finding a root. More specifically, if the student is given $\log_{10} 2=0.301$, $\log_{10} 3=0.477$, and $\log_{10} 7=0.845$ and knows that $\log_{10} 10=1$, he should be able to compute and understand such problems as: $\log_{10} 14=x$; $\log_{10} 2=x$; $\log_{10} 7000=x$; $\log_{10} 21=x$. It is not necessary that the student be thoroughly familiar with scientific notation but such a familiarity can aid in the learning process.

The necessary equipment for performing this exercise consists of a ruler, marked either in centimeters or tenths of an inch (this article is based on the centimeter system), ordinary note book paper having a diagonal at least thirty centimeters long, a table of logarithms, a pair of scissors, and a pencil. A slide rule is not recommended.

First, each student should draw a straight line thirty centimeters long, diagonally across his paper as shown in Figure 1.

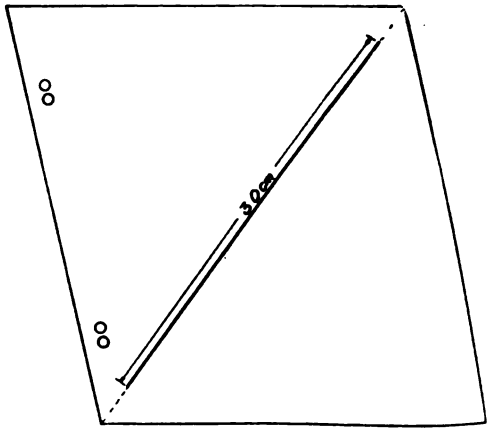


FIG. 1

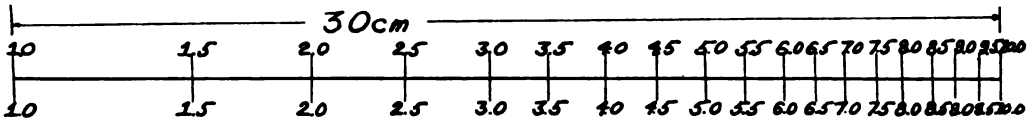


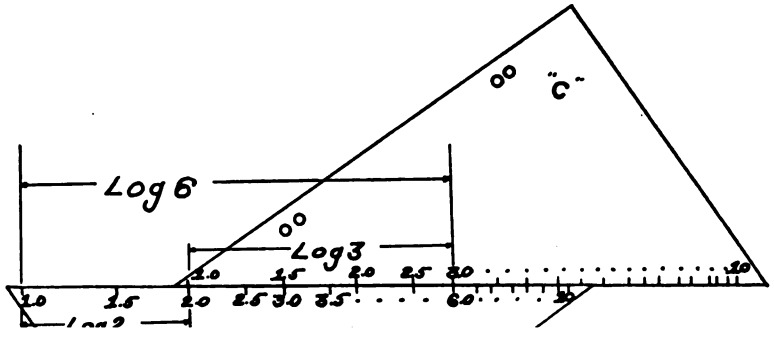
FIG. 2

Next, have the students find the logarithms of 1, 1.5, 2, 2.5, 3, . . . , 9.5, 10; multiply each of these values by thirty and round off the products to the nearest one-hundredth. The class as a whole can participate in finding these values while the teacher records them on the blackboard. If the students have not used the tables previously, the teacher may have to give a short explanation in finding these logarithms. A more detailed slide rule can be made by using more divisions but it is advisable to limit the number of computations to avoid confusion. As each of the products is formed a mark is made along the diagonal to designate centimeters from the end of the diagonal. The marks are numbered as illustrated in Figure 2.

The instructor can sketch the procedure on the blackboard to facilitate his explanations.

The number one is placed above and below the mark at the end of the line because the logarithm of one is zero and would be represented by zero centimeters. The number two is placed above and below the marker which is approximately equal in length to 9.03 centimeters. This value is obtained by multiplying $\log_{10} 2 = 0.301$ by thirty. In this manner we are representing the logarithm of a number by a distance along a line as the decimal part of thirty centimeters. These two sets of numbers above and below the line are referred to as the "C" and "D" scales, respectively.

After the above mentioned values have been located, the paper should be cut along the line separating the C and D scales. Now the student is ready to apply the rules for multiplying and dividing with logarithms. By solving simple problems such as $2 \times 3 = 6$ and $5/2 = 2.5$ the student can readily see that when using the slide rule he is doing nothing more



than adding or subtracting lengths which represent logarithms as shown in Figure 3.

If one wished, he could emphasize the fact that a logarithmic scale is not equivalent to a linear scale by having the students construct a linear scale "slide rule" and noting the products on this scale.

More difficult problems should be introduced as the students become familiar with these two scales. The teacher may ask, "How would you construct a scale to find square roots?" or "How would you construct a scale to find the twelfth power of numbers?". The scales for finding square roots and squaring numbers can be developed by applying the rules used in the work with logarithms. That is, the students could make one scale half the size of the other since we multiply the logarithm of a number by two in squaring.

Mathematics will not be the practical or cultural tool necessary for the development of "correct" understandings until the mysticism of mathematics can be explained in terms of the pupils' experiences. After pupils have had experiences of the kind proposed above, the writers feel that students will find working problems with the slide rule not only an expedient procedure in arriving at an answer but also a fascinating and meaningful exercise.

Secondary School Mathematics

(*Continued from page 391*)

schools have a very considerable influence on the universities, but certainly in England the universities have a very big influence on the schools. Further, both schools and universities reflect the social and economic philosophies of the countries.

11. General Impression

I suppose that I would say that my general impression of mathematics teaching in America is one of disappointment; disappointment that the teaching is considerably less progressive than I had expected. But then as I reflect, this sense of disappointment passes—I was looking for the moon, and naturally I did not find it. Wonderful work is being done; research is being pushed further and further into the best ways of learning and the best ways of teaching. What if this research is a little slow in gaining acceptance? Education is no field for revolutionary changes—the lives of people are at stake. Gradual progress must be our aim. Before we cast off the old we must be sure that the new is better.

One thing I certainly have learned here —that it is possible to make social aims the center of an educational program—that it is possible to make a success of education for citizenship. Perhaps the day is not too distant when education for world citizenship will be the center of education in every country of the world!

REVISION OF BY-LAWS

Revisions of the By-Laws of the National Council of Teachers of Mathematics as printed in the February, 1951 issue of The Mathematics Teacher were adopted at the annual business meeting of the National Council in March. The president, however, asked the committee which had worked on these revisions to continue another year to consider any further changes which might be proposed by the membership. Announcement that the committee would receive these suggestions was made at the annual business meeting and has been sent to all affiliated groups. Suggestions can still be considered for action at the next annual business meeting if received by the committee before November 20.

Members of the committee are George Hawkins, Lyons Township High School and Junior College, LaGrange, Illinois; Mary Potter, Board of Education, Racine, Wisconsin; and Marie S. Wilcox, Chairman, George Washington High School, Indianapolis, Indiana.

The President's Page

MEET OUR EXECUTIVE SECRETARY

LAST May the Board of Directors of the National Council of Teachers of Mathematics appointed Mr. M. H. Ahrendt as our new Executive Secretary. Mr. Ahrendt assumed his duties at our headquarters office at 1201 Sixteenth Street, N. W., Washington 6, D. C., on July 1, 1951. Prior to this date, this position had been filled temporarily by your President, during our first year as a Department of the National Education Association.

Mr. Ahrendt has been an active member of the National Council for several

M. H. AHRENDT

years. Many will remember the fine work he did as General Chairman of our annual meeting in Indianapolis in 1948. We all know him through his good work as Chairman of the Mathematics Kits Committee.

Mr. Ahrendt comes to us from Anderson, Indiana, where he has been for the past four years teaching mathematics and physics at Anderson College. He has had many years of teaching experience in junior and senior high schools in Indiana and has done some work in teacher training and in supervision of practice teaching. Mr. Ahrendt was President of the Indiana Council of Teachers of Mathematics in 1947–48 and was Secretary of the Mathematics Section of the Indiana State Teachers Association in 1949–50. He was Chairman of the Reorganization Committee of the Mathematics Section of the Indiana State Teachers Association in 1949. This committee developed a plan for consolidating the various organizations of mathematics teachers in Indiana. The plan was adopted and is now in operation.

In 1932 Mr. Ahrendt received his M.A. degree at the Univeristy of Wichita, Wichita, Kansas. He did additional graduate work at the University of Cincinnati in 1933–35, at Ball State Teachers College in 1937–39, and at the University of Michigan in 1948. He has done much independent study of advanced and enrichment topics in mathematics, one example of which is his work with "linkages." He has written several articles on mathematics, most of which are to be found in THE MATHEMATICS TEACHER. His hobbies include woodworking, photography, and music. He is married and has a 15-year-old daughter.

Mr. Ahrendt comes to us with high recommendations. He has shown himself to be honest, conscientious, and a hard worker. He has demonstrated originality and initiative. Let's give our loyal help and cooperation to Mr. Ahrendt that he may discharge his duties well, that he may carry on successfully the old services to our members and help to develop new ones, and that he may increase our cooperation with other organizations for the betterment of mathematics teaching.

We wish him happiness and success.

H. W. CHARLESWORTH,
President

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

30. A Second Note on the Pythagorean Theorem[1]

The purpose of this note is to give a proof of the converse of the Pythagorean theorem which is independent of the theorem itself and which uses only the theorems of the first book of Euclid which would have been known to the ancient geometers.

CONVERSE OF THE PYTHAGOREAN THEOREM: *If the square of the number which measures one side of a triangle equals the sum of the squares of the numbers which measure the others, the figure is a right triangle.*

Let ABC (Figure 1) be a triangle the measures of whose sides BC, CA, and AB are respectively a, b, c which numbers satisfy the relation

$$a^2+b^2=c^2. \tag{1}$$

(2) Then $a^2=c^2-b^2=(c-b)(c+b)$.

On BC, AB and BA extended locate points B_1, A_1, D, E, such that $BB_1=c-b$, $BA_1=BC=a$, $AD=-AE=CA=b$. Hence $BE=BA+AE=c+CA=c+b$ from which, using (2), we have

$$\frac{BB_1}{BC}=\frac{c-b}{a}=\frac{a}{c+b}=\frac{BA_1}{BE} \tag{3}$$

which establishes the parallelism of A_1B_1 and EC. From this it follows that angles B_1A_1B and CEA are equal.

Triangle A_1BB_1 is congruent to CBD since they have a common angle B comprehended between equal sides. Further triangle CAE was constructed to be isosceles, hence

[1] See THE MATHEMATICS TEACHER. Vol. XLIII (Oct., 1950), p. 278 for the first note.

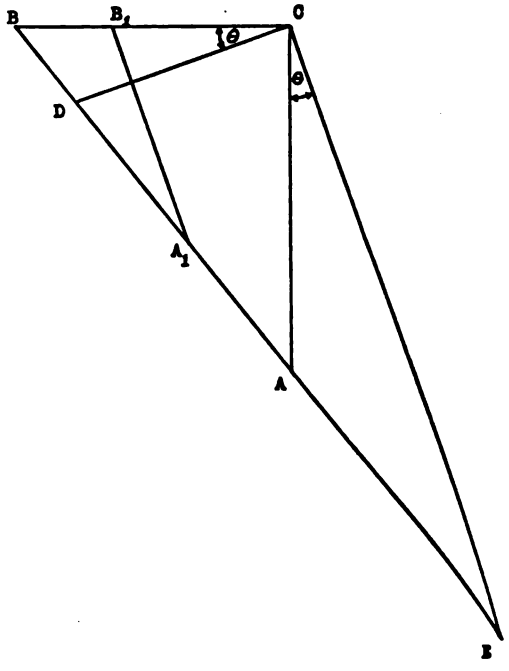


FIG. 1

$$\angle BCD=\angle B_1A_1B=\angle CEA=\angle ECA=\angle\theta.$$

This last shows that $\angle ECD$ if rotated through angle θ would coincide with $\angle ACB$. But $\angle ECD$ can easily be proved to be a right angle since $CA=DA=AE$, or, in other words, the median to DE is $\frac{1}{2}\cdot DE$, and hence, finally, $\angle ACB$ is a right angle which was to be proved.

Victor Thébault,
Tennie (Sarthe) France

31. An Approach to Determinants[2]

Two of the reasons for the difficulties encountered in teaching determinants

[2] The ideas for this note can be traced to Felix Klein, *Elementary Mathematics from an Advanced Standpoint—Geometry*. (New York: Macmillan, 1939), Part I, Section I; T. H.

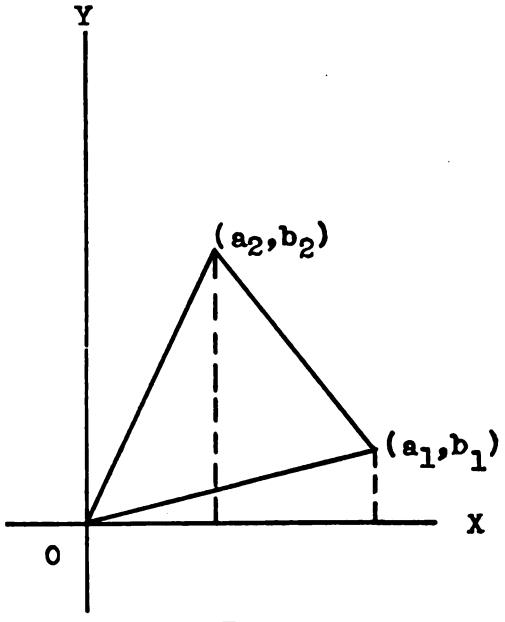


FIG. 2

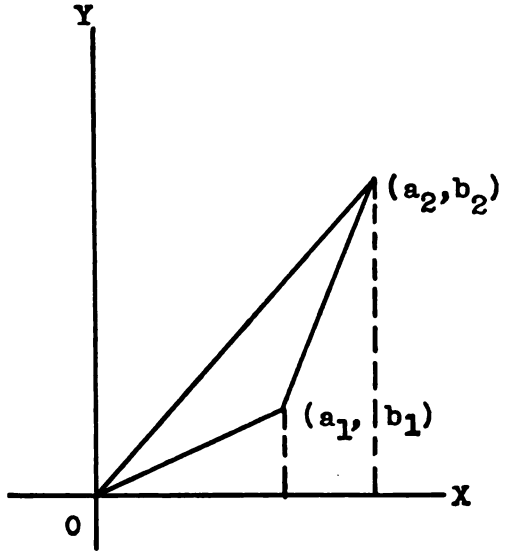


FIG. 3

are (1) the difficulty of motivating their introduction so as to make them appear other than arbitrary, vague, "plucked from thin air," and (2) the reliance for concrete application almost solely upon Cramer's rule which does not seem too important to students since they have other reliable and simple procedures for simultaneous solution of equations.

The following parallel development of algebraic and geometric approaches to determinants partially overcomes these difficulties and even provides some motivation for the derivation of the properties of determinants.

The work begins by suggesting that, as with the quadratic so with simultaneous linear equations, a formula for their solution might be a worthwhile tool if it were simple enough and the problem were to occur often enough. It is easy to derive for $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ that

$$x=\frac{c_1b_2-c_2b_1}{a_1b_2-a_2b_1}$$

and a similar formula for y. At this stage the second order determinant may be introduced as a mnemonic device for recalling this formula.

It may then be suggested that geometric interpretations and applications are often illuminating and that since in each determinant there were two pairs of numbers we might consider a new geometric problem in which we have two points which joined to each other and to the origin form a triangle. If, starting from the origin, we walk with the area of the triangle on our left, and use (a_1, b_1) to represent the first and (a_2, b_2) the second point to which we come, one of figures 2 or 3 must result (limiting ourselves to the first quadrant for convenience). The area in both cases is then easily seen to be $A=(1/2)\ (a_1b_2-a_2b_1)$. For example, in figure 2,

$$\begin{aligned}A&=(1/2)a_2b_2+(1/2)(a_1-a_2)(b_1+b_2)\\&\quad-(1/2)a_1b_1\\&=(1/2)(a_1b_2-a_2b_1).\end{aligned}$$

It takes only a moment for students to recognize this as of the same form found in the solution of our simultaneous equations. They now have two quantities representable by determinants and will be quite ready to agree that there may be

Hildebrandt, "Marginal Notes." *The American Mathematical Monthly*, XXXVI (April, 1929), p. 216 ff. and conversations with various persons, especially Norman Anning and P. S. Dwyer.

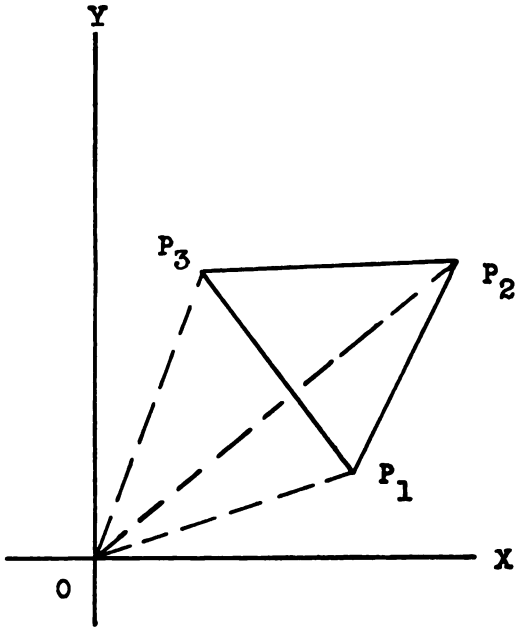


FIG. 4

value in a study of the properties of such arrays.

Next we observe that if the points are reversed in their order of appearance in the area formula then

$$A' = \begin{vmatrix} a_2 & b_2 \\ a_1 & b_1 \end{vmatrix} = a_2b_1 - a_1b_2 = -A.$$

We associate this negative area with walking around the triangle clockwise. Returning to the solution of simultaneous equations we note that this exchange of rows would, formally, correspond to interchanging the order of the equations, and would quite properly not change the solution merely the signs of both numerator and denominator. We now have derived and examined the implications in two situations of the rule about interchanging rows. The result for columns of the second order determinant can be easily derived.

From here we return to the solution of simultaneous equations in three unknowns, obtaining by straightforward elimination

$$x = \frac{d_1(b_2c_3 - b_3c_2) - d_2(b_1c_3 - b_3c_1) + d_3(b_1c_2 - b_2c_1)}{a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)}.$$

The students will recognize the determinants here and the instructor may introduce the third order determinant as a mnemonic for this expression which of course is equivalent to using the expansion by minors as a definition of the third order determinant.

Naturally we next seek a geometric counterpart. Two are available, extension from area to volume or from two to three arbitrary points in the plane. The latter is simpler. Consider the triangle none of whose vertices is at the origin (figure 4). Joining its vertices to the origin we see its area to be given by $OP_1P_2 + OP_2P_3 - OP_1P_3$ or

$$\frac{1}{2}\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}$$
$$= \frac{1}{2}\left\{ 1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} - 1\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} + 1\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\}.$$

This expression is readily seen to be

$$\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

the well-known area formula.

Geometric thinking may then be used to suggest and give concreteness to such theorems as multiplication of a column by a constant, multiplies the determinant by the constant, and two identical columns or rows imply a zero value. Further applications of determinants to plane, solid, and analytic geometry follow naturally from such a beginning as does also their use in the construction of alignment charts.[3] A scheme used by civil engineers

(*Continued on page 400*)

[3] See, for example: Lawrence I. Hewes and Herbert L. Seward, *The Design of Diagrams for Engineering Formulas.* (New York: McGraw-Hill Book Co., Inc., 1923); Maurice Kraitchik, *Alignment Charts.* (New York: D. Van Nostrand Co., Inc., 1944.)

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
135–12 77th Avenue, Flushing 67, N. Y.

The purpose of a mathematical recreation when it is employed in the processes of mathematical instruction is many-fold. A mathematical recreation may be used in order to break the monotony of a classroom study. It may be wrapped in a humorous cloak. The appeal to the pupil's sense of humor may relieve the tension, may spark the waning interest in a specific topic.

A mathematical recreation may be used in order to motivate the development of a topic. Under such circumstances a mathematical recreation serves the purpose of creating the atmosphere which will be conducive to a fruitful and effective evolvement of a mathematical study.

A mathematical recreation may be used for the purpose of arousing the imagination of the pupils. Under such circumstances a mathematical recreation is expected to contribute to the growth of the mathematical experiences of the pupils.

Whatever the purpose of a mathematical recreation may be, its introduction in the classroom situation must consider several important factors which are continuously operative in the processes of mathematical instruction. It should be understood that the last statement implies that the operativeness of these factors may be either positive or negative. Of these two factors the most important are *curiosity* and *interest*. A mathematical recreation is expected to arouse curiosity as well as evoke continuous interest. If these two take place the operativeness is positive. On the other hand, when these two are absent, the effect is totally negative. However, the range between the two effects is such that there may take place some variations from the extremes. Under some circumstances the pupils may indicate a slightly mild interest, that is, they may respond to a mathematical recreation as if it were "just another problem."

In other words, the employment of mathematical recreations as means of instruction has its pitfalls against which the teacher must be always on guard. Justly, a mathematical recreation is an instructional instrument more dangerous than any other means of instruction. The interest of a teacher, the enthusiasm of a teacher is generally not the best criterion for the judgment of the usefulness of a mathematical recreation in a classroom. Furthermore, the standardized and classical types of mathematical recreations, such as (a) the guessing of a number, (b) the guessing of date or of an age, or of a day of a week, and many other mathematical recreations are, in themselves, dull and dry mathematical exercises, replete with algebraic relations which are complicated in their construction and form. Some of these recreations may be the topics of study in a mathematical club, although even then they are devoid of the spontaneity which is the first prerequisite of the evoking of genuine interest.

What is then the correct approach to a mathematical recreation? How can a teacher resort to an introduction of a recreation with a modicum of certainty of success? Is there some definite process and procedure which will insure the teacher against the recreation falling on deaf ears? The answer to these questions requires careful consideration of the psychological effect of a mathematical recreation.

A mathematical recreation presupposes

a *lightness* of approach to a situation as well as a *lightness of the situation itself*. In other words, a mathematical recreation must be as light as a witty epigram. In formulating a mathematical recreation the teacher must consider several very important factors. Fundamentally the purely mathematical factors are of secondary importance when a classroom situation is considered. On the other hand, however, the pupils, their interest, their ages, their command of language, their responsiveness to humorous situations, all these must be carefully weighed and accounted for. In other words, whenever a mathematical recreation is about to be introduced in a mathematics classroom, the teacher should take care to avoid its presentation in purely mathematical form.

Generally speaking, the pupils are, by and large, very sensitive to humorous situations. Laughter is more conducive to effective learning than morbid and rigid seriousness, provided levity is avoided. The psychology of laughter is unfortunately taboo in educational theory. The readers of this department are advised to consult a little book by Henri Bergson on the topic of the psychology of laughter. The reason many of the books on mathematical recreations are usually food for the generals is very simple; those who wrote these books have taken themselves and their subject too seriously in a purely mathematical sense. The sense of humor of a bewhiskered mathematician is not of the same kind as the sense of humor of an average mortal pupil. And the table of mortality or of life expectancy of a pupil in a mathematical classroom is very sad indeed.

The above should not be interpreted as an exhortation to some new approach to mathematical instruction. Nor should it be construed as an appeal to the teachers to become jesters. There is no justification of any extreme. But, if a mathematical recreation is to be effective, it must appeal to the sense of humor of the pupil. And mathematics is not devoid of humor, if a teacher can develop an appropriate satirical technique. This department proposes to indicate some means which the teacher will be able to utilize in classroom situations. This can be accomplished without sacrificing the "greater values" of mathematical instruction, that is, the sacred cow which is known under the collective name of *objectives*.

Mathematics is not a difficult subject. It is, as a matter of fact, no more difficult than the everyday thinking we all are engaged in. The seeming and imaginary difficulty of mathematics may be ascribed to the outmoded traditional approach to the subject. If mathematical instruction is abundantly sprinkled with truly recreational material most of the fears of and for mathematics would vanish as the morning mist disappears under the rays of the rising sun. Recreational material is abundantly available and it beckons both the teacher and the pupil. Much of the recreational material, a greater portion of it, is found not in purely mathematical situations. But, whatever the source of this material may be, it must be clothed in a smile and a timely humorous dress.

Mathematical Miscellanea

(*Continued from page 398*)

and others for computing the area of any polygon from the coordinates of its vertices is also easily derivable. If (x_i, y_i) are the vertices of an n-gon, then its area may be computed from the array

$$A=\frac{1}{2}\left[\begin{matrix} x_1 & x_2 & x_3 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_n & y_1 \end{matrix}\right]$$

by summing all products of the form x_iy_{i+1} plus x_ny_1 and from this subtracting the sum of all products of the form y_ix_{i+1} plus y_nx_1 and finally taking one half of this result.

APPLICATIONS

Edited by SHELDON S. MYERS
Department of Education, Ohio State University, Columbus, Ohio

We begin this first issue of 1951–52 with a renewed appeal for contributions. Those that you readers have submitted in the past have brought many favorable comments and have enabled THE MATHEMATICS TEACHER, through its departments, to enrich the quality of mathematics instruction in this country. Perhaps this knowledge will encourage more of you to shift from passive to active beneficiaries of this journal.

P. G. 4. Gr. 10–12 *Runway Drop at Idlewild International Airport*

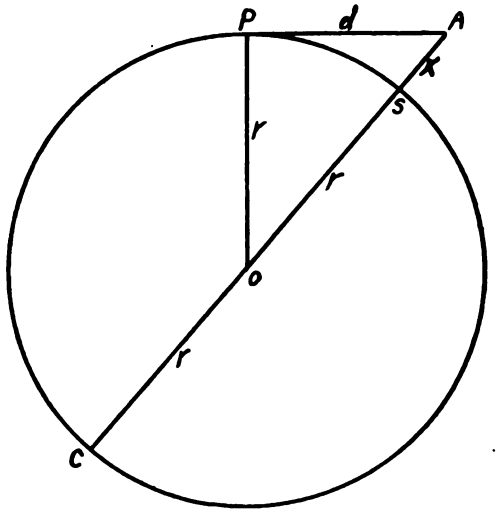
Dr. Maurice Nadler, Head of the Mathematics Department at Jamaica High School, Jamaica, New York, wrote me last April of an interesting application of the theorem in plane geometry which states that a tangent to a circle is perpendicular to the radius at the point of tangency.

In laying out a mile-long runway at Idlewild International Airport, there is a drop of 8″ due to the curvature of the earth. What would this drop be for a two-mile runway?

AP is the apparent level, tangent to the earth, while arc PS is the runway following the curvature of the earth. Since AP is tangent to OP, triangle AOP is a right triangle. Using the Pythagorean Theorem, it can be shown that

$$d^2=(2r+x)x.$$

(Department Editor's Note: After the manuscript was completed, it was discovered that this was not the derivation intended by Dr. Nadler. He wished to point out the use of the following theorem which enables one to derive the above expression in one step: "If from a point without a circle a secant and a tangent are drawn, the secant terminating in the concave arc, the square of the tangent is equal to the product of the secant and its external segment.")



At this point a step occurs with which we would do well to acquaint our students. This is simplification when a very small, insignificant quantity is added to a huge quantity. In this case, x is negligible in comparison with $2r$, thus we have the following simplified expression:

$$d^2=2rx \quad \text{and} \quad x=d^2/2r.$$

Using the value of 7,900 for r, it is found that

$$x=2/3\ d^2,$$

where d is in miles and x in feet.

For one mile, the drop, x, is $\frac{2}{3}$ of a foot or 8″. For two miles, the drop is 32″.

Dr. Nadler adds that a similar application is the development of the approximate formula for visibility from a given height above the earth's surface:

$$d=1.3\sqrt{h},$$

where d is in miles and h in feet.

P. 3 Gr. 10–12 *A Valuable Dog is Lost by a Definition*

Many geometry teachers are interested in how transfer of training and motivation is achieved in Nature of Proof, a progressive method of teaching plane geometry developed by Dr. Harold P. Fawcett and described in the Thirteenth Yearbook of the National Council of Teachers of Mathematics. One of the techniques is to show dramatically the importance and the role of the definition in geometric and non-geometric situations. Teachers should be on the alert for newspaper clippings which will serve this purpose. Following is an Associated Press dispatch of June 6 from Sesser, Illinois, which might be used:

> Virgill Robbins decided to have his valuable pointer dog vaccinated against rabies last week but didn't have time to leave his hardware store. He asked a small boy to have it done and gave him a dollar for the fee.
>
> The lad took the dog to the City Hall and told Police Chief Wilford Day that Mr. Robbins wanted the dog "shot." "Surely not," Chief Day said, but when the boy pulled out the dollar for the job the chief guessed Robbins meant it. The police chief said he'd shoot the dog but would take no money.
>
> The boy returned to tell Mr. Robbins what he had done. The merchant rushed to the City Hall and found that his dog, for which he had spurned a $300 offer, had been destroyed.

A teacher could use this clipping to point up the following ideas in Nature of Proof:

1. The influence, on the conclusion, of a definition in an argument.
2. Definitions are human agreements, not eternal truths.
3. The need in mathematical and non-mathematical situations of group agreements about definitions and other assumptions in order to establish a common ground for communication and understanding.
4. The logical difficulties that might arise from implicit assumptions.

T. 2 Gr. 9–12 *Interpolation in the Laboratory*

One of the skills which is more often taught in trigonometry than in other areas and which has wide use in industry and science is interpolation. It is important that students do not conclude from too limited experience with interpolation that its use is confined to logarithms and trigonometric functions. Other uses can easily be shown to high school students. Here is one involving the immersion refractometer which determines the refractive index of liquids like oil in order to get a line on their density.

The immersion refractometer is immersed in the oil, and the bending of a a ray of light by the oil is measured by the instrument. The amount of bending, or refractive index, is a measure of the density of the oil. However, the refractometer readings are in terms of scale divisions and must be converted to refractive index. This conversion may be done with the aid of a table in Lange's *Handbook of Chemistry*.[1] This table gives refractive indexes ranging from 1.32539 to 1.36640 for instrument readings ranging by whole numbers from −5 to 105. Here is a sample interpolation for an instrument reading of 20.8:

Scale Divisions	Refractive Index
20.0	1.33513
20.8	?
21.0	1.33551

For the purpose of interpolating, it is correct to assume, in this interval, that refractive index is linearly related to instrument reading. Therefore, since 20.8 is .8 of the way between 20.0 and 21.0, the refractive index corresponding to 20.8 must be .8 of the way between the first and second refractive index, or .8 of .00038. This is .00030 units of refractive index beyond the first one, or 1.33543. As in all mathematics teaching, students should be taught how to generalize processes and how to use them in different situations. Experimental studies have shown that this is the best way to assure transfer of training.

(*Continued on page 404*)

[1] Norbert Adolph Lange, *Handbook of Chemistry*. (Sandusky, Ohio: Handbook Publishers Inc., 1941. p. 1087.)

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, N. Y.

Roman Numerals

THE TOPIC of Roman numerals appears in textbooks in arithmetic for its informational value. It is handy to have a second set of numerals, and tradition links us to the Roman numerals for this auxiliary set. Custom dictates their use on the face of a clock. Convenience causes their use after the identification number on a United States Savings Bond to indicate the maturity value,—L on the $50.00 bonds, C on the $100.00 and D on the $500.00 (The $25.00 bond is marked Q.)

Pupils studying these numerals have a tendency to experiment with them, sometimes departing widely from the accepted pattern. Indeed instances may be found of out-of-school eclectic uses of the numerals. In 1949, for example, an Italian postage stamp was issued in honor of the physicist Volta. It bore the dates MDCCIC and MCMIL.

Should the innovations made by the pupils be accepted? and how may we answer the question "Is there a way to write really *big* numbers?" Prudence dictates that for testing purposes, the pupil should stay with the textbook and its rules. Common sense says that there must have been ways to write large numbers for the Romans would have needed them in their censuses and in other statistics. Reference to Professor Florian Cajori's *History of Mathematical Notations*, vol. I (Chicago, 1928) and to Professor D. E. Smith's *History of Mathematics*, vol. II, (Boston, 1925) shows a wide variety of forms used by the Romans. Listing here, however, is no indication of the vogue of a symbol nor is the date of its occurrence an indication of the time when it was first introduced. Professor Smith notes that the Romans appear to have avoided the symbol IV for 4 preferring IIII instead. Professor Cajori states that a line above a numeral to indicate multiplication by 1000 appears in an example of the 4th century A.D. Among the special instances listed is the numeral XXCIII instead of LXXXIII.

Roman numerals were in active use for a considerable period in Europe. They were used in an inventory of stores in the English navy in the second quarter of the seventeenth century. It is said that they were used in government accounts in France until 1789 at the official beginning of the French Revolution. This, however, should be checked for it would seem incredible. It perhaps was the fact that the numerals were in frequent use as matters of record that led certain of the writers of arithmetics to omit mention of them. In this group are Tonstall and Humphrey Baker in the sixteenth century and Hodder and Leybourne in the seventeenth. In the arithmetics printed in the United States, beginning with Pike's of 1788, Roman numerals are generally given at least half a page although the topic is dropped at this point. One of the most complete treatments is given by Roswell C. Smith in his *Arithmetic on the Productive System* (Rochester, 1846). His treatment of the subject lists the numerals without giving rules for their formation except in the case of two short footnotes.

The topic is headed

NUMERATION

"NUMBER, which shows how many are

DETERMINANTS-A POWERFUL TOOL

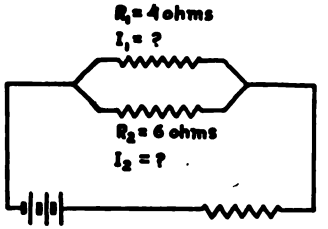
THE SOLUTION OF SIMULTANEOUS EQUATIONS IN ELECTRICAL CIRCUITS

THE MATH:

$$13x + 9y = 10$$
$$4x - 6y = 0$$

$$x = \frac{\begin{vmatrix} 10 & 9 \\ 0 & -6 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = \frac{-60}{-114} = .526$$

$$y = \frac{\begin{vmatrix} 13 & 10 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = \frac{-40}{-114} = .351$$



R_1 = Resistance in top branch
I_1 = Current " " "
R_2 = Resistance in bottom branch
I_2 = Current " " "
R = Resistance in main line

ITS APPLICATION TO THE ELECTRIC CIRCUIT:

FROM CIRCUIT THEORY

$$I_1 R_1 + (I_1 + I_2) R = E$$
$$I_1 R_1 - I_2 R_2 = 0$$

SUBSTITUTING THE VALUES IN THE DIAGRAM

$$13I_1 + 9I_2 = 10$$
$$4I_1 - 6I_2 = 0$$

$$I_1 = \frac{\begin{vmatrix} 10 & 9 \\ 0 & -6 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = .526 \text{ amperes}$$

$$I_2 = \frac{\begin{vmatrix} 13 & 10 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = .351 \text{ amperes}$$

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SLOPES IN MAP READING

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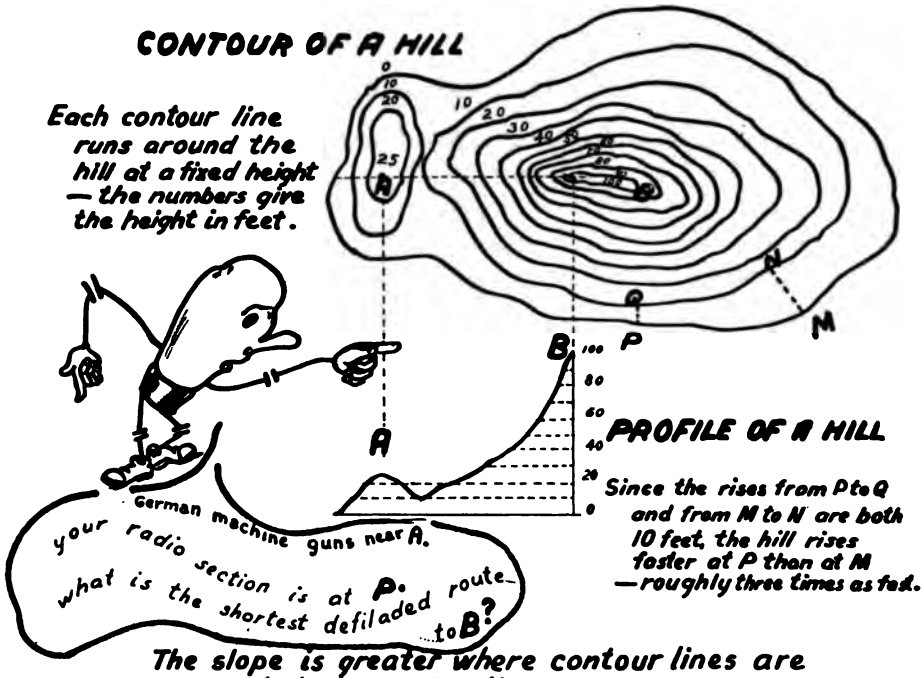
A knowledge of slopes is useful in obtaining the outline of a hill from its contour lines in the map.



The slope is greater where contour lines are crowded closer together.

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POSTER # 7

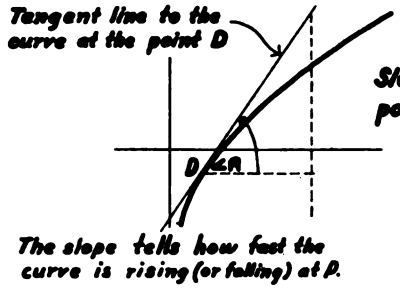
THE MATHEMATICS OF RADIO SLOPES

An example of the slope of a curve is the grade of a road.

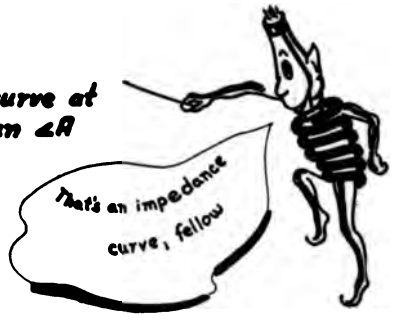
$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \tan \angle A$$
(or grade)

The slope tells how steeply the road is rising or falling.

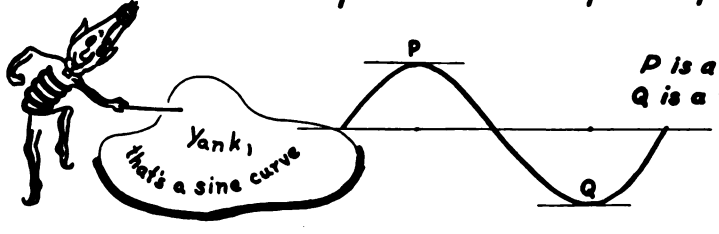
For a general curve we use tangent lines to find the slope.



Slope of curve at point D = tan ∠A



An important use of slopes is to find maximum points (peaks) and minimum points (valleys) of a curve.
At maximum and minimum points of curve, the slope is zero.



P is a maximum
Q is a minimum

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the MATHEMATICS of RADIO

GRAPHS

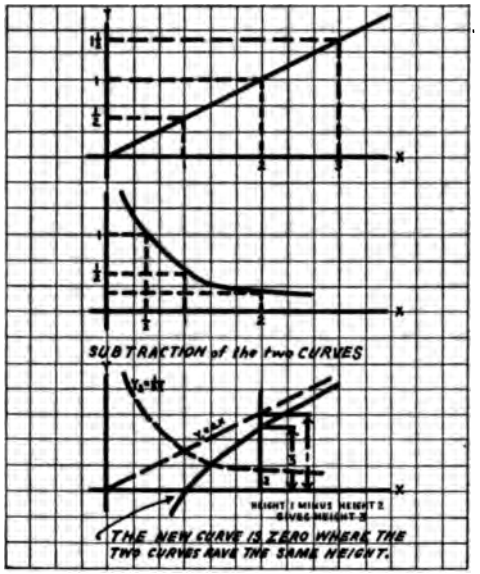
the EQUATION

$Y_1 = aX$

$Y_2 = \frac{1}{bX}$

$Y_1 - Y_2 = aX - \frac{1}{bX}$

the GRAPH



PLOTTING the CURVE

Let $a=\frac{1}{2}$ for this case

IF X IS	THEN Y IS
0	0
1	$\frac{1}{2}$
2	1
3	$1\frac{1}{2}$

Let b=2 for this case

IF X IS	THEN Y IS
0	∞ (infinity)
$\frac{1}{2}$	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

For each X, we get the height of the new curve by subtracting the height of curve 2 from the height of curve 1.

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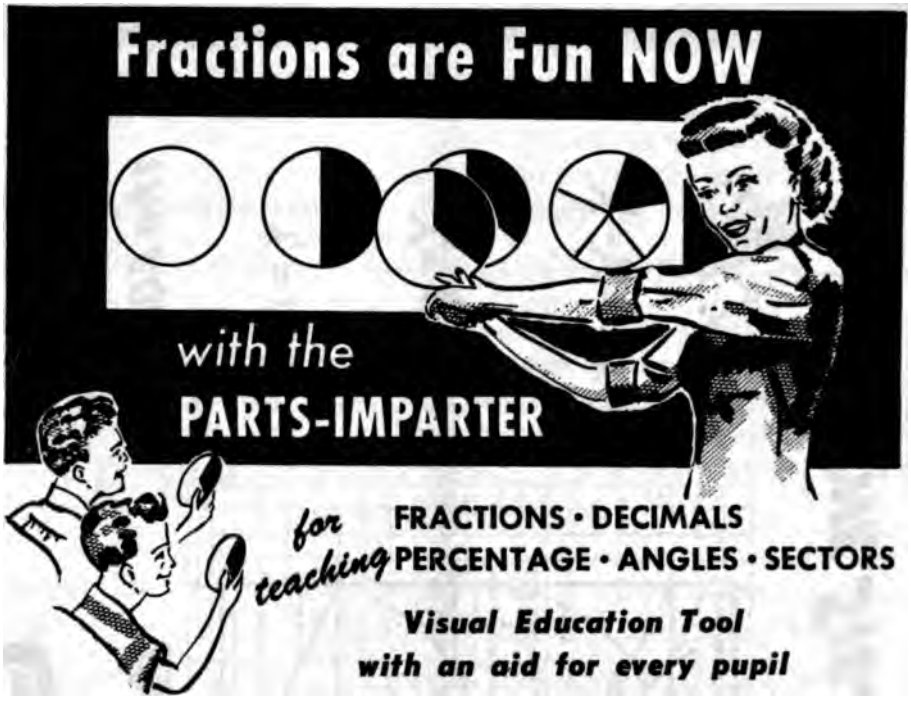

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Edited by JOHN R. MAYOR
The University of Wisconsin, Madison, Wisconsin

IN THIS issue this Department is given over entirely to data recently made available by the Federal Security Agency, Office of Education. The tables given below are taken from the Statistical Circular, No. 294, National Summary of Offerings and Enrollments in High School Subjects, 1948–49, by Mabel C. Rice, Survey Statistician. The study was carried out under the joint auspices of the Research and Statistical Service and the Division of Elementary and Secondary Education. This section includes those parts of Tables 1 and 2 which refer to enrollments in mathematics, except for the 7th and 8th grades, and a limited selection from other parts of these Tables so that enrollments in mathematics may be compared with those in other areas. The enrollments in 7th and 8th grade mathematics are

TABLE 1.—*Number and Percentage of Pupils Enrolled in High-School Subjects, and Number of States* Reporting Each Subject, 1948–49*

(All public secondary day schools, continental United States.)

Subject-field and subject	Number of States* reporting subject	Enrollment	
		Number	Percent[1]
1	2	3	4
Mathematics			
Elementary algebra	49	1,042,451	15.1
Intermediate algebra	49	372,152	5.4
General mathematics	49	649,810	9.4
Plane geometry	49	599,336	8.7
Solid geometry	49	93,944	1.4
Trigonometry	49	108,551	1.6
Advanced or college algebra	46	34,363	.5
Advanced general mathematics	44	42,600	.6
Mathematics review	24	12,332	.2
Analytics	9	468	[2]
Calculus	3	185	[2]
Other mathematics	20	1,617	[2]
English			
Ninth grade English	49	1,564,358	22.6
Tenth grade English	49	1,397,897	20.2
Eleventh grade English	49	1,198,018	17.3
Twelfth grade English	49	855,617	12.4
Speech and public speaking	49	246,213	3.6
Social Studies			
U.S. history, advanced	49	1,231,694	17.8
World history (grades 9–12)	49	876,432	12.7
World geography (grades 9–12)	49	271,969	3.9
American government or advanced civics	48	431,916	6.3
Problems of democracy	48	282,971	4.1
Economics	49	254,770	3.7
Science			
Ninth grade—general science	49	1,073,934	15.5
Biology	49	989,756	14.3
Chemistry	49	406,662	5.9
Physics	49	278,834	4.0

* In this study, the District of Columbia was counted as a State.
[1] Based on the total enrollment of 6,907,833 pupils in all public secondary day schools, 1948–49.
[2] Less than .05 percent, or less than 1 pupil in 2,000.

omitted since these grades are included in this study only when they are organized as a part of the secondary school.

Subjects other than mathematics, which show noticeable increases and decreases for the sixteen year period, have been selected from Table 2. The study reveals decreases on a percentage basis in enrollments in ancient and medieval history, physics, Latin, French, bookkeeping, shorthand, and economic geography, and increases over the same period for U. S. History, world history, geography, problems of democracy, general science, biology, Spanish, typewriting, home economics, agriculture, industrial subjects, physical education and music.

The following paragraphs, descriptive of coverage and procedures in the study, are quoted directly from The Statistical Circular, No. 294.

This circular presents national information on offerings and enrollments in high-school subjects for all public secondary day schools for the school year 1948–49 (Table 1). In addition,

(*Continued on page 421*)

TABLE 2.—*Number and Percentage of Pupils Enrolled in Subjects Taught in the Last 4 Years of High School in 15 or More States,*[1] *1948–49, and Percentage Enrolled in 1933–34*

(Public Secondary Day Schools, Continental United States)

Subject	1948–49		1933–34
	Number	Percent[2]	Percent[3]
1	2	3	4
Mathematics			
Algebra	1,448,966	26.8	30.4
General mathematics	704,742	13.1	7.4
Plane geometry	599,336	11.1	15.1
Solid Geometry	93,944	1.7	2.0
Trigonometry	108,551	2.0	1.3
Other Subjects			
U.S. history, advanced	1,231,694	22.8	17.3[4]
World history	876,432	16.2	11.9
Ancient and/or medieval history	79,473	1.5	11.0
Geography	301,652	5.6	2.1
Problems of democracy	282,971	5.2	3.5
General science	1,121,980	20.8	17.8
Biology	995,930	18.4	14.6
Physics	291,473	5.4	6.3
Spanish	443,995	8.2	6.2
Latin	422,304	7.8	16.0
French	255,375	4.7	10.9
Home Economics	1,304,846	24.2	16.7
Agriculture	364,185	6.7	3.6
Industrial subjects	1,434,302	26.6	21.0
Physical education	3,747,220	69.4	50.7
Music	1,625,235	30.1	25.5
Bookkeeping	472,163	8.7	9.9
Typewriting	1,216,142	22.5	16.7
Shorthand	421,635	7.8	9.0
Economic geography	90,045	1.7	4.0

[1] In some instances the elementary, applied or advanced courses included with a basic subject (see note) were not taught in as many as 15 States.

[2] Based on total enrollment of 5,399,452 in the last 4 years in public secondary day schools in the United States.

[3] Based on total enrollment of 4,496,514 in the last 4 years in 17,632 public secondary day schools in the United States reported in the 1933–34 survey.

[4] Includes enrollments in grades 9–12. Data for 1948–49 are for grades 10–12 only.

NOTE.—For improved historical comparability, the enrollments in all levels of a subject (elementary, applied, or advanced) have been combined with the enrollment in the basic subject. Thus, the enrollment in "algebra" includes the enrollments in elementary algebra, intermediate algebra, and advanced or college algebra.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

AN ANGLE BISECTOR DEVICE[1]

The angle bisector device suggested here involves the same principles for bisecting an angle as are employed in the usual ruler and compasses construction. The device itself is of interest because it helps add a note of generality to the usual geometrical construction, and also because it offers possibilities for several nice applications.

The device consists of four bars which may be cut from cardboard, wood, metal, or plastic. (See Fig. 1.) If cardboard is used the device can be assembled with brass paper fasteners. If the heavier materials are used $\frac{1}{8}''$ brass bolts and washers should be used. Suggested dimensions for the device are as follows: OR and OT, each $20''$ long; AF and BF, each $9''$; and OS, $24''$. As is illustrated by the diagram, bar OS must have a slot cut in it. In order that the device may be used to bisect a large range of angles, the slot in OS should be made as long as the strength of the material will permit.

To assemble the device, first locate A and B on bars OR and OT respectively so that $OA = OB = 7\frac{1}{2}''$. Drill holes at O, A, B, and F on the various bars as needed, insert bolts and washers as needed, and

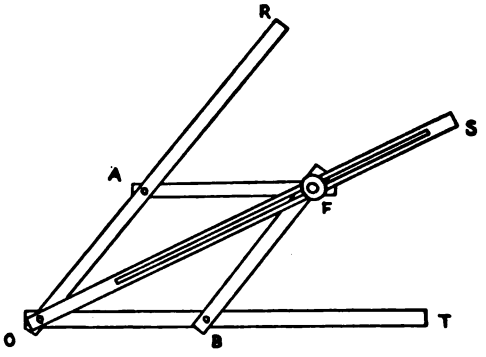

FIG. 1

the construction is complete. Since triangles BOF and AOF are congruent for all positions of F along OS, the device can be used to bisect any angle AOB within the limits allowed by the mechanical features of the instrument.

An examination of the device will reveal that it can also be used to demonstrate the principles involved in constructing the perpendicular bisector of a line segment. If A and B are placed at the extremities of a given line segment, then OF will be its perpendicular bisector. As another application the device may be used to find the center of a circular object. Suppose that we wish to find the center of the circular end of a log. If arms OR and OT are closed until both are tangent to the circle, then OS will pass through its center. By making a pencil mark along the slot, rotating the position of the device around the log, and then making a second mark again along the slot, the center of the end of the log is easily located. The proof of this fact depends upon the truth of the theorem which states that the bisector of the angle formed by two tangents to a circle from a point outside the circle passes through the center of the circle.

[1] The three devices described in this issue were designed and produced by tenth grade plane geometry students in the Mathematics Laboratory, Monroe High School, St. Paul, Minnesota.

Incenter Demonstrator

By applying the mechanical principles of the angle bisector device to the problem of bisecting the three angles of a general triangle simultaneously, it is possible to develop a second device which will show that the three angle bisectors are concurrent. This is not difficult, nor is the new device as complicated as it might appear from a casual inspection of the diagram. (See Fig. 2.) The device consists essentially of three angle bisectors, one mounted at each of the vertices of the general triangle *ABC*.

Fig. 2 the following dimensions are recommended: *DF*, *EF*, *HK*, *MK*, *RS*, and *ST*, each 4″ long; *AG* and *BN*, each 10″; *PC*, 14″; *AB*, 12″; *AC*, 17″; *BX*, 22″; and *TC*, 3″. The dimensions listed are generous enough so that holes needed to assemble the device may be drilled $\frac{1}{4}$″ from the ends of the bars. To facilitate reference to the diagram during construction all bars should be lettered to correspond with the diagram as soon as they are cut to size. All holes drilled in the various bars should be large enough to accommodate a $\frac{1}{8}$″ brass bolt.

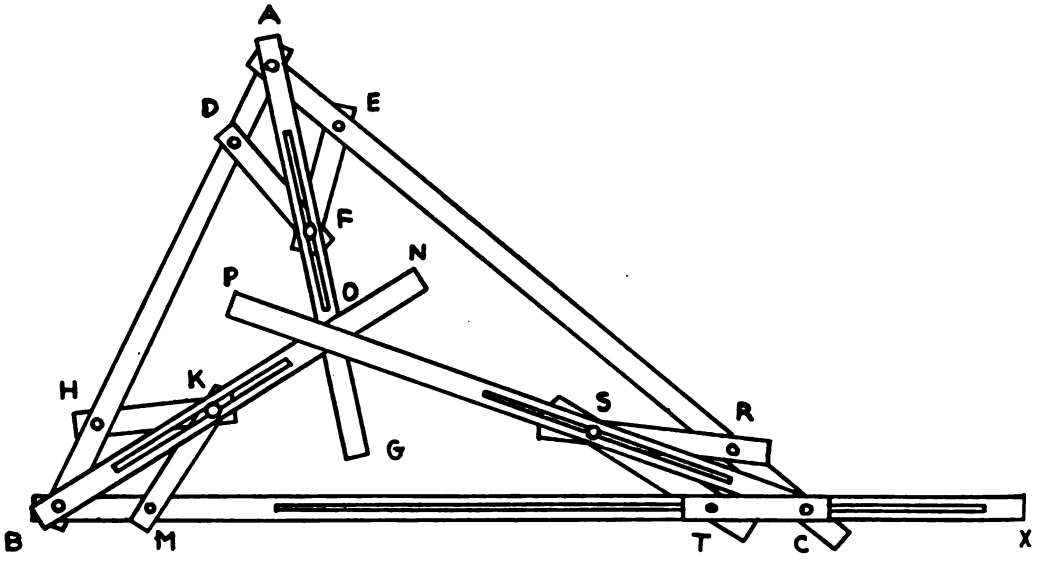

Fig. 2

Because of the rather large number of movable parts involved, the most suitable materials to use in constructing the incenter demonstrator are plastic and metal; however, it is also possible to use wood. An enterprising student in the department editor's plane geometry class last year built the device with laths which he obtained from a dismantled wooden venetian blind. The movable parts worked beautifully.

Unfortunately it is not very practical to use the dimensions suggested in the previous article to build the incenter device, because it would become too large to manipulate. For the device illustrated in

In drilling holes at *D* and *H* in bar *AB* the distances *DA* and *HB* must be made $2\frac{1}{2}$″ from center to center. The same holds true for the lengths *MB*, *CR*, *TC*, and *AE*. Slots cut in bars *AG*, *BN*, and *CP* need not be made more than 4″ long, nor need they be cut closer to *A*, *B*, and *C* than $2\frac{1}{2}$″.

The device can be assembled in much the same manner as the angle bisector device. There is only one difference; bar *TC* is a sliding bar. Both its ends slide along the slot in bar *BX*. This feature, however, presents no special difficulty. If all parts have been cut according to instructions, the single sliding bar *TC* will

fit into position automatically. To demonstrate that the angle bisectors are concurrent for any triangle which can be produced with the device simply slide the vertex C along the slot in BX.

A Harmonic Divider

Another device which may be built by applying the basic ideas of the angle bisector device is what might be called an harmonic divider. Its construction is based on a direct application of the theorem which states that the bisector of an interior angle of a triangle and the bisector of the adjacent exterior angle divide the opposite side harmonically. For triangle ABC this means that if AE and AF are respectively the bisectors of angles BAC and CAH, then $BQ/QC = BP/PC$. (See Fig. 3.)

The trick in the construction is to develop a system of movable parts such that AE and AF will bisect angle BAC and the adjacent exterior angle simultaneously, while at the same time leaving bar AD free to intersect bar BM anywhere along its length.

It's really quite simple—almost as simple as building the angle bisector device alone. An inspection of the diagram will reveal that the two bars AE and AF are all one piece. They are joined together in the same manner as the arms of a carpenter's square. If a hole is drilled in the "square" at A, then both arms will pivot together always 90° apart. But the bisectors of an interior and adjacent exterior angle of a triangle are perpendicular to each other, so if AE can be made to bisect angle BAC for all positions of C along BM, then AF will automatically bisect angle CAH. Angle BAC can be bisected mechanically for all positions of C by mounting an angle bisector device at A which uses bar AE of the "square" as its slotted bisector bar.

The "square" may be constructed of wood, sheet metal, or plastic. If wood is used the two bars AE and AF may be screwed to the "leg" edges of a small draftsman's triangle in order to form the necessary right angle, or they may be screwed to the arms of a small flat angle iron. If sheet metal or plastic are used, the "square" can be cut out as one piece in the same manner as the carpenter's square. Incidentally, sheet aluminum works excellently. Convenient dimensions for AE and AF are 10″ and 20″ respectively. Suggested dimensions for the other bars of the device are as follows: BH and AD, each 12″ long; OR and OS, each $3\frac{1}{2}$″; and BM, 24″.

(*Continued on page 421*)

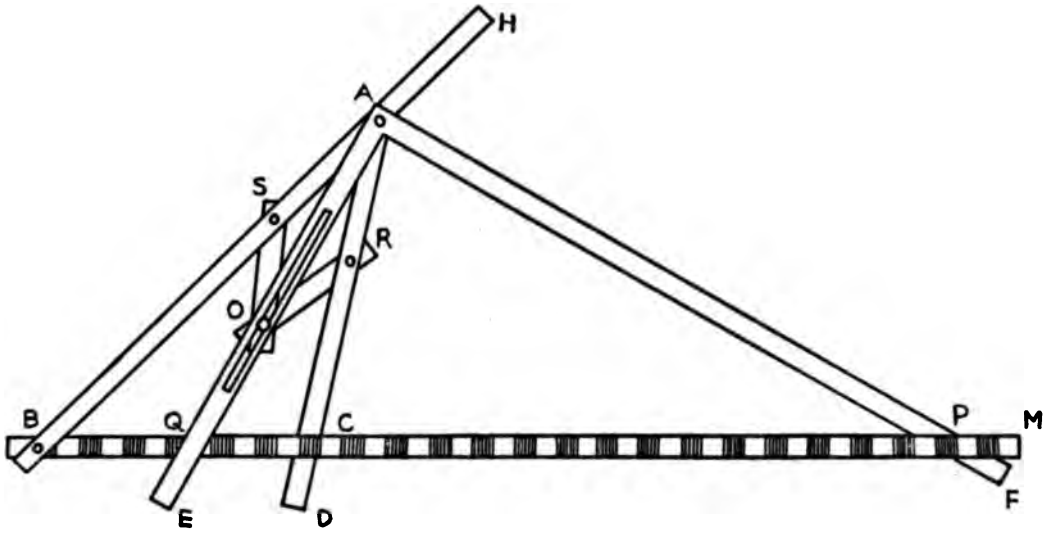

Fig. 3

BOOK SECTION

Edited by JOSEPH J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

ELEMENTARY SCHOOL

Row-Peterson Arithmetic, Primer, by Harry G. Wheat, Geraldine Kauffman, and Harl R. Douglass. With Workbook. Paper, 64 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. Primer, $0.60; Workbook, $0.40.

Row-Peterson Arithmetic, Book One, by Harry G. Wheat, Geraldine Kauffman, and Harl R. Douglass. With Workbook, Paper, 96 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. Book One, $0.72; Workbook, $0.40.

Manual, Row-Peterson Arithmetic, Primer and Book One, by Margaret Leckie Wheat and Harry G. Wheat. Paper, 144 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $0.60.

Row-Peterson Arithmetic, Book Two, by Harry G. Wheat, Geraldine Kauffman and Harl R. Douglass. With Workbook. Cloth, 240 pages, 1951. Row-Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $1.72; Workbook, $0.48.

Manual, Row-Peterson Arithmetic Book Two, by Margaret Leckie Wheat and Harry Grove Wheat. Paper, 188 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $0.60.

HIGH SCHOOL

1. Special Courses in Mathematics

Machine Shop Mathematics, Second Edition, by Aaron Axelrod, New York University. Cloth, xi+359 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $3.60.

Practical Mathematics, Part IV, Trigonometry and Logarithms by Claude I. Palmer, late Professor of Mathematics and Dean of Students, Armour Institute of Technology; and Samuel F. Bibb, Illinois Institute of Technology. Fifth Edition. Cloth, xii+193 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $2.60.

COLLEGE

1. Algebra

Algebra for Commerce and Liberal Arts, by Alvin K. Bettinger, and Wendell A. Dwyer, The Creighton University. Cloth, xi+225 pages, 1951. Pitman Publishing Corporation, 2 West 45th Street, New York, N. Y. $3.00.

Intermediate Algebra, by Paul K. Rees, Louisiana State University; and Fred W. Sparks, Texas Technological College. Cloth, viii+328 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $3.25.

College Algebra, by Henry L. Rietz, late of the University of Iowa; Arthur R. Crathorne, late of the University of Illinois; Fifth Edition revised by J. William Peters, University of Illinois. Cloth, xv+387 pages, 1951. Henry Holt and Company, 257 Fourth Avenue, New York, 10, N. Y. $2.95.

2. Analytic Geometry

Analytic Geometry, Second Edition, by John W. Cell, North Carolina State College. Cloth, xii+326 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.75.

Brief Course in Analytics, Revised, by M. A. Hill, Jr., and J. B. Linker, University of North Carolina. Cloth, xi+224 pages, 1951. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.40.

3. Statistics

Index Numbers, by Bruce D. Mudgett, University of Minnesota. Cloth, x+135 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.00.

4. Advanced Mathematics

Differential Equations, Third Edition, Revised, by H. B. Phillips, Massachusetts Institute of Technology. Cloth, viii+149 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.00.

Linear Computations, Paul S. Dwyer, University of Michigan. Cloth, xi+344 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $6.00.

Introduction to the Theory of Algebraic Functions of One Variable, by C. Chevalley. Mathematical Surveys Number VI. Cloth, xi+188 pages, 1951. American Mathematical Society, 531 West 116th Street, New York. $4.00.

Tables Relating to Mathieu Functions, by the National Bureau of Standards. Cloth, xlvii +278 pages, 1951. Columbia University Press, 2960 Broadway, New York 27, N. Y. $8.00.

Fourier Transforms, Ian N. Sneddon, University of Glasgow. Cloth, xii+542 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $10.00.

TEACHING OF MATHEMATICS

How to Teach Arithmetic, by Harry G. Wheat, West Virginia University. Cloth, vii+438 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $3.00.

Teaching the New Arithmetic, Second Edition, by Guy M. Wilson. Cloth, xiv+438 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $4.50.

REVIEWS

Cotton Mill Mathematics (Second Edition), Thomas H. Quigley and W. S. Smith. Atlanta, Turner E. Smith and Co., 1930. xiv+322 pp., $2.67.

This text is prepared for use in the classroom by school age as well as adult students who have an intimate knowledge of the cotton mill but who have little mathematical background and yet wish to further themselves in the industry.

The book is divided into two parts. Part I deals with basic arithmetic. It covers sequentially fundamental operations from addition of whole numbers through fractions, decimals and percentage. Throughout this part there are generous practical application problems concerning the various specializations of a cotton mill.

Part 2 has no set sequence but is divided into chapters to permit selection of more technical topics posed by each of the specializations in a cotton mill.

There is no index, but there is a table of contents listing the chapter titles. These chapters are numerous and each one covers a very limited phase of the text. Answers to all exercises are included.—JOHN T. LADD, Highland Park High School, Highland Park, Michigan.

Essentials of Business Arithmetic (Third Edition), Edward M. Kanzer and William L. Schaaf. Boston, D. C. Heath and Company 1950. viii+476 pp., $2.36.

This attractive and well-written text is planned for a full-year course of study but may be used conveniently for a shorter course. The first half of the book embodies the subject matter of elementary business arithmetic developed in one-lesson units which can be completed in approximately 85 class periods. These units are introduced by means of simple, concrete problems based on actual business activities; much care is taken to familiarize the student with business procedure and terminology in order to increase understanding and insight. The second half of the book contains important supplementary material.

A feature of the text is a wealth of good exercises. These are classified into A, B, and C assignments to provide for individual differences. Topical reviews and cumulative reviews are included at frequent intervals. General review problems and specimen examinations are given at the end of the book.

The reviewer recommends this text to the attention of all teachers of business arithmetic. —HAROLD D. LARSEN, Albion College, Albion, Michigan.

Plane and Spherical Trigonometry (Fifth Edition), Claude I. Palmer, Charles W. Leigh and Spofford H. Kimball. New York, McGraw-Hill Book Company, Inc., 1950. xiii+368 pp., $3.25.

The order of topics has been maintained, in this fifth edition, including the early introduction of radian measure, inverse functions and equations. This is a commendable feature, as many teachers follow the text and if these topics are included in the latter part of the book they are omitted from the student's experiences.

Instead of reducing angle to the explicit form $n \cdot 90° + \theta$ in finding values of functions of positive angles larger than 90°, or negative angles, use is made of the acute angle between the terminal side of the angle and the x-axis. The treatment of the tan 90° is very well handled and should avoid the situation in which the student believes tan 90° exists and is equal to the number infinity, which is not the case.

The text is well organized, diagrams are plentiful and well labeled, important concepts are given in bold type, formulas are derived for the students, there are many practical applications and related problems included throughout the text with an entire chapter devoted to this phase of the work and greater emphasis is given to the curves of the functions and less to line representation.

The discussion of logarithms is the final chapter in the text. This will necessitate a change in the sequence of teaching as logarithms are used in solving triangles in previous chapters. An alert and skillful teacher will have no difficulty with this situation. The book does not stress logarithms, which is so common in many trigonometry texts.

The book could be used by a skillful teacher in high school, but I believe it is better suited for more mature minds.—GEORGE L. KEPPERS, Iowa State Teachers College, Cedar Falls, Iowa.

Plane and Spherical Trigonometry (Revised Edition), John A. Northcott. New York, Rinehart and Company, Inc., 1950. ix+234 pp., $3.50.

Dealing with a book that is in its revised edition, we know that changes that were made have been suggested by experience. This text certainly shows the experience of many teachers who have suggested that more be done to help students realize the great importance of the analysis of trigonometric functions.

Here the plotting of trigonometric curves as well as the inverse functions are given their rightful importance. An abundance of figures and well-worded and clear explanations further enhance the worth of this volume for those stu-

dents who plan to continue their mathematical studies in analytic geometry and the calculus.

Problems in the solution of triangles, although given later in this text than in many others, are plentiful and will give sufficient practice to those who plan to use a knowledge of trigonometry almost immediately after completing the course.

The section dealing with spherical trigonometry has been left almost the same as in the first edition. Excellent treatment of the topics chosen in this section is what undoubtedly suggested that this section remain as it was in the beginning.

Students who use this text will be helped greatly in reaching the important first steps of what is so often called "Mathematical maturity."—REV. BONAVENTURE KNAEBEL, St. Meinrad Minor Seminary, St. Meinrad, Indiana.

Plane and Spherical Trigonometry, H. L. Rietz, J. F. Reilly, and Roscoe Woods. New York, The Macmillan Company, 1950. xiii+204 pp., $3.00. Without tables, $2.75.

The authors of this text recommend its use in technical schools and first year college programs. It would seem possible that high school teachers could use this book to advantage in any class having capable students. No attempt is made to create interest in trigonometry through appeals to the student; instead, significant ideas are presented in problems designed to arouse the native interests of the mathematically inclined. College instructors will find simplified presentations to be within the grasp of the majority, whereas this is not necessarily true for high school-age groups. The teacher will need to supply items of historical and contemporary nature having interest value. Items of explanation do not attempt applications, but are devoted to brief, concise, discussions of various topics. Many exercises pertain to physics and problems workmen might encounter. Selectivity is possible because each set of exercises contains more problems than are necessary. Answers to the odd numbered problems are included.—RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Topology of Manifolds, R. L. Wilder. New York, American Mathematical Society, 1949. ix+402 pp. $7.00.

This book is volume 32 in the Colloquium Publications of the American Mathematical Society. As is customary for books of this series, the reader is expected to have reached a rather advanced stage of mathematical maturity. However, it is true that the book is essentially self-contained in that no technical knowledge is assumed as a prerequisite.

There are twelve chapters of which at least half are introductory in the sense that they develop the concepts and techniques necessary for stating and solving the problems of interest to the author. The first chapter constitutes an introduction to topology and also contains historical remarks on topology. Chapter V introduces the algebraic topology needed for later chapters including a discussion of homology theory, which is basic for what follows. The results in the final chapter are almost entirely due to the author and most of them are new. An appendix on unsolved problems and detailed bibliographical comments are included.—D. H. POTTS, Northwestern University, Evanston, Illinois.

The Location of Critical Points of Analytic and Harmonic Functions, J. L. Walsh. New York, American Mathematical Society, 1950. viii +384 pp., $6.00.

This book is volume 34 of the Colloquium Publications series. The author uses the term critical point to include both zeros of the derivative of an analytic function of a complex variable and points where the two first partial derivatives of a harmonic function vanish. The problem investigated is: given a function, determine the regions in the complex plane which contains all (or none) of the critical points. The subject has its historical origin in the work of Gauss. A typical example of the results is the theorem of Lucas: Let $p(z)$ be a polynomial and let R be the smallest convex set on which the zeros of $p(z)$ lie. Then all the zeros of the derivative $p'(z)$ also lie on R.

The avowed purpose of the author was to assemble and unify much of the known material on the subject, but results which are new have also been included. Although many of the problems are of a topological nature the methods employed are those of classical analysis. Accordingly the requisite background is a course on functions of a complex variable.—D. H. POTTS, Northwestern University, Evanston, Illinois.

Mathematics of Relativity, G. Y. Rainich. New York, John Wiley and Sons, Inc., 1950. vii +173 pp., $3.50.

This book is an extremely fine introduction to the mathematics of relativity. All mathematical tools beyond elementary differential equations are developed within the book. However, a certain amount of background in college physics is essential. An outline of the presentation can roughly be given as follows. In Chapter 1 certain basic mathematical expressions of classical physics are put into a new form. This new form suggests a four-dimensional geometry, which is then developed in Chapter 2. Chapter 3 is entitled "Special Relativity" and explores the consequences of the application of four-dimensional geometry to physics. Defects of this application are pointed out and the generalization to curved spaces is made in Chapter 4 to rectify the situation. The application of the geometry thus obtained to physics is carried out in Chapter 5 on "General Relativity."

As the author points out, most of the material is not new. But there is one important exception. It is shown, contrary to previous beliefs, that the original Einstein theory is ade-

quate for both gravitational and electromagnetic phenomena. The question of quantum phenomena is still left open.

Although the reviewer is in agreement with the author on the point that "complete clarity in presenting the theory of relativity can be achieved only by stressing the mathematical aspect," it was felt that the physical and philosophical aspects could have stood some amplification. Certainly a knowledge of the historical background of the theory of relativity would increase the reader's grasp of the situation.—D. H. Potts, Northwestern University, Evanston, Illinois.

Theory of Sets. E. Kamke. Translated from the second edition of *Mengenlehre* by F. Bagemihl. New York, Dover Publication, 1950. viii+152 pp. $2.45.

This is an excellent translation of a very fine book. It is a readable introduction to the general theory of sets, and is liberally supplied with examples. Even an almost total lack of background would not prevent an interested student from learning a great deal about the subject.

After discussion of operations with sets, and of the concepts of enumerability and non-enumerability, the author takes up in succession arbitrary sets and their cardinal numbers, ordered sets and their order types, well-ordered sets and their ordinal numbers, and operations with ordinal numbers. At the close are some remarks on concepts which lead to classical paradoxes (such as the set of all sets which do not contain themselves as elements), and on methods of avoiding these paradoxes.—T. C. Holyoke, Northwestern University, Evanston, Illinois.

What Is Going on in Your School?

(*Continued from page 414*)

the enrollments in subjects reported in the last 4 years of high school in 15 or more States in 1948–49 are compared with enrollments in the same subjects in 1933–34—the last previous year for which national data are available (Table 2). A complete report of the findings of the present survey is scheduled for publication in the near future.

In February 1949, questionnaires were mailed to all public secondary day schools (except the evening and ungraded) enrolling 500 or more pupils, and to one-half of the schools enrolling fewer than 500 pupils. In all, 3,615 large secondary schools and 10,134 of the smaller secondary schools were circularized. Usable returns were received from 91.8 per cent of the larger schools and from 75.1 per cent of the smaller. This represents a 79.5 per cent response from all schools to which the questionnaire was sent.

The enrollments in the field of mathematics and in the field of science exceed 50 per cent of the pupils in secondary schools (Table 1). Elementary algebra enrolls a greater per cent than any other single subject in mathematics. Among the science courses the largest percentage enrollments occur in general science, biology, chemistry, and physics in that order.

By making appropriate adjustments for sampling and non-response, the 1948–49 survey presents data for *all* public secondary day schools in continental United States. The 1933–34 study, on the other hand, was based on replies from 17,362 secondary schools or approximately 70.2 per cent of the public secondary schools in the United States at that time.

Devices

(*Continued from page 417*)

Holes should be drilled in bar BH at B, S, and A so that $BA = 10''$ and $AS = 3\frac{1}{2}''$. Bar AD needs holes at A and R; AR must be $3\frac{1}{2}''$ from center to center. Bars OR and OS must each have holes drilled so that $OR = OS = 3''$. The only hole needed in bar BM is at B. All holes should be large enough to accommodate $\frac{1}{8}''$ brass bolts. The slot in bar AE need not approach A nearer than $1\frac{1}{2}''$, nor need it be more than $5\frac{1}{2}''$ long. The device can be assembled by inserting $\frac{1}{8}''$ brass bolts at B, A, S, R, and O, and securing them loosely with washers and nuts.

If bar BM is marked off into half inches or quarter inches, the meaning of harmonic division for segment BC can be demonstrated concretely for all positions of C and the corresponding positions of Q and P along BM. It is of interest that when $BQ = QC$, then AQ is perpendicular to BC and AF is parallel to BM. For this position of Q the ratio of internal division is unity ($BQ/QC = 1$), and the external division point is the point at infinity.

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Laboratory Mathematics

A PROMINENT development during the past few years has been a lively interest in laboratory methods, involving the use of experiments, models, instruments and a variety of equipment. In this connection, the Eighteenth Yearbook (1945) of the National Council of Teachers of Mathematics contains a wealth of source material and bibliographic data. The purpose of the following notes is to supplement this data and bring it up to date with a minimum of repetition. References to visual aids, in the sense of pictures, charts, slides, film strips, motion pictures, etc., have been deliberately excluded.

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(Continued on page 428)

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
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The Question: What is the place of "mental arithmetic" in the modern teaching of arithmetic?

The Study: Boulware, C. Elwood. *The Emerging Concept of Mental Arithmetic.* Teachers College, Columbia University. 1950.

The purpose of this study was "to show the emerging concept of the meaning and place of mental arithmetic in computation and problem-solving and to consider its teaching possibilities as an instrument of mental growth and development" (p. 19). From an analysis of recent textbooks and professional literature Dr. Boulware decided that "mental arithmetic consists of the various processes by which related fundamental operations in arithmetic are performed by use of the nature of the number system and without the use of algorisms" (p. 7).

In the course of his investigations the author examined seventy-five textbooks printed in the nineteenth and twentieth centuries. He discovered a kind of cyclic trend. Before 1821 the emphasis was on mechanical performance of computations; from 1821 to about 1871 or 1881 he found the stress was on so-called intellectual arithmetic. From about 1891 to 1925 the texts swung back to the operational emphasis. Since 1930 or 1935 the amount of attention given to the development of meaning and understanding in the learning of arithmetic has reflected again a sort of intellectual emphasis.

Dr. Boulware noted that the modern teaching of arithmetic had been influenced considerably by the twentieth century development of the psychology of learning. The conception of the learning process as first involving the perception of gross wholes followed by a stage of differentiation in which refinement and increased clarity of insight appeared was felt to have its counterparts in that arithmetic pedagogy which dealt with grouping and regrouping instead of counting, which sought the development of general ideas instead of special skills and which called for wise judgments, the seeing of relationships and the weighing of values. The use of relational thinking was illustrated in the case of operations by introducing the notions of commutability, associativeness and distributiveness; in the case of number relationships per se the discovery of 9×5 from 10×5 was given as an example.

The new psychology not only presented the notions of gross wholes in the initial stages of learning followed by differentiation but also included the idea of integration. The most important instance of this was the use of ten as the base of a place-value number system; this concept permeated all of the operations on the natural numbers. A second instance was in the various forms which unity could take especially in dealing with fractions.

Finally, a synthesis of the notions of mental arithmetic and the newer learning emphasis was portrayed through five case studies in which students reasoned through problems in their own way, demonstrating unusual original perception of arithmetical relationships through a mental arithmetic approach and revealing the implementation of the newer psychology of learning.

The Question: Do teachers of mathematics approve and practice pupil-centered methods of teaching?

The Study: Syer, Henry W. *Pupil-Cen-*

tered Methods of Teaching Mathematics. Harvard University. June 1950.

During the past twenty-five years the educational spotlight has increasingly played on the learner as a participant and active doer. At least, Dr. Syer found that leaders in education writing in general books on education, in treatises on methods of teaching mathematics, in reports of commissions and committees and in research studies had described approvingly the use of pupil-centered methods. These procedures emphasized sense perception and experimentation in learning mathematics, demanded that concrete experiences precede verbal generalizations, preached that verbalism was not learning, pointed out that physical activity went with mental activity, stated that learning took place in social situations and that methods should be adapted to individual differences. The principal purpose of Dr. Syer's study was to determine whether these ideas were accepted and practiced by the classroom teacher of mathematics.

In gathering his data for grades seven through twelve the investigator sent a questionnaire to 784 of the nations outstanding schools selected with the aid of several organizations considered competent in making the necessary judgments. Both teachers of mathematics and administrators were reached. In the New England area 72 classes were visited and 354 pupils and 75 teachers of this section of the country were interviewed. As part of the study 61 research studies comparing pupil-centered with conventional methods were examined and analyzed for their findings.

Findings

Noting that the methods known as the lecture, recitation, discussion, supervised study and laboratory reflected, in that order, an increasing degree of student participation Dr. Syer found from the questionnaire, to which approximately sixty per cent of the schools responded, that about eighty-six per cent of class time was spent in lecture and recitation, about ten per cent in pupil-centered activities and about six per cent in clerical matters. In the case of the classroom observations the discussion method consumed about four per cent of the class time, the laboratory method less than one per cent and supervised study three to fourteen per cent depending on the definition of the term. From the interviews with the teachers Dr. Syer found that the teachers felt that the pupil-centered methods had value but that the need to change the methods in vogue was not pressing. The teachers reported that the principal sources of their methods were their colleagues and "experience." Methods courses ranked third as a source. Among the factors impeding change the teachers named, in order, the teaching situation, themselves and the pupils. Fifty-two out of seventy-five did not feel that they were restricted in their freedom to vary their methods. Size of class was a dominant element in the complaint about the teaching situation. The pupils who were interviewed were aware that the methods used needed improvement but their suggestions did not veer far from the climate of method that they had contacted.

The sixty-one research studies yielded two hundred seventy-nine measures of outcomes. One hundred fifty of these favored the pupil-centered methods, sixty the conventional and sixty-nine showed no statistically significant advantage. Dr. Syer found that the quality of the research varied widely; in fact, he felt that he could classify only twenty of the studies as excellent or good. Omission of important data, failure to match the groups being compared and the use of instruments of unknown reliability and validity were common.

Finally, by use of chi square techniques the investigator sought to find signs that might reveal where a school stood with respect to pupil-centered methods. Disclaiming any causal implications and suggesting that further investigation was

needed Dr. Syer listed fifteen descriptive factors and fifty-three methods factors as possible indicators. Among the former were the section of the country, attitude toward discussing methods, the budget of the mathematics department and importance attached to individual differences in learning rates. Among the latter the questionnaire data analysis revealed the presence of a mathematics collection, the formation of a mathematics club, the use of mathematics plays and exhibits and the practice of making community surveys of the uses of mathematics as guideposts to the kinds of methods used.

Contemplating the results of the study the reader will either be swept with pessimism or stirred by the challenge depending on his personal characteristics.

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(*Continued from page 425*)

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ATTENDANCE RECORD OF THE TWENTY-NINTH ANNUAL MEETING

Hotel William Penn, Pittsburgh, Pa.
March 28-31, 1951

Name of State	No.	Name of State	No.	Name of State	No.
Alabama	3	Kentucky	1	Oklahoma	2
Arkansas	1	Louisiana	2	Pennsylvania	624
California	1	Maryland	17	Rhode Island	2
Colorado	2	Massachusetts	16	South Carolina	2
Connecticut	4	Michigan	20	Tennessee	8
Delaware	2	Minnesota	1	Texas	2
District of Columbia	14	Missouri	3	Virginia	14
Florida	5	Nebraska	2	West Virginia	27
Georgia	4	New Hampshire	3	Wisconsin	9
Illinois	23	New Jersey	21	Canada	8
Indiana	13	New York	66		
Iowa	5	North Carolina	4	35 States and Canada	
Kansas	1	Ohio	96	Total	1026

AIDS TO TEACHING

Edited by

HENRY W. SYER
School of Education
Boston University
Boston, Massachusetts

and

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.62—Credit for Consumers

B.63—Loan Sharks and Their Victims

Public Affairs Pamphlets, 22 East 48th Street, New York 16, N. Y.

Booklets; 5½″×8½″; 32 pages each; 1–9 copies are $.20 each, discounts on larger quantities.

Description of B. 63: This booklet, by LeBaron R. Foster, discusses the various sources of consumer credit, how to use credit to advantage, how to tell the rate you are paying, why credit is expensive, and outlines a program for social protection. There is a summary of warnings at the end and an excellent bibliography.

Description of B. 64: This publication by William T. Foster describes the devious ways used by small loan companies to charge exorbitant interest rates for small loans. It describes the various ways in which loan sharks operate to evade the law and to prevent effective laws from being passed. The effect of these illegitimate business enterprises is contrasted with the beneficial results of enforceable laws. It concludes its message by giving a series of specific suggestions on how to avoid being "duped" by a loan shark. A bibliography for further reading is included.

Appraisal of B. 63 and B. 64: These descriptions are more general than some booklets concerning credit and will appeal to a more mature audience. Although such consumer mathematics is more prevalent in junior high school courses, these booklets are written for the senior high school and adult level. Since general mathematics is appearing in grades ten, eleven and twelve, they should be very helpful. By using the facts outlined in these booklets, pupils can investigate the loan practices of each local community, with advantage to both themselves and the community.

B. 64—Know Your Money

U.S. Secret Service, Treasury Department, Washington, D. C.

Booklet; 6″×9″; 32 pages; Free.

Description: This useful booklet is filled with many extremely interesting pictures. It is based upon the idea that "the best check to any evil is always that provided by an educated public." By telling the public how to detect counterfeit money and forged government checks, the Secret Service hopes to cut down such illegal practices. There is first a short history of money and description of our paper currency. Throughout there are excellent enlarged pictures of good and of counterfeit money. Then there are stories from the Secret Service files; a description of what to do when money burns or wears out; the scientific side of detection; how to detect counterfeit coins; and a section on government checks.

Appraisal: This book makes interesting reading even disregarding its educational value. Certainly copies should be around for supplementary reading. There is no reason why time could not be spent on a very interesting unit on counterfeit money with talks by bank experts, displays of counterfeit money, contests to see who can detect the difference in specimens displayed. The old cry that this is not in the

curriculum of the mathematics department will not deter those who see the usefulness of this topic and who see that the justification is just as strong in mathematics as in some other subjects. If social studies or science teachers are already doing this, it may be left alone, but if not we should reach for some of the lively topics to brighten our work. The classes will pounce on it.

B. 65—What Everybody Ought to Know about this Stock and Bond Business

Merrill Lynch, Pierce, Fenner & Beane, 70 Pine Street, New York 5, N. Y.

Pamphlet; $8\frac{1}{2}'' \times 11''$; 6 pages; Free.

Description: This pamphlet was written to answer the most common questions asked stock brokers about stocks and bonds. It answers questions such as: What are stocks and bonds? Why should anybody buy stocks? Who may buy stocks and bonds? When should you buy stocks and bonds? How do you buy and sell stocks and bonds? What are the different kinds of stocks? What are the factors that affect the prices of stock on the stock market? Its main objective is to promote the careful purchase or sale of stocks and bonds for investment purposes. It suggests that this investment be made in an understanding manner with surplus funds that are not needed for necessary expenditures, insurance or savings sufficient to meet emergencies.

Appraisal: This pamphlet is written in a simple, concise manner so that it is readable to the man on the street. Thus, it is of an appropriate level for junior or senior high school students. Although it is written to promote the sale of stocks and bonds, it emphasizes the need for caution, such as "—no one should speculate unless he can afford to take risks" or "investigate —then invest" or "—not everybody should buy stocks and bonds." It also emphasizes the purchase of stocks and bonds for investment purposes rather than for speculation. It does not include any mathematical problems, but does use many terms and figures such as par value and stock market quotations.

B. 66—Vernier Tools

Brown & Sharpe Manufacturing Co, Providence 1, R. I.

Pamphlet; $6'' \times 9''$; 3 pages; Free.

Description: This brief pamphlet was written to teach one how to read a vernier. After describing the principle of the vernier, it gives sample problems with drawings to illustrate how to read the vernier. The examples show why the vernier reading gives a certain decimal value. The examples include English and metric units of measure. Additional instructions are given on how to operate vernier tools to attain maximum accuracy.

Appraisal: This pamphlet should be used with a large model of a vernier to give instruction in measurement and the reading of a vernier. The print is small and the examples somewhat complex for immature high school students. However, the teacher will find it valuable in showing how to teach the principle of the vernier.

EQUIPMENT

E. 43—Arithmetic Readiness Cards: Grouping

Scott, Foresman and Company, 114 East 23d Street, New York 10, N. Y.

Set of cards; $6\frac{1}{2}'' \times 8\frac{1}{2}''$; number cutouts; teacher's guidebook; in box; $3.20 (educational discount).

E. 44—Arithmetic Readiness Cards: Number System

Scott, Foresman and Company; 114 East 23d Street; New York 10, N. Y.

Set of cards; $6\frac{1}{2}'' \times 8\frac{1}{2}''$; number cutouts; tabulation cutouts; teacher's guidebook; in box; $3.20 (educational discount).

Description of E. 43: This set contains 54 tagboard cards with printed pictures on both sides, 9 diecut cards which are separated into 81 cutouts each printed

with a number from 2 to 10, and a teacher's guidebook. The pictures are numbered 1 to 108. Those numbered from 1 through 54 show organized groups of objects arranged in smaller sub-groups, whereas those numbered from 55 through 108 show objects in unorganized groups. Each card has a cut corner which aids in arranging the cards so that all pictures or organized groups or all pictures of unorganized groups will be face up. Each card also has two slots into which a number cutout can be inserted. These cards aim to supply the child with pictorial experiences in the transfer from recognition of small number groups to the understanding of abstract numbers.

Description of E. 44: This set contains 54 tagboard cards with printed pictures on both sides, 9 diecut cards which are separated into 135 cutout pictures of sticks or tally marks, 9 diecut cards which are separated into 135 cutouts each printed with a number digit 0 to 9, and a teacher's guidebook. The pictures are numbered from 1 to 108. Those numbered from 1 through 54 show organized groups of objects arranged in sub-groups of ten. These sub-groups are arranged in pyramid formation to facilitate identification of a group of ten without counting. Pictures numbered from 55 through 108 are arranged in a similar manner except that groups of ten are shown as bags, bundles, boxes, etc., which the child is required to accept as a group of ten. Each card has a cut corner which aids in arranging the cards so that all groups employing pyramid formation will be face up. Each card also has four slots, into which two tabulation cut-outs or two number cut-outs may be inserted, with the exception of 9 cards which have six slots which allow three insertions. These cards aim to supply pictorial material useful in developing the basic concepts of our number system.

Appraisal of E. 43 and E. 44: These sets of cards provide attractive material useful in developing the concepts and relationships of abstract numbers. They may be used independently or correlated with the common procedure of employing disks, sticks and other objects. They are appropriate for teaching methods which allow the pupils to discover the ideas involved in grouping and in our number system.

The cards appear to be reasonably durable. They are soiled easily, but can be cleaned readily with a gum eraser. The variety of brightly colored drawings of toys, animals and similar objects should be familiar to most children in the early grades. (Reviewed by Robert H. Wyllie, Boston University, Boston, Mass.)

E. 45—Experimenting with Numbers

Houghton Mifflin Company, 2 Park Street, Boston 7, Mass.

Number Kit (See below); Designed by Catherine Stern; $24.00 net to teachers and schools.

Description: A set of materials that represents a new approach to number readiness. The box of materials contains:

1 Counting Board with Number Guide
1 Unit Box filled with Unit Blocks (two each of Block 1–9; one 10-Block)
1 Unit Box filled with 100 Cubes
1 Set of 10 Number Cases
1 Extra Set of 10 Unit Blocks
2 Sets of 10 Number Markers
1 Zero, 1 Plus Sign, 1 Equal Sign
A Teacher's Manual

The Counting Board contains 10 vertical grooves into which blocks of the corresponding size may be inserted. Each block is scored to show its cube-units: the 1-block is 1 unit long, the 2-block 2 units, the 3-block 3 units and so on up to the 10-block which is 10 units long. Each block is a different color. These Unit Blocks are used to represent the number range from 1 to 10. The grooves in the counting board are marked like the blocks. At the top of each groove is a removable marker which has the corresponding number symbol on it.

The Pattern Boards contain from 1 to 10 blanks in two columns. These boards are made to receive the single cubes. The first board contains 1 blank, the second

has 2 and the last one has 10 blanks. These boards may be used to teach patterns of certain configurations. The pattern of one number is related to the pattern that precedes and follows it. The Pattern Boards give meaning to odd and even numbers and aid in the mastery of some addition and subtraction facts.

The Unit Box is a case 10 units square. It is used to hold the block pairs such as 2 and 8, 6 and 4, etc., that tell the story of 10. The Number Cases from 1–9 are also square. These cases help the pupil in discovering the block pairs (combinations) that fill each case.

The Number Markers 1 to 10 and symbols for plus, equal, and zero are used to record the addition facts that the children discover in the number cases. The oral addition stories can be recorded in this manner before the actual writing of numbers is taught.

Appraisal #1: Experimenting with Numbers is the beginner's course in Structural Arithmetic which is based on measuring rather than counting. (This method is described fully by the author in her book *Children Discover Arithmetic*, Harper and Brothers, 1949). When a child adds a 3-block to a 4-block, he sees that the two blocks can be equal only to the length of the 7-block.

The Teacher's Manual contains a carefully developed sequence of demonstrations, experiments, and games that enable the child to discover number relationships for himself. On the first level, the children make experiments without using the number names or their written symbols. On the second level the names of the numbers are introduced and the children learn the expressions and meaning of oral addition. On the third level, the number symbols from 1 to 10 are taught and the children learn to let numbers stand for blocks and patterns. The Number Markers and symbols may now be used to express the addition facts that the children discover.

Although this set of materials is largely used in the kindergarten and first grade, the aids could be used to great advantage by the second and third grade teachers in getting mastery of the primary addition and subtraction facts.

Firsthand experiences with the blocks. cubes, and patterns which are true representations of the numbers, lead to learning by insight which provides an understanding of numbers that can never be obtained by rote memorization or drill. (Reviewed by Mrs. Ida Mae Heard, Southwestern Louisiana Institute, Lafayette, La.)

Appraisal #2: This material is so functional in purpose that one should not approach a developmental number concept curriculum to provide individualized instruction for the child without this material. Qualities, which stimulate interest for exploring to discover number relationships for himself, promote mastery of Number Concepts by being able to progress from the use of concrete materials with attractive colors, interesting shapes, convenient sizes particularly adaptable to comparing varying number groups; to the semi-abstract, by using number names to develop familiarity with expressions and meanings of the oral concept; and by using number symbols which leads to the expression of the number concept, thus providing for three levels to make every step in his learning meaningful. Structural Arithmetic proves to be a means of completely eliminating remedial cases, in that the devices are self-corrective, motivating the child by experiencing success to the extent of his potential ability to meet the level of perception of his maturation. Making implications of the instructional program of Structural Arithmetic within the classroom, promotes the child's progress in continuous developmental growth in Number Concept through better guidance and observation techniques. (Reviewed by Alice Funfar, University of Minnesota, Minneapolis, Minn.)

FILMS

F. 59—Motivating the Class

McGraw-Hill Publishing Co., 330 West

42nd Street, New York, N. Y.

Film; 16 mm.; Black and white; Sound; 2 Reels.

Description: This film shows a student teacher teaching a mathematics class. His first lesson is a lecture on the nature and development of mathematics and is a failure due to lack of interest by the students. In a conference with his supervising critic teacher, suggestions for meeting the varied goals of the class members are discussed. Shots of subsequent lessons show the student teacher using practical applications, field work, motion pictures, projects, graphs of progress and posters to stimulate the successful study and learning of mathematical principles.

Appraisal: As in most teaching films, this one must cover a vast topic in a short twenty-minute portrayal. However, it succeeds in giving the viewer many practical ideas for adding interest and meaning to his instruction. In order to do this in such a short time some exaggerated and unrealistic situations are used. Although the goals of students are the prime motivators of learning, and are discussed by the supervising teacher, the subsequent development shows very little of this intrinsic motivation. However, the film is effective in showing how instruction is improved and interest developed by the use of a variety of materials. The selection of a practice teacher to illustrate a poor lesson and a changed approach is very appropriate. This is a useful film for the training of teachers in any field and is most usable in the training of teachers of mathematics.

F. 60—A Thousand Hours

Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11, Mich.

16 mm. film; Black and white; Sound; 10 minutes; 1 reel; $36.50.

Description: The opening scene shows a boy coming out of school and entering a clubhouse which has a sign on it "Invention Shop." Two other boys are in the shop working. A discussion follows concerning the value of mathematics.

The captain of one of the great airline transports enters. The boy that had been seen coming out of school expresses his desire to quit school. The captain inspires the boys with an interesting explanation of the background of the sciences necessary to pilot training. One of the captain's trips is used in this explanation.

The following are a few of the subjects the captain covers: advanced geography used in looking up weather reports; foundation of physics used in looking over plan of flight; radio used for the flight officer's report while flying; and trigonometry and calculus used in making the reports.

A change in the boys' attitudes is noticeable as the film ends.

Appraisal: Twenty-two teachers were together to evaluate this film. Considering the entire film they found that it can be most effectively used as a motivating device. Thirty-nine per cent rated the film as good when used to introduce new material, but 22% said that it was poor for this same purpose. The same situation existed when they considered using the motion picture to augment explanations. The teachers believed this film good for providing a common experience but it definitely is of no use in developing skills or for review purposes. The previewers checked the grade level as usable from the 7th to the college grades, with marked predominance in the 8th, 9th, and 10th. Also, it can be used in all mathematics courses and especially in algebra. Over 90% scored the speed of development of ideas and duration of scenes as moderate. Three of the evaluators found some inaccuracies in the mathematical content while the remainder found none. Just under 60% viewed the film as completely holding the interest of students while "partially" was pencilled by the others. Given three alternatives for rating the teaching methods in the film as conducive to learning, they scored 32% as "completely," 63% as "partially" and 5% as "not at all."

The photography was good. The sound of the movie was given 9 for poor, 5 for fair, 7 for good and 1 for very good. The effectiveness of characterizations and background objects was positively good. There was complete agreement that the presentation would encourage further pupil activity in mathematics and in other subjects. All checked the captions as being of appropriate length and pertinent. The dialogue was definitely effective. The arrangement of topics was in a logical sequence and the amount of material is reasonable for comprehension in one showing. All said that there was coordination between sound and picture. Of the entire group, 71% did not believe that the content could be just as effectively and efficiently presented in some other way. The teachers agreed that the film attempted to supplement rather than replace the instructor. Nineteen remarked that they would use this film. Some comments expressed were, "This film is good for motivating further pupil activity in mathematics"; "It will also have application as an occupational guidance film." (Reviewed by Anthony di Luna, Raymond Fleet and Kenneth Hatheway.)

INSTRUMENTS

I. 32—Blackboard Compass

Milton Bradley Company, Springfield 2, Mass.

Compass; No. 8076; 16″ legs; $2.00.

Description: The compass is made of two varnished wooden legs which are connected at one end by a wing screw which enables the user to fix the setting for drawing circles with desired radii. One leg of the instrument contains a tiny rubber tip whose function is to prevent the compass from sliding while circles are being drawn. The other leg contains an opening for chalk and a sliding brass ring is used to hold the chalk firmly in place.

This excellent compass has been purchased by many schools because it is both simple and durable.

Appraisal: The blackboard compass is a valuable instrument in any mathematics course which includes the study of circles. The price of this instrument has increased greatly in the past ten years, but is worth $2.00 nevertheless. (Reviewed by Bernard Singer, Hyannis, Mass.)

PICTURES

P. 11—Portraits

Yerkes Observatory, Williams Bay, Wis.

Pictures; see below.

Description: In 1903, the Yerkes Observatory of the University of Chicago started to reproduce lantern slides, transparencies, and prints of astronomical phenomena. Their collection includes negatives of astronomers who were also notable mathematicians. Portraits of the men listed below are available in several forms —3¼″×4″ glass slides, 8″×10″ unmounted prints, and 8″×10″ window transparencies. The price per slide is $.75, $.50 per print, and $2.00 per transparency.

F. 8	Tycho Brahe	*F. 42*	Laplace
F. 17	Albert Einstein	*F. 50*	Newton
F. 23	Galileo	*F. 53*	Poincare
F. 24	Gauss		

Appraisal: Since the photographs are to be used for the creation of a mathematical atmosphere, the prints are superior to the slides which cannot be projected for a long period of time and the transparencies which are too expensive. Discussions of the mathematical significance of these men will have to be brief since students of secondary mathematics are hardly in the position to appreciate specific advantages of higher mathematics. The prices of the slides, prints, and transparencies are all within reason but the prints serve the desired purposes and are relatively inexpensive. (Reviewed by Bernard Singer, Hyannis, Mass.)

LETTERS

Order of Operations

Miss Edith Wood of Anchorage, Kentucky, calls attention in the February, 1951 issue of THE MATHEMATICS TEACHER to a difference among authors about the order of operations, addition, subtraction, multiplication, and division. Of course there is no disagreement if proper symbols of aggregation are used.

The difference occurs in a problem like $12+6\div2\times3$. There is general agreement that multiplication and division should be performed first, and then addition and subtraction. One group of authors say that, where there is multiplication and division, the multiplication should be first performed and then the division. This group would say that the correct answer to the problem $12+6\div2\times3$ is $12+6\div6$, or 13. The other group would say that $12+6\div2\times3 = 12+3\times3 = 12+9 = 21$. Of course if proper signs of aggregation are used, the question becomes entirely academic.

Chrystal* in his *Algebra*, does not agree with the first group. He says on page 14 "$\times a\div b\times b = \times a$ and $\times a\times b\div b = \times a$." Some authors would have the first of these equal to $\frac{a}{b}$ and the second equal to a.

Chrystal says again: "Another notation for a quotient (other than $a\div b$) is sometimes used, namely, $\frac{a}{b}$ or a/b. As this is the notation for fractions, and therefore has a meaning already attached to it in the case where a and b are integers, it is incumbent upon us to justify its use in another meaning. To do this we have simply to remark that b times $\frac{a}{b}$, that is, b times a of the bth parts of unity, is evidently a times unity, that is, a; also, by the definition of $a\div b$, b times $a\div b$ is a. Hence we conclude that $\frac{a}{b}$ is operationally equivalent to $a\div b$ in the case where a and b are integers."

Again on page 15, in establishing the law of commutation he shows that

$$a\times b\div c = a\div c\times b = \frac{ab}{c}.$$

Others would have

$$a\times b\div c = \frac{ab}{c} \quad \text{and} \quad a\div c\times b = \frac{a}{bc}.$$

A difficulty arises in a problem like $a\div ab$. Probably both groups would give the answer to this as b. If, however, the same expression is written as $a\div a\times b$, there is a disagreement which cannot be settled by any rule about order of operations. It seems evident that the form ab is not exactly equivalent to the form $a\times b$. ab has the force of a parenthesis, that is, the multiplication must be performed before other operations are. This is not true of $a\times b$.

A similar situation arises with the form $\frac{a}{b}$. Thus, probably both groups would say that $a\div\frac{a}{b}$ is equal to b.

But Chrystal, on page 15, in establishing the law of commutation, says that $a\div a\div b$ is equal to $\frac{1}{b}$. Probably both groups would agree with this result.

Thus, despite Chrystal, ab is not entirely equivalent to $a\times b$ nor is $\frac{a}{b}$ equivalent to $a\div b$ when there are continued operations.

A statement which correctly covers all cases is this:

I. Operations enclosed in symbols of aggregation, either expressed or implied, (as ab, $\frac{a}{b}$, $\sqrt{\ }$, ( ), [ ], { }, or the vinculum) should first be performed.

II. Multiplications and divisions should then be performed in the order in which they occur from left to right.

III. Finally additions and subtractions in any order.

VIRGIL S. MALLORY
Head, Department of Mathematics
State Teachers College
Montclair, N. J.

* G. A. Chrystal, *Algebra, An Elementary Textbook for the Higher Classes of Secondary Schools and for Colleges* Part I, Fifth Edition. Adams and Charles Black, London, 1910.

Report on the Second Delegate Assembly

JOHN R. MAYOR, *Chairman*
Committee on Affiliated Groups
University of Wisconsin, Madison, Wisconsin

THE AGENDA of the Second Delegate Assembly of The National Council of Teachers of Mathematics, held in Pittsburgh, March 29 and 30, 1951, was determined by questions proposed by Affiliated Groups during the year and reported in this section of THE MATHEMATICS TEACHER and in the *Newsletter of the Affiliated Groups*. Among the topics given consideration by the Delegates were clarification of requirements for affiliation, relationships between Affiliated Groups and State Representatives, distribution and sharing of materials prepared by the various Groups, problems of publicity, Speakers Bureau, budget of the Committee on Affiliated Groups, possibilities for a traveling exhibit, and contest sponsorship.

The program of the Assembly also included a welcome to the Delegates by President Harry Charlesworth, a ceremony in recognition of the eleven Groups affiliated since the First Delegate Assembly in Chicago in 1950, the annual report of the Committee on Affiliated Groups, and reports on activities of Affiliated Groups of the National Council for the Social Studies and the National Council of Teachers of English.

All recommendations of the Second Delegate Assembly were subsequently approved by the Board of Directors.

Copies of the Minutes of the Second Delegate Assembly will be sent upon request addressed to John R. Mayor, North Hall, Madison 6, Wisconsin.

REQUIREMENTS FOR AFFILIATION

The Second Delegate Assembly, interpreting the actions of the First Delegate Assembly[1], voted that it be required that each Affiliated Group:

1. Have a written constitution and by-laws.
2. Maintain a list of its members with teaching positions and addresses and with indication of members of the National Council, and file a copy of the list with the Chairman of the Committee on Affiliated Groups.
3. Renew its affiliation annually.
4. Renew affiliation and pay annual dues each year between October 1 and December 31.
5. Pay annual dues to the National Council under the plan

   less than 150 members $3.00
   151 to 250 members $4.00
   251 to 350 members $5.00
   over 350 members $6.00

   except that annual dues are waived for groups with 75% or more of their members also members of the National Council, and new Groups are not required to pay affiliation dues during the school year in which they become affiliated.

Point 5 implies that, at least for the present, no minimum has been set for the number of members in an Affiliated Group.

DELEGATE ASSEMBLY

In regard to the Delegate Assemblies it was decided that:

1. One vote be allowed each Group regardless of size.
2. No Delegate be allowed to vote at a Delegate Assembly if the annual dues for renewal of affiliation have not been paid.

SHARING OF TEACHING AIDS

The Delegates voted that:

Copies of all printed and duplicated materials of each Group be sent to the Chairman of the Committee on Affiliated Groups and to the Washington office of the National Council.

FUNDS

It was ruled that the expenditure of funds collected from the Groups through affiliation dues be left to the chairman of the Committee on Affiliated Groups. The discussion revealed that the Groups favored provision for financial support for a Speakers Bureau, and for the establishment of a travelling exhibit, and, if necessary because of cost, reduction in the number of issues of the *Newsletter*.

[1] John R. Mayor, "The National Council and Its Affiliated Groups," THE MATHEMATICS TEACHER, XLIII, 345–347.

Future Plans

While no official action was taken on other topics on the agenda the Assembly received with favor proposals for better Affiliated Groups and National Council publicity, early establishment of a Speakers Bureau as a service agency to Affiliated Groups, establishment of a traveling exhibit for meetings of Affiliated Groups and possible use by schools, encouragement of mathematics contest sponsorship, and an annual luncheon for Delegates of Affiliated Groups.

New Groups

Recognition was given to the following eleven groups, which completed affiliation since the First Delegate Assembly:

The Colorado Council of Teachers of Mathematics
Affiliated May 27, 1950; Charter no. 84.

Mathematics Department, Alabama Education Association
First Affiliated April 10, 1936; Charter no. 27. Renewed July 8, 1950.

Oklahoma City Mathematics Council
First Affiliated April 12, 1936; Charter no. 28. Renewed November 1, 1950.

Arkansas Council of Teachers of Mathematics
Mathematics Section, Arkansas Education Association, Affiliated November 3, 1939; Charter no. 48. Renewed under new name, December 13, 1950.

Pennsylvania Council of Teachers of Mathematics
Affiliated February 11, 1951; Charter no. 85.

Pinellas County (Florida) Council of Teachers of Mathematics
Affiliation renewed February 12, 1951; Charter no. 86. (date of earlier affiliation not determined)

Association of Teachers of Mathematics of Philadelphia and Vicinity
Affiliated February 19, 1951; Charter no. 87.

Mathematics Section of South Carolina Education Association
Affiliated May 15, 1939; Charter no. 60. Renewed March 16, 1951.

Mathematics Section, South Dakota Education Association
Affiliated March 16, 1951; Charter no. 88.

Benjamin Banneker Club of Washington, D. C.
Affiliated April 3, 1938; Charter no. 42. Renewed March 24, 1951.

Utah Council of Teachers of Mathematics
Affiliated March 24, 1951; Charter no. 89.

Official Delegate List

Mary C. Rogers and John R. Mayor served as co-chairmen of the Second Delegate Assembly. Elaine Rapp was secretary of the Assembly with Alice M. Reeve as assistant secretary. Special reports were presented by Jackson B. Adkins, Exeter, New Hampshire; Margaret A. Striegl, Wauwatosa, Wisconsin; Mary C. Rogers, Westfield, New Jersey; Madeline Messner, Roselle, New Jersey; Houston Karnes, Baton Rouge, Louisiana; Kenneth Brown, Knoxville, Tennessee.

The official Delegates were:

Mathematics Department, Alabama Education Association
Mrs. E. L. Asbury, Holt

Arkansas Council of Teachers of Mathematics
R. W. Young, Arkadelphia

California Mathematics Council
Charles C. Fabing, Los Angeles

The Colorado Council of Teachers of Mathematics
Harry W. Charlesworth, Washington and Denver

Mathematics Section, Eastern Division, Colorado Education Association
Lillian M. Sullivan, Denver

Florida Council of Teachers of Mathematics
W. A. Gager, Gainesville

Dade County Council of Teachers of Mathematics
Verna Kimler, Miami

Hillsborough County Council of Mathematics Teachers
Howard L. Gallant, Tampa

Georgia Council of Teachers of Mathematics
Gladys M. Thomason, Sandersville
Bess Patton, Atlanta (Alternate Delegate)

Illinois Council of Teachers of Mathematics
Henry Swain, Winnetka

Chicago Elementary Teachers' Mathematics Club
Joseph J. Urbancek, Evanston

Men's Mathematics Club of Chicago and Vicinity
M. D. Oestreicher, Chicago

Women's Mathematics Club of Chicago and Vicinity
Martha Hildebrandt, Maywood

Indiana Council of Teachers of Mathematics
K. Eileen Beckett, Lebanon

Iowa Association of Mathematics Teachers
Henry Van Engen, Cedar Falls

Kansas Association of Teachers of Mathematics
Gilbert Ulmer, Lawrence

Kentucky Council of Mathematics Teachers
Edith Wood, Anchorage

Louisiana-Mississippi Branch of The NCTM
Houston T. Karnes, Baton Rouge

Mathematics Section of the Maryland State Teachers Association
Margaret L. Heinzerling, Baltimore

Detroit Mathematics Club
William Slemer, Detroit

Minnesota Council of Teachers of Mathematics
Florence Collins, St. Paul
The Association of Teachers of Mathematics in New England
Jackson B. Adkins, Exeter, New Hampshire
Association of Mathematics Teachers of New Jersey
Mary C. Rogers, Westfield
Association of Teachers of Mathematics of New York City
Barnett Rich, New York
Nassau County Mathematics Teachers' Association
Elaine Rapp, Freeport, New York
Alice M. Reeve, Rockville Centre (Alternate Delegate)
Suffolk County Mathematics Teachers Association
Edmund Miles, Amityville, Long Island, New York
Greater Cleveland Mathematics Club
Charles E. Scott, Cleveland
Oklahoma Council of Teachers of Mathematics
James H. Zant, Stillwater
Tulsa Mathematics Council
Muriel Lackey, Tulsa
Ontario Association of Teachers of Mathematics and Physics
Robert E. K. Rourke, Newmarket
Pennsylvania Council of Teachers of Mathematics
Lee E. Boyer, Millersville
Mathematics Teachers Association of Western Pennsylvania
L. McClure Lanning, Pittsburgh
The Association of Teachers of Mathematics of Philadelphia and Vicinity
M. Albert Linton, Jr., Philadelphia
Mathematics Section, South Carolina Education Association
Margaret W. Huff, Columbia
Mathematics Section, South Dakota Education Association
Joy Hamrin, Sioux Falls
Mathematics Section, East Tennessee Education Association
Kenneth Brown, Knoxville
Mathematics Section, Texas State Teachers Association
Ida May Bernhard, San Marcos
Dallas Elementary Mathematics Association
Ruth Clough, Dallas
Mathematics Section, Virginia Education Association
Burton F. Alexander, Petersburg
Richmond Chapter, The NCTM
Mary E. Hawkins, Richmond
Benjamin Banneker Mathematics Club, Washington, D. C.
Guinevere D. White, Washington
The West Virginia Council of Mathematics Teachers
Frances Grimm, Huntington
Wisconsin Mathematics Council
Margaret A. Striegl, Wauwatosa

Groups Not Yet Affiliated

New York State Council
Alice M. Reeve, Rockville Centre
Ohio Council of Teachers of Mathematics
Ona Kraft, Cleveland

NEWS NOTES

The annual **Convention of the Central Association of Science and Mathematics Teachers** will be held in Cleveland, November 22–24 of this year. The Hollenden Hotel has been selected for headquarters. Interesting general meetings are scheduled for Friday morning and Friday afternoon, November 23, and Saturday morning, November 24. Among the featured general speakers are Dr. Keith Glennan, President of Case Institute of Technology and member of the Atomic Energy Commission; Dr. Elvin C. Stakman, Chief of the Division of Plant Pathology and Botany, University of Minnesota; and Dr. David Dietz, Science Editor, Scripps Howard Newspapers. Dr. Dietz will speak at the banquet Friday evening. Section meetings in Elementary Mathematics, Elementary Science, General Science, Mathematics, Biology, Chemistry and Physics will be held Friday. Group meetings in areas of Elementary Schools, Junior High Schools, Senior High Schools, and Conservation will be held Saturday.

In Quest of Truth is the title of a short historical pageant on mathematics written by Daniel B. Lloyd, of Wilson Teachers College in Washington, D. C. It is suitable for reading or dramatization by junior or senior high school students, for classroom use, and for assembly or club programs. Sample copies may be obtained from the author for 25¢ or in quantities at $1.00 a dozen.

The Treasury Department is distributing to school administrators a 6-page folder entitled DEFENDING AMERICA: SCHOOL SAVINGS IN THE NATIONAL EMERGENCY. This folder restates the basic educational purposes of the Treasury's School Savings Program—to achieve long-range training and practice in personal thrift and the wise use of natural and other resources. The folder also asks for a patriotic expansion of School Savings in this time of national emergency.

MINUTES OF ANNUAL BUSINESS MEETING

National Council of Teachers of Mathematics
Hotel William Penn, Pittsburgh, Pennsylvania

FIRST SESSION

Thursday, March 29, 1951, 3:30–4:30 P.M.

The meeting was called to order by President H. W. Charlesworth, with 78 people present.

No minutes of the previous meeting were read.

The resignation of Dr. W. D. Reeve as Editor-in-Chief and the recent affiliation with the N.E.A. necessitated the appointment of a new Editor-in-Chief for THE MATHEMATICS TEACHER, and an Executive Secretary. In order to fill these appointments a revision of the By-Laws is required. The following Committee was appointed to serve for this purpose:

George Hawkins, Lyons Township High School and Junior College, La Grange, Illinois
Mary Potter, Board of Education, Racine, Wisconsin
Marie S. Wilcox, Chairman, George Washington High School, Indianapolis, Indiana.

The proposed By-Laws, as they appeared in the February, 1951 issue of THE MATHEMATICS TEACHER were presented by Mrs. Wilcox. She called attention to each article proposed, and changes in the old By-Laws were noted with the reasons for changing. At the end of the reading and presentation, Mrs. Wilcox explained that acceptance of the By-Laws would not prohibit minor changes and that the Board of Directors had authorized the continuance of the Committee for one year to consider any suggestions for further revision. Mrs. Wilcox made it clear that letters with suggestions would be welcome. She then moved that the proposed By-Laws as printed in the February issue of THE MATHEMATICS TEACHER be accepted. The motion was seconded by Dr. R. R. Smith.

The discussion which followed brought forth the following questions:

1. The advisability of limiting the nominee for President to a person who has served on the Board within the preceding five years.
2. The consideration of restricting Board members to one term of three years, with a specified lapse of time before they would be eligible for re-election.
3. The possibility of increasing the size of the Board of Directors.
4. The advisability of having a future president elected for one year as a non-voting member of the Board.

Dr. Rosskopf moved that the By-Laws be tabled. There being no second to this motion, no action was taken.

The vote was then taken on the original motion, which was approved. There was one dissenting vote.

Dr. Christofferson moved that a vote of thanks be extended to the members of the Committee on the Revision of the By-Laws. This was seconded by Dr. R. R. Smith and met with much applause.

Mr. Schreiber gave the results of the recent elections. 1,015 votes were tallied.

Vice-President representing College Mathematics
James H. Zant
Members of the Board of Directors
William A. Gager
Lucy E. Hall
Henry Van Engen

Congratulations were offered to these members and each was introduced.

There being no further business, Mr. Schreiber moved the meeting be adjourned. This motion was duly seconded and passed.

Respectfully submitted,
AGNES HERBERT, *Acting Secretary*

SECOND SESSION

Saturday, March 31, 1951, 11 A.M.

The meeting was called to order by the President with 125 present.

Announcement of previous Board action on the place for the next two annual meetings was given; Nebraska, Iowa and Atlantic City had all extended invitations. It was decided that the 1952 meeting should be held in Des Moines, Iowa and that the invitation of Atlantic City should be accepted for 1953.

The question was then raised of the position of the National Council in the present National Emergency situation. A Committee has been appointed to formulate a platform and reaffirm some of the objectives in the Commission Report for May, 1945.

Dr. Hildebrandt urged those present to return to their own localities and emphasize two things:

1. Every student should take as much mathematics as possible.
2. Students with special aptitude should take every part of mathematics available in order to be useful in scientific work.

Mr. Lloyd asked that we stand pat on what has been done and the Committee be congratulated on their careful thinking of this program.

Mr. Charlesworth told of the plans on smaller publications and Year Books.

Mr. Christofferson moved the adoption of

the following tribute to Dr. Kenneth Brown, to be published in THE MATHEMATICS TEACHER and a copy of which is to be sent to Mr. Brown:

The National Council of Teachers of Mathematics has become really a national council with a spirit of devoted service to teachers and children because of the unselfish work of many men and women. Conspicuous among these persons has been the man who for several years has been the Chairman of State Representatives. His devotion to the Council, his vision of the importance of the organization, his tireless work in keeping a host of representatives alert and active have been constant influences in building the Council to one of the strongest teacher organizations in the country.

His retirement from this office in the reorganization of the Council will be a serious loss. His influence will continue and his activity in promoting the Council will increase. We express gratitude to Kenneth Brown for his selfless and effective service. We thank him and look forward to his continued influence in the Council.

This was seconded by Mr. Peak and carried.

Mr. Shuster moved the adoption of the following tribute to Mr. Edwin W. Schreiber, to be published in THE MATHEMATICS TEACHER and a copy of which is to be sent to Mr. Schreiber:

During the long and varied life of the National Council of Teachers of Mathematics many Presidents, Vice-Presidents, Editors, and Board Members have played their brief part on the stage and have then passed on to constitute the growing list of former leaders of the National Council. However, during all these long years, one Charter Council Member remained constantly on the Board. He has had charge of the purse strings of the Council all these years and has been a Rock of Gibraltar in resisting any enthusiastic but unwise scheme to deplete the treasury. He has been the pilot who has constantly kept the Constitution of the Council before him like a chart and has steered the Council on a course that has avoided all the legal, diplomatic, and financial rocks and shoals

It is unnecessary to say that this firm, wise, and kindly man who has rendered such long and valuable service to the Council is our Secretary-Treasurer, Edwin W. Schreiber.

We wish continued good health and happiness to him and we hope we shall have him as a regular attendant and contributor to our meetings for many, many years.

This was seconded by Dr. Bakst and carried.

Dr. Bakst moved that we offer our thanks to Dr. Hildebrandt as Editor-in-Chief of THE MATHEMATICS TEACHER for the excellent job that he has done thus far. This was seconded by Mr. Swain and carried.

Mr. David Kotler presented the following resolution, which after the reading was seconded by Mr. T. S. Klein:

WHEREAS, The National Council of Teachers of Mathematics is a Department of the National Education Association, and

WHEREAS, The National Education Association adopted at its summer 1950 meeting held in St. Louis the following resolution: "Members of the Communist Party shall not be employed in our schools. Communist organizations and communist front organizations should be required by law to register with the Attorney General of the United States." And

WHEREAS, The National Council fully endorses the above resolution of the National Education Association, and

WHEREAS, The National Council of Teachers of Mathematics, an organization of teachers who are loyal and devoted to our system of government and society, deems it its duty to implement the above resolution of the National Education Association,

THEREFORE, Be it resolved that

First: The National Council of Teachers of Mathematics will deny any elective and appointive office (including committees) to those who are:

(a) Members of the Communist Party;
(b) Members of organizations which are known and which have been declared subversive;
(c) Members of organizations which have been declared Communist controlled and directed as well as members of organizations and associations which are affiliated with Communist controlled and Communist directed organizations.

Second: The Board of Directors of the National Council of Teachers of Mathematics is hereby empowered and directed to implement and enforce this resolution.

Dr. Christofferson moved that it be referred to the Board. General discussion followed: Mr. Charlesworth had previously conferred with Dr. Givens of N.E.A. concerning their policy. Mr. Urbancek then read the original N.E.A. policy, in full. This resolution follows:

Preservation of Democracy:

The National Education Association strongly asserts that all schools have an obligation to teach the rights, privileges, and the responsibilities of living in a democracy.

The responsibility of the schools is to teach the value of our American way of life, founded as it is on the dignity and worth of the individual; our youth should know it, believe in it, and live it continuously.

As a measure of defense against our most potent threat, our American schools should teach about communism and all forms of totalitarianism, including the principles and practices of the Soviet Union and the Communist Party in the United States. Teaching about communism does not mean advocacy of communism. Such advocacy should not be permitted in American schools.

Members of the Communist party shall not

(*Continued on page 446*)

Program

The Twelfth Christmas Meeting The National Council of Teachers of Mathematics

Oklahoma Agricultural and Mechanical College Stillwater, Oklahoma*

December 27, 28, 29, 1951

THURSDAY, DECEMBER 27, 1951

9:00 A.M.—12:00 NOON. Meeting of the Board of Directors—Meeting Room 9

2:00 P.M.–5:00 P.M. Meeting of the Board of Directors—Meeting Room 9

10:00 A.M.–5:00 P.M. Registration—Main Lobby, Student Union

7:30 P.M. Entertainment—Student Union

FRIDAY, DECEMBER 28, 1951

8:00 A.M.–5:00 P.M. Registration—Main Lobby, Student Union

The Friday morning meetings are centered around the theme, "The Place of Experimentation and Discovery on the Part of Pupils in the Learning Process in Mathematics."

9:00 A.M.–10:15 A.M. General Session—Ballroom

Presiding: HARRY W. CHARLESWORTH, President of the National Council of Teachers of Mathematics, East High School, Denver, Colorado

Helping Pupils Discover Mathematical Concepts, F. G. LANKFORD, JR., University of Virginia, Charlottesville, Virginia

10:30 A.M.–12:00 NOON. Elementary Section—Varsity Room

Presiding: EDITH STEANSON, University Elementary School, University of Oklahoma, Norman, Oklahoma

The How and Why of Discovery in Elementary School Arithmetic, ESTHER J. SWENSON, University of Alabama, University, Alabama

Some Practices Concerning Experimentation and Discovery in Learning in Mathematics in the Elementary School, W. I. LAYTON, Stephen F. Austin State College, Nacogdoches, Texas

10:30 A.M.–12:00 NOON. Junior High School Section—Pioneer Room

Presiding: AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland

Student Discovery of Algebraic Concepts, OSCAR F. SCHAAF, University School, Ohio State University, Columbus, Ohio

Geometric Readiness. IRVIN H. BRUNE, Iowa State Teachers College, Cedar Falls, Iowa

10:30 A.M.–12:00 NOON. Senior High School Section—Howdy Room

Presiding: MARIE WILCOX, Washington High School, Indianapolis, Indiana

The Pupil Discovers Algebra, EUNICE LEWIS, University School, University of Oklahoma, Norman, Oklahoma

Experimentation and Discovery in Geometry, PHILLIP S. JONES, University of Michigan, Ann Arbor, Michigan

10:30 A.M.–12:00 NOON. College Section —Pow-wow Room

Presiding: GILBERT ULMER, University of Kansas, Lawrence, Kansas

Student Discovery on the College Level by Means of Construction of Models, C. B. READ, University of Wichita, Wichita, Kansas

Confidence, Confusion, Completion, RUTH LANE, Sam Houston State Teachers College, Huntsville, Texas

* All meetings will be held in Student Union Building.

10:30 A.M.–12:00 Noon. Teacher Education Section—Corral Room

Presiding: WILLIAM A. GAGER, University of Florida, Gainesville, Florida

Best Training for Future Teachers, ZEKE LOFLIN, Southwestern Louisiana Institute, Lafayette, Louisiana

Numerical Concepts of Prospective Elementary Teachers before Taking a Course in Professionalized Content in Arithmetic, ELLA MARTH, Harris Teachers College, St. Louis, Missouri

12:15 P.M.–1:30 P.M. Get-Acquainted Luncheon—Cafeteria

2:00 P.M.–4:00 P.M. *Elementary Section*—Varsity Room

Presiding: DALE PATTERSON, Colorado State College of Education, Greeley, Colorado

Panel Discussion: *The Role of Textbooks in Teaching Arithmetic in the Elementary School*, FOREST FISCH, Colorado State College of Education, Greeley, Colorado; JACK HALL, Laboratory School, State College of Education, Pittsburg, Kansas; LUCY L. ROSENQUIST, Colorado State College of Education, Greeley, Colorado (Emeritus)

2:00 P.M.–4:00 P.M. Secondary Section—Howdy Room

Classroom Experimentation Utilizing the Meaning-Discovery Approach to Mathematics. This section is sponsored by the Wichita Mathematics Association, Wichita, Kansas

Presiding: LUCY E. HALL, High School North, Wichita, Kansas

The Teacher's Role as Guide and Director, LOTTCHEN L. HUNTER, Wichita Public Schools, Wichita, Kansas

The Slide-Rule as a Means of Extending Number Concepts in Junior High School, DAVID L. HOLLAND, Robinson Intermediate School, Wichita, Kansas

New Approaches to Learning, T. OTHO COTT, Allison Intermediate School, Wichita, Kansas

The Discovery Approach to Geometry, CELIA CANINE, High School North, Wichita, Kansas

A Second Course in Basic Mathematics, KENNETH N. NICKEL, High School East, Wichita, Kansas

2:00 P.M.–4:00 P.M. College Section—Pow-wow Room

Presiding: C. B. READ, University of Wichita, Wichita, Kansas

What the College Mathematics Department Can Expect from Their Entering Freshmen, and What to Do about It, ESTHER F. GIBNEY, University of Houston, Houston, Texas

Our Neglect of Mechanics, C. E. SPRINGER, University of Oklahoma, Norman, Oklahoma

Learning Theory Applied to College Mathematics, LYLE J. DIXON, University of Kansas, Lawrence, Kansas

2:00 P.M.–4:00 P.M. Teacher Education Section—Corral Room

Presiding: H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio

What Do Good Mathematics Teachers Do Better than Poor Mathematics Teachers? ROBERT E. PINGRY, University of Illinois, Urbana, Illinois

Four Types of Number Activity that Teachers Should Develop. HOLMES BOYNTON, Northern Michigan College of Education, Marquette, Mich.

A Program for Training Teachers of Arithmetic in the Primary Grades. EDWINA DEANS, Arlington Public Schools, Arlington, Virginia

2:00 P.M.–4:00 P.M. Research Section—Pioneer Room

This section is sponsored by the Research Committee of the National Council of Teachers of Mathematics.

Presiding: HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa

The Teaching of Statistics in Secondary School Mathematics, MAX BEBERMAN, University High School, University of Illinois, Urbana, Illinois

Mathematics in Meteorology, with Its Implications for the Teaching of Secondary and College Mathematics, MAX KRAMER, New Mexico College of

Agriculture and Mechanic Arts, State College, New Mexico

Influences in the Development of Geometry as a Subject in the Secondary School, G. H. LUNDBERG, Vanderbilt University, Nashville, Tennessee

4:00 P.M.–5:00 P.M. Affiliated Groups Section—Howdy Room

Presiding: JOHN MAYOR, University of Wisconsin, Madison, Wisconsin

Panel Discussion: *What the Affiliated Groups Are Doing*, REPRESENTATIVES OF THE GROUPS IN OKLAHOMA AND NEIGHBORING STATES

4:00 P.M.–5:00 P.M. Mathematics Films—Audio-Visual Center, Little Theater

6:30 P.M. Banquet—Ballroom

Twentieth Century Mathematics Teaching—Retrospect and Prospect, J. O. HASSLER, University of Oklahoma, Norman, Oklahoma

SATURDAY, DECEMBER 29, 1951

8:00 A.M.–3:30 P.M. Registration—Main Lobby

8:00 A.M.–9:00 A.M. Mathematics Films—Audio-Visual Center, Little Theater

9:00 A.M.–10:15 A.M. General Session—Ballroom

Presiding: JOHN MAYOR, University of Wisconsin, Madison, Wisconsin

The Evaluation of Learning in Mathematics, MAURICE HARTUNG, University of Chicago, Chicago, Illinois

The general session will be followed by discussion groups in which the topic of the address will be discussed with reference to the various instructional levels. Members are urged to register for the group at the level in which they are most interested.

10:30 A.M.–12:00 NOON. Discussion Groups

The Evaluation of Learning in Mathematics

Group 1. *In the Elementary School*—Meeting Room 2

Leader: GLENADINE GIBB, Iowa State Teachers College, Cedar Falls, Iowa

Group 2. *In the Junior High School*—Meeting Room 3

Leader: H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio

Group 3. *In the Senior High School*—Meeting Room 5

Leader: GILBERT ULMER, University of Kansas, Lawrence, Kansas

Group 4. *In the College*—Pow-wow Room

Leader: HAROLD TRIMBLE, Iowa State Teachers College, Cedar Falls, Iowa

Group 5. *In Teacher Education*—Meeting Room 6

Leader: NATHAN LAZAR, Ohio State University, Columbus, Ohio

10:30 A.M.–12:00 NOON. General Mathematics Section—Howdy Room

This section is sponsored by the Colorado Council of Teachers of Mathematics.

Presiding: FOREST FISCH, Colorado State College of Education, Greeley, Colorado

Trends and Improvements in the Teaching of General Mathematics, ALBERT N. RECHT, University of Denver, Denver, Colorado

RUTH IRENE HOFFMAN, Denver Public Schools, Denver, Colorado

C. F. BARR, University of Wyoming, Laramie, Wyoming

10:30 A.M.–12:00 NOON. General Section—Pioneer Room

Panel Discussion: *The National Emergency and the Teaching of Mathematics*

Chairman: JAMES H. ZANT, Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma

Participants: M. H. AHRENDT, Executive Secretary, National Council of Teachers of Mathematics; LYLE J. DIXON, University of Kansas, Lawrence, Kansas; L. W. LAVENGOOD, Tulsa Public Schools, Tulsa, Oklahoma

10:30 A.M.–12:00 NOON. Curriculum Section—Corral Room

This section is sponsored in part by the Colorado Council of Teachers of Mathematics.

Presiding: F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

The Unified Kindergarten through Twelfth Grade Mathematics Program of the Denver Public Schools, BURNETT SEVERSON, Denver Public Schools, Denver, Colorado

Recent State Curriculum Developments in Mathematics, DANIEL SNADER, University of Illinois, Urbana, Illinois

1:30 P.M.–3:30 P.M. Elementary Section—Varsity Room

This section is sponsored by the Dallas Elementary Mathematics Association, Dallas, Texas.

Presiding: CARRIE DENSON, Dallas Public Schools, Dallas, Texas

The Teaching of Long Division, RUTH WHELESS, Dallas Public Schools, Dallas, Texas

Discussion

1:30 P.M.–3:30 P.M. Secondary School Section—Howdy Room

This section is sponsored in part by the Beaumont Mathematics Association, Beaumont, Texas.

Presiding: KENNETH BROWN, University of Tennessee, Knoxville, Tennessee

Problem Solving and Mathematical Concepts in the Secondary School, ROSALIE DUNHAM, Wilson Junior High School, Tulsa, Oklahoma

An Aid in the Development of Independent, Orderly, and Logical Proofs, VELMA B. OAKES, Siloam Springs High School, Siloam Springs, Arkansas

Panel Discussion: *Aids in Teaching High School Mathematics*, ALICE McCALL, Beaumont High School, Beaumont, Texas; NATALIE DINAN, Beaumont High School, Beaumont, Texas; KATHRENE BAILEY, South Park High School, Beaumont, Texas; PEARL BOND, Beaumont High School, Beaumont, Texas

1:30 P.M.–3:30 P.M. College Section—Pow-wow Room

This section is sponsored by the Arkansas Council of Mathematics Teachers.

Presiding: OLIN HUGHES, Arkansas State Teachers College, Conway, Arkansas

The Faculty Looks at Mathematics for General Education, H. H. HYMAN, Florida State University, Tallahassee, Florida

Mathematics Intangibles, JOHN ABERNATHY, Arkansas Polytechnic College, Russellville, Arkansas

1:30 P.M.–3:30 P.M. Teacher Education Section—Corral Room

Theme: *Advancing the Cause of Mathematical Education*

Presiding: NATHAN LAZAR, Ohio State University, Columbus, Ohio

Improving the Training of Teachers of Secondary Mathematics, A. C. MADDOX, Northwestern State College, Natchitoches, Louisiana

Improving the Training of Teachers of Elementary Mathematics, F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

1:30 P.M.–3:30 P.M. Enrichment Section—Pioneer Room

Presiding: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey

Books for Mathematics—Recreational and Supplementary Reading, MARTHA HILDEBRANDT, Proviso Township High School, Maywood, Illinois

The Mathematics of Crystals, AARON BAKST, Flushing, New York

Mathematics for the Most Capable Students, H. W. CHARLESWORTH, East High School, Denver, Colorado

PROGRAM COMMITTEE

Chairman, Lenore John; Ruth Clough, Mary Lee Foster, Ida Mae Heard, Agnes Herbert, E. H. C. Hildebrandt, Jessie May Hoag, L. W. Lavengood, John R. Mayor, Philip Peak, Henry Van Engen, Marie Wilcox, James H. Zant.

ANNOUNCEMENTS

Registration

The registration fee is fifty cents for

members of The National Council of Teachers of Mathematics, members of the Mathematical Association of America, and teachers in elementary schools. The fee for non-members and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press, and commercial exhibitors are not charged a registration fee but should register. You are urged to register in advance, using the Advance Registration and Reservation Form below. Registration will be in the Main Lobby of the Student Union Building.

Room Reservations

The Union Club, an integral part of the Student Union, will accommodate a limited number of guests in regular hotel rooms, each with bath, at the following rates.

One person to a room	$4.25
Two persons to a room	6.25
Three persons to a room	7.75
Four persons to a room	9.25
A dormitory holding 20 people	2.00 per person

Reservations for rooms in the Union Club should be sent directly to Mr. Arthur H. Taylor, The Union Club, Oklahoma A. & M. College, Stillwater, Okla.

The following hotels and motor courts in Stillwater are available: Going Hotel, Grand Hotel, Aggie Courts, Anglin Motel, Curran-Au-Tel, Rosa Courts and Sloan's Tourist Courts. Reservations for rooms in these should be sent directly to the hotel or motor court.

Any overflow will be housed in the College Dormitories.

Food Service

The Cafeteria in the basement of the Student Union Building will serve meals from Thursday evening, December 27, through Saturday noon, December 29. The Student Union Coffee Shop will also be open for all meals.

The Banquet will be held Friday evening, December 28, in the Ballroom. Reservations ($2.50) should be made in advance on the Registration and Reservation Form.

The Get-Acquainted Luncheon Friday noon, December 28, will be cafeteria style. No reservation is necessary.

Discussion Groups

Please register in advance for the Saturday morning discussion groups, so that plans may be made for additional groups if the registration is large. Please use the Registration and Reservation Form and indicate first and second choices.

Entertainment and Recreation

Since the Council will have the complete facilities of the Student Union Building at its disposal, all entertainment and recreation will be arranged within the building.

Exhibits

There will be exhibits of mathematical models, instruments, teaching aids and other classroom materials. Teachers are invited to bring materials for exhibit. Textbooks and commercial teaching aids will also be on exhibit. Inquiries should be addressed to James H. Zant, Department of Mathematics, Oklahoma A. &. M. College, Stillwater, Oklahoma.

Supplies and Equipment

Speakers and other participants in the program who need projection equipment or other materials should communicate not later than December 10 with James H. Zant, Oklahoma A. & M. College, Stillwater, Oklahoma.

Films and Film Strips

Mathematics films will be shown as indicated in the program. Persons wishing to suggest films to be shown should communicate, before December 1, with Lenore John, Laboratory School, University of Chicago, Chicago, Illinois.

Transportation

Stillwater cannot be reached efficiently by rail. Those who come from Chicago

ADVANCE REGISTRATION AND RESERVATION FORM

Fill out completely and mail to Mr. Raymond L. Caskey, Department of Mathematics, Oklahoma A. & M. College, Stillwater, Oklahoma, before December 10, 1951.

Mr., Miss, Mrs. _____
Last name First name Initial

Address _____
Street and number City State

Registering as member of N.C.T.M. _____; M.A.A. _____; Student _____

Elementary teacher _____; Exhibitor _____; Non-member _____. Members of families and friends of those attending the meetings are welcome to all social and recreational events.

Registration fee: $.50 or $1.50 Amount _____

Room Registration

For the Union Club, send directly to
Mr. Arthur H. Taylor, Union Club, Oklahoma A. & M. College,
Stillwater, Oklahoma

_____ Banquet Reservations at $2.50 each _____

Total $_____

Make checks payable to Raymond L. Caskey

Discussion Group Registration
State discussion group number

First Choice: Group _____

Second Choice: Group _____

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and the East and North should purchase tickets to Perry, Oklahoma, which is 25 miles from Stillwater. When bus service is not available, transportation from Perry to Stillwater will be furnished by the College, provided notice is received by the Department of Mathematics before December 15, if possible. Those arriving at Oklahoma City or Tulsa will find bus service available. It is probable that air service will be available to Stillwater by December 1; at present air passengers must travel from Oklahoma City or Tulsa by bus. Persons driving their own cars will find excellent highways leading into Stillwater from all directions.

Minutes

(*Continued from page 440*)

be employed in our schools. Communist organizations and communist front organizations should be required by law to register with the Attorney-General of the United States.

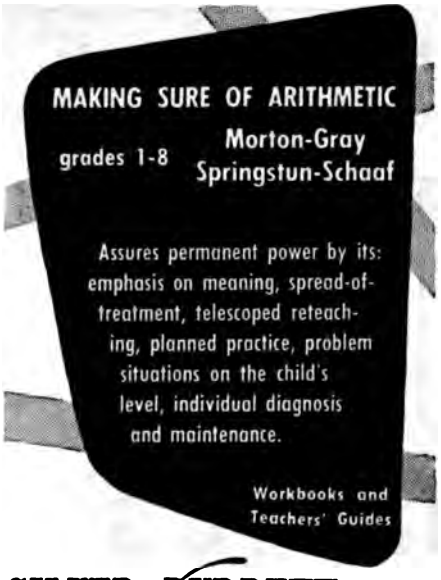
The Association charges the teaching profession with the obligation of providing the best defense of democracy through full participation in making democracy really live and work.

The Association again reminds the public of the repeated pronouncements of our military leaders and statesmen that education is the basis of our national security and that a well-planned, adequately supported system of free public schools is fundamental to the perpetuation of our American way of life.

Mr. Belding moved that this Body go on record concurring with the N.E.A. Resolution. This was seconded by Mr. Urbancek and carried.

There being no further business the meeting adjourned.

Respectfully submitted,
AGNES HERBERT, *Acting Secretary*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THE MATHEMATICS TEACHER

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What Permanent Values Have Courses in Mathematics for Students Who Will Make No Professional Use of Them?*

By NORMAN MILLER
Queen's University, Kingston, Ontario

THE SUBJECT which is broached in my title today is not a new one. Indeed it figures largely, in one guise or another, in the program of this meeting. Further discussion of the topic is suggested by the emphasis which is currently being placed on general education. There are educators who regard mathematics, beyond the elements of arithmetic, as a specialty, and would give it scant space in a general education. On the other hand, this meeting bears witness to the belief of many people that mathematics has *general* as well as *special* values and that its total contribution to a general education is at least as great as that of any other subject. If we hold this belief, our ability to translate our faith into argument convincing to the non-mathematical educators will be the measure of our success in retaining for mathematics its traditional honored place on the curriculum.

I ask you to think, then, of the man or woman who is twenty years out of school or college and has made no specific use of mathematics beyond elementary arithmetic. If there is truth in the old cliché that "education is what remains after all that we have learned is forgotten", then what is the contribution of *mathematics* to education? A metaphor which is apposite if it is not pressed too far is to the effect that it is not the water which remains in the mill which grinds the grain, but the water which passes through the mill. What, in this metaphor, is pertinent to education may be expressed by the statement that a student who has undergone the discipline necessary to master any subject is thereby better equipped for life, even though he forgets the details of the subject—a point of view that used to be insisted on by the late President Lowell of Harvard. As teachers we are naturally anxious to see results of our work and disappointed when these do not measure up to our expectations. We might take comfort from a remark of Sir Richard Livingston of Oxford to the effect that "the most precious fruits of a good teacher's work are those that he is never likely to see".

I shall review briefly what seem to me to be some of the permanent values of the study of mathematics, which justify its being given an important place in general education. By isolating for study the re-

* Read at the 28th Annual Meeting of the National Council of Teachers of Mathematics, Chicago, Illinois, April 14, 1950.

sults which should be achieved, we may throw some light on teaching methods by which we may more effectively achieve them.

Training in Language

It is my belief that a training in the exact and effective use of one's mother tongue should be one of the important permanent values of the study of mathematics. To realize this result requires a clear objective and no little patience on the part of the teacher. He should require that a pupil should at all times say exactly what he means, with an economy of words but with no fuzziness or indefiniteness of meaning. He should insist relentlessly that a student should never equate two things which are not equal—an angle to its sine or a number to its logarithm. How much clarity would be gained in the thinking and speaking of everyone if they never said or implied the equivalence of two things which are demonstrably not equivalent!

In a mathematical argument one's mind may take leaps. A good student often sees the conclusion before you reach it. But it is an important part of his training to marshal his arguments in orderly fashion so that there are no gaps and every step carries conviction. This habit, if insisted upon, is surely one that should remain a permanent possession of an adult after he has forgotten the rules of logarithms or the formulas of trigonometry.

Language and thought are so closely interdependent that one can hardly imagine improvement in one without a corresponding improvement in the other. Careful teaching in mathematics should make for economy and conciseness and against prolixity in the use of words. When I meet an adult who has a greater than usual power in the spoken and written word, the conclusion always leaps to my mind, "He must have been a good student of mathematics." Again, when I hear a preacher or other speaker who wraps up a subject in an endless proliferation of words, some of them inept and inappropriate, I groan: "He should have paid attention to mathematics." In Canadian parliamentary life some three decades ago there was a particularly keen debater who was known for the effectiveness and incisiveness of his language. It did not surprise me to learn that as an undergraduate he had majored in mathematics.

The objection may be raised that, if improvement in speaking and writing is desired, more time should be spent in formal instruction in English, at the expense of other subjects. The answer is two-fold: 1) English is not the only subject which is benefiting in mathematics classes; indeed the improvement in English is a by-product; 2) The learning of a subject is often more effective when it is shown to have meaning and use in other subjects. I recall one discussion of suggested curriculum changes in which a teacher of English vigorously objected to the large amount of time which it was proposed to devote to English. His argument was that English composition should be the concern of all teachers and that to devote an undue amount of time to formal classes in English was to invite other teachers to reject any responsibility in the matter.

My final remark on this subject of language must be that if worthwhile results are to be achieved they must be worked for. It is a temptation for a teacher, always faced with the spectre of a final examination, to concentrate on the facts and formulas of the course and to pass over inaccuracies of statement with the remark, "I see what you mean", or "You have the right idea but you haven't expressed it very well". I urge you to withstand the temptation, to take time off until the pupil has expressed his idea well, until he not only has the correct idea in his mind but also the correct expression for it in words. Let us remember that these facts and formulas will be forgotten in five or ten years but they will have served

their purpose if they have been the means of inculcating a habit of meticulous accuracy in the use of words.

Training in Reasoning

I have mentioned that improvement in one's reasoning power and in his ability to express his thoughts go hand in hand. It is trite to remind you of the age-old claim that it is in mathematics that pupils meet their first and clearest example of a logical argument. It is one of the most satisfying experiences in a child's intellectual growth when he acquires the conviction that some things have to be accepted and believed, not because his parents or his Sunday-school teachers say so but because he feels that the conclusion is inevitable. His ability to reach a conclusion by carrying through an argument requiring two, three, or more steps is a measure of his intellectual growth.

The implication of this for us teachers of mathematics is that, for the purpose of general education, it is not the conclusions in themselves that are the important things. To paraphrase a well-known remark of Robert Louis Stevenson, the exercise which we get in the course of our journey is a more important objective than the destination which we reach at the end of it. We should not be in too much of a hurry to reach the conclusion that the angle in a semicircle is a right angle or that a polynomial in x is divisible by $x-1$ if the sum of its coefficients is zero. If a student memorizes the statements and even the proofs of all the theorems in the text-book, but does not acquire an ingrained feeling of the inevitability of the arguments which led to these theorems, he will have little of his mathematics course left after twenty years. He will forget the theorems but, with proper teaching, his mathematics courses should make a difference to the way his mind reacts to the problems which he will meet in his family life, in business, in politics, and in religion.

Another implication of which teachers are becoming more and more conscious is that the traditional material of geometry can usefully be enlarged by the introduction of other situations related to the pupil's experience, on which his reasoning powers can be exercised. Geometry is the closest approach that most pupils will ever get to the study of formal logic. Indeed some formal logic may find its way into school courses in geometry.

I realize that in urging as a permanent value of a training in mathematics the improvement in one's reasoning power I am treading on the fringe of the controversy on the transfer of training. I suggest to you that teachers of mathematics with vision and enthusiasm have never been in doubt on this question.

Training in Historical Perspective

Some educators are in the habit of dividing academic subjects into two classes one of which, labelled the humanities, is supposed to have the greater value for the development of the student's personality, his outlook on life, and his feeling for his fellowmen. In the second category of inhumanities are grouped mathematics and science. I do not see much value in this classification. All depends on how a subject is taught. If such a division must be made, then mathematics has a good claim to be classed with the humanities—as the Greeks and Romans did in their quadrivium of arithmetic, geometry, astronomy, and music. I am not alone in making this claim. According to Sarton, the historian of science, the history of mathematics should really be the kernel of the history of culture.

If we make such a claim for mathematics, then it is for us to substantiate the claim in our teaching. This point of view was developed in an address in 1912 by Professor C. J. Keyser on The Humanization of the Teaching of Mathematics. "It is sufficient and it is necessary", he urged, "greatly to enlarge, to enrich, and to vitalize our sense of what mathematics, regarded as human enterprise, signifies."

The more we illuminate our subject with historical or inspirational material, the greater value will it have as a part of general education and the more will it contribute to the student's adult mental equipment.

One way of enriching our teaching is by placing ideas and discoveries in their historical perspective. Since much of high school mathematics originated with the Greeks, it is possible to give the student an appreciation of and admiration for the accomplishment of this ancient people, pointing out the handicap under which they worked on account of the lack of a supple algebraic notation and the clear-sightedness of their reasoning about things which they could visualize. Euclid's proof that the number of primes is infinite, Pythagoras's proof that the diagonal of a unit square is measured by no rational number, Hero's formula for the area of a triangle—these are achievements which can hardly fail to kindle the imagination of a student and possibly to stimulate him to further reading of his own.

To take a leap of twenty centuries, we may mention another outstanding period in the history of mathematics—the seventeenth century. This is the century which saw the invention of logarithms, of analytic geometry, and of calculus. A student who reaches these subjects in his study of mathematics will acquire a deeper sense of the significance of this century than he will by a study of its political history.

Let me illustrate this point. When logarithms have been taught and when the students realize that their power arises from a judicious use of indices, it is illuminating to point out that Napier, their inventor, made no use of indices; indeed indices came into use after Napier's time. If this information prompts questions from a few eager students as to how this was possible, they may be directed to the address of Lord Moulton in the Napier Memorial Volume. The student who has thought about this question and read this essay will have an appreciation of greatness in a human being which will not depart from him when he forgets the rules of logarithms.

A greater opportunity for the humanizing of our subject is furnished by the introduction of calculus. Why did a new and revolutionary method in mathematics come into being in the seventeenth century? Why were the methods which served Archimedes and Apollonius, or at least those which served Fermat and Descartes, not sufficient for Newton? In answer to these questions, one may point to the rebirth of learning in Europe which, beginning in the late fifteenth century, stimulated literature first and science somewhat later. By the middle of the seventeenth century a great deal of information in science had been accumulated and the subject was in a healthy ferment. The names of Galileo, Tycho Brahe, Kepler, Descartes, Pascal, Harvey, Napier, are enough to suggest the discoveries which were being made in various countries of Europe. What kind of world did these discoveries reveal? Borrowing a word from Newton, we may call it a world in a state of flux. The science of dynamics stems from Galileo and the early seventeenth century. Now the mathematics of the Greeks and of the Schoolmen in the Middle Ages was good enough to describe a static world, a world at rest,—good enough for the moon, but not for the earth. For ours is a changing world, a dynamic world, a world in a state of flux. The phenomena in which Newton was interested were winds and tides, the motion of the moon and of the planets, gravitation, light, chemical changes, and for these he found the older mathematics inadequate. Mathematics for a world in motion, a world of change, a dynamic world, a world of variables,—this is the essence of the calculus. A student to whom this has been pointed out, at greater length than I have time for here, will retain an appreciation of one of the greatest mathematicians, Sir Isaac New-

ton, and of the time in which he lived, which will remain when he is no longer able to carry out the differentiation of the sine of x. Moreover, a teacher who fails to take advantage of this humanizing possibility of his subject and introduces the calculus by plunging at once into the formal process of differentiation is neglecting a valuable opportunity of general education.

Let us continually vitalize our teaching from the history of mathematics and from the biographies of great mathematicians. By imparting, along with the technique of mathematics, an appreciation of the great periods of discovery in the history of thought, we establish mathematics as one of the greatest means of a permanent and humanizing education and repudiate the term *dry-as-dust* which should have no right whatever to be associated with mathematics.

The Applicability of Mathematics

In speaking of applications of mathematics, I am not thinking here of engineering or statistics or actuarial work. There is no question of the necessity of mathematics for these fields. The pupils we have in mind are those who will make no professional use of the mathematics they learn. I take my cue from a remark of Henry Adams in his autobiography, *The Education of Henry Adams*. "Nothing in education," he says, "is so astonishing as the amount of ignorance it accumulates in the form of inert facts." By what precautions can we ensure that mathematics does not become for our pupils an accumulation of inert facts? I have already mentioned some of these. It remains to emphasize that what remains in memory after twenty years is largely a function of the connections that have been established between the mathematics he learns and the experience of the learner, whether this experience is acquired either in or out of school.

One implication of this is that connections are to be sought between topics in different subjects. The fact that a pupil is likely to receive a stronger impression from what he sees than from what he hears is the justification for the use of graphical illustrations, wherever possible, in arithmetic and algebra. A graphical study of the quadratic function gives reality to the algebra of quadratics. A graphical or experimental demonstration of the resultant of two forces is not likely to be forgotten and the same applies to the principle of the lever, the working of pulleys, and the effect of friction. The wind-rose of the meteorologist and his barometric pressure charts showing *low* and *high* areas are further examples of graphical aids. Complementary to the use of graphs in the study of algebra is the fact that algebraic proofs of geometric theorems serve again to tie the two subjects together and to provide suggestions to the student in his search for solutions of his own problems.

Opportunities are to be sought also to connect mathematics with the daily life of the pupils. For those who come from farms one can use examples involving such items as troughs and tanks, feeding mixtures, fertilizer ratios, and butterfat content of milk. For city pupils examples are readily available in buildings and machinery, budgets, and cost-of-living index numbers. The seventeenth yearbook of the National Council is a useful source of material for teachers who do not make mathematics a collection of inert facts. Not only do such applications impress the meaning of the subject on the pupil, but they also provide the basis for an appreciation of the scientific age in which we live.

One great teacher who stressed the importance of applications was Felix Klein. He was, moreover, wise enough not to exalt them to the exclusion of the main aim of mathematics. His advice still has point for us: "We want a quickening of mathematical instruction by the introduction of applications but we do not wish that the pendulum, which in former decades may have inclined too much toward

the abstract side, should now swing to the other extreme; we should rather pursue a proper middle course."

The teacher who has his eye on long range values of his subject will strive for concreteness in his own language. The teacher's experience and the width of his reading show up in the language of his teaching. Along with concreteness and contributing to it, let me mention the saving grace of humor which is as wholesome in the teaching of mathematics as in other situations. Lewis Carroll was a mathematician as well as a writer of delightful fiction. Teachers of mathematics today will probably profit more from his fiction than from his mathematics. I take one example. Mathematicians have the practice, in naming mathematical concepts, of using common words and giving them definite meanings which do not agree precisely with their meanings in common use—such words as *real, imaginary, function, set, group, ring, field.* What more apt comment on this practice than the statement of Humpty Dumpty to Alice in Wonderland regarding his use of words: "I pay them extra and make them mean what I like"!

The Personality of the Teacher

You cannot have failed to notice how often I have mentioned the role of the teacher. I shall close on this note.

Felix Klein, whom I have already mentioned, is responsible for the statement that teaching is a function of two variables, the subject taught and the pupil. This dictum implies an ideal teacher. In this imperfect world I would add a third variable—the teacher. What does the pupil remember from his mathematics courses after twenty years? One answer, whether we like it or not: he remembers us, his teachers. How often does it happen that, when adults reminisce about their school days, their talk is mainly of their teachers! How important, then, that the impression made by the teacher's personality should be such as to emphasize the importance and dignity of his subject and its value as a part of our cultural heritage! Whether the residue in the adult's mind from his study of mathematics has associations which are pleasant or unpleasant, inspiring or dull, depends as much as anything on the personality of the teacher.

I have just referred to one great teacher. I refer now to another. When his pupil, anxious to arrive at the value of what he was learning, demanded: "What do I get from learning this?", the teacher told a servant: "Give him a penny, since he must make a profit from what he learns." There is a moral to this story but I have never thought that this impatient answer was given in one of Euclid's most inspired moments. A greater teacher than Euclid gave a more illuminating answer. His disciple also was in a hurry to fix upon the core of his teaching and demanded: "Show us the Father and it sufficeth us." You remember the answer; it illustrates my point about the importance of the teacher and his personality. The answer was: "Have *I* been so long time with you, and have you not seen the Father?"

The "CARE-for-Korea" campaign voted by the General Federation of Women's Clubs at Houston will open on November 12th and end on Thanksgiving Day—in time to save Korean women and children from hunger and cold this winter. CARE's package program for Korea has been approved by the U. S. State Department's Advisory Committee on Voluntary Foreign Aid and has been endorsed by President Truman and General Ridgway. Funds raised in the campaign will be applied toward six types of CARE packages compiled to meet specific Korean needs: woolen blanket, cotton textiles, $7 each; food, woolen fabric, underwear, $10 each; knitting wool, $13. Delivery will be made to the most destitute orphanages, refugee families and other civilian war victims.

Flying Saucers—A Project in Circles

By NINA OLIVER
Ardmore High School, Ardmore, Oklahoma

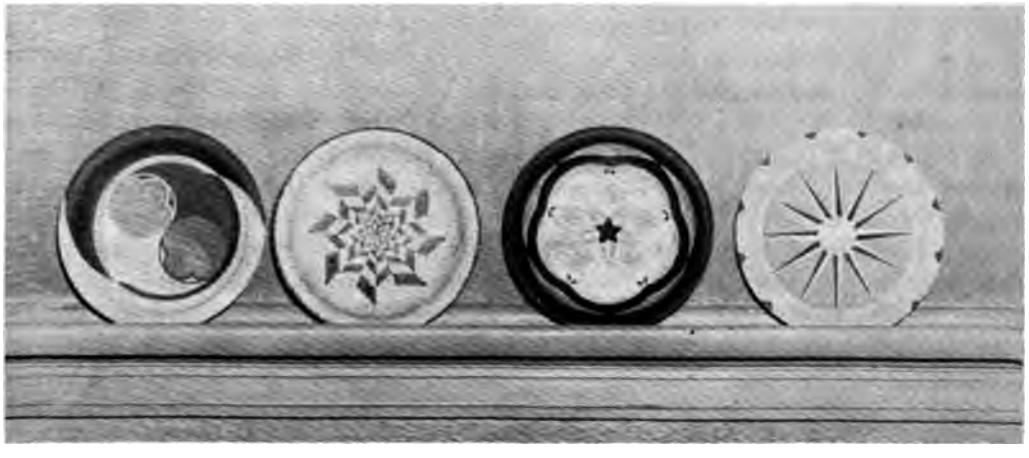
"YOU are now entering the Hall of Geometrical Flying Saucers. Famous stars, well-known squares, and scandalous triangles dwell here. . . ." This announcement, written by a plane geometry student with a flair for the theatrical, topped ballots handed to guests entering the Open House exhibit of our Flying Saucers project in circles.

Until we began the project in our three plane geometry classes at Ardmore High School, geometry theorems on determining the center of a circle, on central angles and their arcs, and on regular inscribed polygons were just another routine. Now it's a different story, as students thrill to the realization that geometry moves in the best of circles, compatible with art, design, architecture, and other professional fields.

As my plane geometry classes were approaching the study of circles this year, I began searching for a new idea to vitalize the subject. After using compasses and straight edge for several months, we started preliminary work on vocabulary by discussing construction of windows, arches, and columns of some of the attractive church buildings in our town. To transfer such structural ideas over into the classroom and to experiment mathematically and artistically, we evolved a unique plan. We decided to use paper pie plates of exactly the same size, creating designs for these plates by applying theorems of geometry.

Each student chose his own medium to use—water color, oil paint, ink, poster paint, airplane dope, enamel, shellac, pastel, poster-pastellos, crayolas, charcoal, or embroidery. He was also free to inscribe any type of polygon, arc, sector, star, etc., but had to be able to explain how his work was constructed. Students developed their own designs and color schemes and, when all the plates were in, we had a most attractive Flying Saucers display. Even some of the slower geometry students had top rating designs in superb color combinations. The designs ranged from extreme formal patterns to freedom of modern art and architectural designs.

This six-week project, to be done in marginal time, was required of every student. When designs arrived, they were put on display on the plate-rail above the



Detail of several plates entered in the Flying Saucers Contest

Students of Ardmore High School holding their winning designs are from left to right: Marie Edens, Don Butcher (who won both second and eighth places), Harold Burton, Mary Hunt, Jimmi Shortes, Fanelia Chadwell, Joe Gillian, and Max Montgomery. Mrs. Nina Oliver is holding the plate of one of the winners who was absent at the time this picture was taken.

blackboard. It became a matter of daily suspense to see just what new pattern each day would bring. Students were eager to explain their creation, and to tell how they had used training in art, mechanical drawing, home economics, or science in completing projects.

One student who is especially interested in atomic studies had the nucleus of an atom centering his plate, with spherical circles illustrating the protons, and electrons revolving around it. Another, interested in art, had a most artistic creation using an orange flower-petal design against a black background to give the plate an Oriental touch.

As plates were brought in, students explained how they constructed 72 degree angles, Gothic arches, spirals, concentric circles, five-pointed stars, octagons, or decagons. They discussed reasons for combining certain colors and lines, warning other students against hazards of difficult types of figures or media.

Scientific students released imagination concerning atomic structure and fission, appreciation of light spectrum and the color circle, and knowledge of the points of the compass. One young philosopher showed mankind pondering the fate of civilization in the atomic age, with a prediction of a happy outcome. His plate showed The Thinker with an atomic cloud hovering overhead, and flame edging an inner circle just inside the green outer edge of the plate. Individuality was portrayed by each design, and at the conclusion of the six-week period we were very proud to open our room for the Flying Saucers exhibit.

A boy and girl from each class served on the arrangements committee for the Open House. They made posters advertising the "Hall of Flying Saucers," wrote and presented over the loud-speaker a skit inviting students and faculty to the "Flying Saucers Premier." Refreshments were served to the 500 students, teachers, parents, and professional men and women who attended. The committee in charge had to call for additional help, and before the day was over almost every student had served as guide, explaining the geometry and medium of the various saucers.

Each guest was given a ballot, and was asked to vote on the ten plates he liked best. In addition to the blurb already quoted, guests were admonished to "please examine our space-ships carefully and vote for the ten that fly the highest with you." Among parents and professional people who came to visit and to vote were two outstanding architects, an art teacher, the high school principal, and the superintendent of schools.

The students were proud of their creations, and when the project was completed they felt they had gained a new conception of the meaning of geometry. Now they see more than circles and angles and theorems. They see cathedral windows, interior decorations, patterns for printed materials, designs for furniture, and a whole group of other possibilities still unexplored.

Texas' First Mathematics Institute was held on the campus of the University of Houston from July 30 through August 2 and was sponsored by the College of Education, the Mathematics Department and the College of Business Administration of the University. The purpose of the Institute was to stimulate interest in the teaching of mathematics, to show its practicality as a subject in the curriculum by showing its uses in business and industry and to help teachers of the subject to acquire ideas as to the improvement of their teaching.

Dr. William A. Brownell, Dean of the College of Education, University of California at Berkeley, was the guest speaker and consultant in the field of arithmetic. Dr. E. H. C. Hildebrandt, Professor of Mathematics at Northwestern University, Evanston, Illinois, served in the same capacity in the field of secondary mathematics teaching, and Dr. E. T. Bell, Professor of Mathematics at the California Institute of Technology, Pasadena, contributed in the area of college mathematics. These authorities in their fields greatly stimulated the thinking of all who participated in the Institute.

An outstanding feature of the Institute was a series of study groups held each afternoon where classroom teachers could gather and discuss down-to-earth problems that occur in their daily teaching. Eight of such groups were conducted under the leadership of Texas people, including Miss Cecile Foerster, Supervisor of Kindergarten, First and Second Grades, Houston Public Schools; A. E. Wells, Superintendent of Schools, Abilene, Texas; Miss Lillian Smythe, Sixth Grade Teacher, Pasadena Public Schools; Dr. I. I. Nelson, Professor of Curriculum and Instruction, University of Texas, Austin; Dr. Ruth Lane, Associate Professor of Mathematics, Sam Houston State Teachers College, Huntsville; Miss Ruby K. Jones, Mathematics Teacher, Odessa, Texas; Dr. C. O. Mitchell, Chairman, Business Administration Department, East Texas State College; Dr. W. I. Layton, Head of Mathematics Department, Stephen F. Austin State College; and Dr. W. L. Porter, Head of the Mathematics Department, A. and M. College at College Station.

The Institute was opened by an address by Dr. M. M. Slotnick, Chief Mathematician of the Humble Oil and Refining Co. of Houston, who spoke on "The Place of Mathematics in Industry." It was interesting to know that mathematics occupied such an important position in all aspects of business, industry, and life in general.

The Institute did not neglect the social aspects of living either. Mathematics teachers enjoy human relationships as do all people. A luncheon was held on Monday at which people became acquainted and mingled socially. That evening a social hour featured "The Singing Hicks," and square dance demonstrations. On Wednesday a banquet was held in the ballroom of Oberholtzer Hall. The speaker for the evening was George Carmack, Editor of The Houston Press. Under the direction of Dr. C. F. Hiller, Vice-President in Charge of University Development and Public Relations, who acted as Master of Ceremonies, the affair provided humor, stimulation, and relaxation as a fitting climax for the Institute.

Attendance was not confined to local people. Three hundred twelve people registered, some coming from Arkansas, Louisiana, West Virginia and Ohio.

Proceedings of the Institute were published early in September. Any one interested in obtaining a copy may send his order with check or money order for $1.50 to C. B. Rader, Mathematics Department, University of Houston, Houston 4, Texas.

ESTHER F. GIBNEY

The National Council welcomes invitations from groups willing to serve as hosts to any of its meetings, and where the facilities are adequate. If your organization will sponsor a meeting, send your invitation to the President for consideration and action of the Board.

Arithmetic for Kindergarteners

By LORETTA U. BOYCE
Philadelphia, Pennsylvania

I AM always interested in the comments of people when they learn that I teach kindergarten. Immediately they seem to feel that I am "lucky" or relieved of many responsibilities encountered by the grade teachers. The majority of people feel that kindergarten is a place where children cut, sing, paste, and enjoy rhythms until they reach the age of six. Unfortunately, many kindergartens serve for that purpose alone. Ideally, the kindergarten serves a far greater purpose. It is a place where children develop as individuals and prepare for future studies.

Experience demonstrated that trips can be exceedingly meaningful to youngsters as they try to understand better the world in which they live. Trips have helped them to create original stories and songs for our collections. The pupils have often relived their trip experiences in block building, clay modeling, paper work, and during discussion periods. After one of our weekly trips in the neighborhood, the class decided that they would like to have a store in the classroom. We discussed the matter to be sure that it was more than a passing fancy and decided that we would have a grocery store. Immediately each pupil decided and made known what he would like to contribute to the self-appointed committees. Within a relatively short period of time our store was constructed and a plan prepared whereby the job as store clerk would rotate weekly.

When the store began to function smoothly, various questions were asked about prices and making change for the customers. Then I was actually confronted with the problem: "How much arithmetic should be included in the kindergarten curriculum"? I had never given the group any type of formal examinations but I was convinced that they were ready for the very early lessons in arithmetic.

According to Wheat,[1] testing of ability on this age level should be informal and I followed through on this idea. I asked each child to count as far as he could and then listed next to each name the abilities and difficulties manifested. I learned that the youngsters could count objects ranging from ten to one hundred. I realized that the ability to count was not indicative of a readiness for arithmetic for counting might have been learned through a meaningless drill or under the influence of a strong paternal or maternal hand.

When we speak of readiness I am always tempted to ask "Just how do we really know when a child is ready?" I am cognizant of the fact that a child will show a readiness after he has mastered what has gone on before or when he shows an anxiousness for new assignments; but readiness, as Brownell[2] defines it, includes all aspects of ability. He further states that involved in this ability are the dynamic factors of motivations and the social factors of needs. I can go along with Brownell for it has been my observation that children learn to count or build up number concepts as it serves a need and interest. When I was able to make clear to the youngsters the need for counting as the store clerk, as the customers, or just as a person, they realized the importance and were motivated to count. I further explained and showed the class how we used counting for block building, table setting, giving out supplies and various

[1] H. G. Wheat, *The Psychology and Teaching of Arithmetic* (Boston: D. C. Heath and Company, 1937), page 168.

[2] W. A. Brownell, "Readiness and the Arithmetic Curriculum," *Elementary School Journal*, 38 (January 1938), 344.

other activities in the classroom.

Although children might show an outward readiness for arithmetic we must consider the inner man. The article by Dickey[3] provoked much thought and contributed to my thinking out this problem of arithmetic readiness in young children. "Readiness," as defined by Dickey, "is a concept designed to tie together the overt behavior (symptoms) of the learner with the inner dynamics of maturation." He further states that "it (i.e. readiness) is the bridge connecting the observable behavior of the learner with the organized covert behavior operating on his level of maturation." Readiness includes the physical, emotional, mental and social status of the child. It is also important to remember that learning is an orderly process based on maturation.

This entire concern for readiness on the part of the pupil before presenting new facts and ideas is the result of a change in our philosophy of education. There is now a broader emphasis on growth and development of the personality including social-utility objectives.[4] Psychologically we have changed our concept also for now we have the broad organismic concept of the learning and creative processes which emphasizes the roles of maturation. The goals and insights into the problems are always in view.

In view of the fact that the children were eager to use arithmetic and possessed the ability and readiness, I started an orderly systematic study of arithmetic that has led to a clear, exact and definite learning. We counted tongue depressors individually and collectively, picking up a stick with each number. We counted the stairs as we ascended to the roof and descended to our classroom. Each day one child was permitted to count the number of children present and give the number to the porter who in turn would put down the cots for our afternoon rest period. Occasionally I would allow an error to go through and then the experience would be more significant when the person that counted discovered that there were too many cots or not enough cots. When children discover their own errors the correction is more meaningful.

Although I was not firmly convinced that I was actually doing the right thing when I included orderly arithmetic lessons in my curriculum, I felt that it would be more harmful to let the ideas and skills possessed by the pupils in the group lie dormant and unused, or unchallenged.

I have been convinced as I work with five-year-olds that we underestimate the ability of children in the kindergarten to use numbers. It is not necessary to hold back the teaching of arithmetic but it is necessary to bring forth some meaningful and useful teaching of arithmetic in the early grades.

This experience with numbers has not been an unhappy experience for anyone for I stand alert watching for clues of disturbances and repeated failures. Furthermore, the children are not required to count but they are motivated to learn because it will subsequently mean assignment to the tasks involving counting. There are several youngsters in the class who require individual attention and others who cannot count beyond twenty but in neither case is it an indication of failure.

I can recall three definitions I learned some time ago which are significant in such a discussion. 1) Maturation is the growth of a structure as it reacts to various stimulations surrounding it. 2) Learning is growth in response to the functioning of a structure. 3) Development is the perfection of functions. They all remind us of one important factor . . . a child must follow the laws of his own growth although he may not show a readiness equal to other members of his class.

Development was encouraged through many experiences. The store was not the

[3] J. W. Dickey, "Readiness for Arithmetic," *Elementary School Journal*, 40 (April 1940), 593.
[4] *Ibid.*, p. 592.

only activity that involved the use of numbers but also such daily tasks as setting the tables, giving out paper, or scissors, taking attendance, or selecting the four records for use before lunch. Many times the class formed a line and I asked my helper to send four children to the washrooms. As a safety measure we limited the number of children in any particular area according to space. For example: three children could use the work bench together, not more than six could use the dramatic play area, and as many as five children could use the science materials. Many times I found more than the limited amount and asked the children to count and see whether we didn't have too many in the group.

After the pupils learned to count they would apply their knowledge to every conceivable aspect of the classroom activity. One day I saw some of them clustered around our assignment calendar which gives names (and an identifying sticker next to each name) of the persons responsible for watering the plants, feeding the fish, and setting the tables each day. They were busily and accurately counting "how many days before I water the plants."

Many of our games involved the use of arithmetic. I often counted aloud to see how fast they cleaned up after an activity period. Subtraction (a topic initiated by a pupil) was demonstrated as merely a matter of regrouping. I used the game of musical chairs to illustrate the point. We started with all the group on one side of the room. We counted the number and then placed a chair in the center of the room for each child except one. As the music played the group would dance around the chairs but when the music stopped everyone grabbed a chair. In view of the fact that we were short of one chair someone that wasn't quite fast enough didn't get a chair. The child would go to the other side of the room. After I had enough children on both sides of the room I would explain the matter of subtraction as nothing more than regrouping.

It is axiomatic that we can teach a child to count through drill. I have taught successfully to a large number of youngsters such mathematical terms as *half*, *almost all*, *large*, *small*, *biggest*, *widest*, *highest*, etc., but I have not forced the remainder of the class to learn the material. A child will learn when he is ready, interested or feels the need. Above all, a child will learn when he understands.

I can agree with some of the results found in the MacLatchy study of the preschool child as described by Carolyn Stotlar.[5] Children under six can asquire a considerable understanding of numbers and some ability in dealing with number situations. The important factor in all presentations of arithmetic problems is to teach with meaningful concrete objects, especially to young children, and to stress understanding. The abstract number problems should enter the program after a firm foundation has been tried and established. Abstract numbers in the initial arithmetic lessons are void of any understanding.

Number concepts are important and should be developed as early as possible for they provide the ideas necessary to understand facts and processes. In Grades I and II concepts are vital for one aspect leads to accuracy in counting and another shows the relationship between cardinal number names and ordinal ideas. Still another aspect deals with the idea of tens and ones and another shows the relationship between number names, numerals and quantities. We in the kindergarten age group are limited to the number of concepts we can build up but I feel that our discussions of arithmetic and its purposes and uses will be helpful in preparing children for Grade I. At least they won't be stunned by reading and arithmetic simultaneously. It is gratifying to hear a pupil ask for a whole sheet of construction paper "So I can make a big house" or to

[5] Carolyn Stotler, "Arithmetic Concepts of Pre-School Children," *Elementary School Journal*, 46 (February 1946) 342–345.

have a pupil ask for a half of a sheet and then ask whether he might fold it in half.

Recently I looked over at the block-building area and saw Ned standing forlorn, looking at his incomplete building. I went over to find out just what could have happened to cause his helpless appearance. He told me with deep regret that he couldn't finish his building because all of the long blocks were taken and he needed more. Before I could help him solve his problem Bob who usually has little to offer (constructively) shouted, "Use two short blocks like we did and it makes a long block." I didn't teach that fact to Bob nor any other youngster and couldn't accurately tell how he discovered that correct observation but it takes me back to a statement made by Florence Reid,[6] "Rational counting is an outgrowth of the child's development."

Occasionally it is felt that the task of teaching kindergarteners to count and build up number concepts is an easy one. Quite to the contrary; the task is not easy but can be achieved. I have assured the children that academic skills are secondary and have placed emphasis on their security and happiness in the group.

The following is a brief concise record of the progress of one child made over a four month period and will indicate the possibility for development if only the opportunity is given:

February 8—C's first day as store clerk. "Teacher how many pennies should I get for eggs?" (I answered fifty pennies.) C began to count and went to 20 without requiring any assistance. Continued task without trouble for articles less than 20 cents.

February 16—C's turn to set the table. Counted the children one by one and set the table for 18 children. At lunch he discovered that he needed two more settings and went to the kitchen for additional supplies. "Teacher, I made a mistake." I explained to him that a mistake is very possible and he should recount for the correct number. C had quite a bit of difficulty for the children just would not stay together.

February 19—C compared his building with his friend's building and decided that he needed "this many more blocks" holding up seven fingers. C sang a counting song as he added blocks to his building.

February 23—C asked to serve as store clerk because the appointed child was absent. He counted his money and then turned to me. "This is how my grocer counts." He yelled at M, "This store is not open yet—don't you see I'm counting my money?" Later M walked in and gave C what she thought was enough money for her purchase. C took a long time counting each penny as he placed it in the register. Finally, he discovered that he was short and ran out of the store into the doll corner where M was playing the role of mother. "You are a gypper. You didn't give me all of my pennies. Don't you think I can count?" She disagreed and C called me to the scene. We agreed that we couldn't check the amount M had given because other money was in the register at the time. So we suggested that hereafter C should count the money before putting it in the register. C continued as clerk and counted the money slowly before permitting the customers to leave the store.

February 28—"Teacher, there are no more days in February." I asked C how many days there were in February and he counted with unwanted but definite assistance from classmates. "Now we must go to the *next* page of the calendar."

March 1—"Miss Loretta this is a long month." "Why, C?" I asked. "Because there are *more* days than we had last time." C counted 31 days to me without trouble although he slowed up from 29 to 30. C went to tell and prove to the other children that March would be a long month.

March 6—C counted with excitement the number of baby guppies we have. (He now counts rapidly and with a great deal of self confidence. He loves to count and goes around counting every object in the room.) I was particularly interested in his progress because in the beginning he tended to shy away from our very first counting experiences.

March 7—C counted the children for the porter and gave him the correct number. "Be sure you put down 23 cots, Tim. Boy, will we need a lot of room for cots today . . . everybody came to school!" C counted each cot as it was put in place and I was impressed that he was able to tell Tim that he needed three more cots after 20 had been set up.

I don't know whether C's progress is evident to the reader but he has gone a long way to appreciate and use numbers. To him arithmetic is fun and he enjoys counting to see how far he can go. C told me that he wanted to be a number man when he grows up and I chuckled for I thought of the common connotation of the

[6] Florence Reid, "Incidental Number Situations in First Grade," *Journal of Educational Research*, 30: 36.

title. (A number man is one who collects numbers for gambling purposes.) I explained to C the many ways we use numbers and no matter what he really decides to be when he grows up, he will use numbers.

March 19—C returned after a short illness. "Miss Loretta, did I miss any new counting games?" "Yes," I replied, "but we will play them again so you may learn them." "Gee, I'll be so glad." I have often wished that I could have recorded all of C's expressions with all the zeal that he showed. C went over to the crayon can later in the day and counted each piece of crayon and grouped the pieces according to colors. "Look Miss Loretta, we have more yellows than reds. The *next biggest* color is green." C ran out into the hall and came back shouting, "Teacher, when the long hand moves to the next number it will be clean-up time." "Thank you, C." C could not tell time but he knew the position of the hands for clean-up time, lunch time, and snack time.

March 28—We had visitors and C put on a perfect show for them. "Do you know my teacher's name? Her name is Miss Loretta. Do you know what my name is? My name is C. Miss Loretta has a lot of children today and they are so noisy. Do you want me to count for you?" C counted, but I lost track to go to another child's rescue.

April 10—C went over to the calendar and counted how many days before the holiday. His number was correct. "Teacher, did I ever count to 100 for you?" "Why no," I answered, "would you like to right now?" C started to count but I stopped him to get the tongue depressors. He started and counted to 100 taking out a depressor for each number.

In summation, by this time (early May) C can count to 100 and enjoys counting. He wants to do subtraction and addition the way "daddy" does but I assured him that the first-grade teacher will teach him more number work. He is anxious and ready for new work and I feel he has benefited from his early experiences in the kindergarten group. He learned the early elements of number work in a free relaxed group and it made quite a difference.

I wasn't sure that kindergarten was the right place to begin getting children arithmetic conscious, but I am convinced that there is a place for arithmetic in kindergarten if it is taught with meaning. The important factor to remember is that the children must show a readiness for the subject matter and must know the social value of the subject matter.

ANNOUNCEMENT OF 1951 NCTM CHRISTMAS MEETING

The 1951 Christmas meeting of the National Council of Teachers of Mathematics will be held at the Oklahoma A. & M. College, Stillwater, Oklahoma, December 27, 28, and 29. Plan to attend and profit by the excellent program prepared by Miss Lenore John, vice-president of the Council. The complete program may be found in the October issue of THE MATHEMATICS TEACHER.

Travel Information

Those traveling by train should arrive in either Perry, Tulsa or Oklahoma City, Oklahoma and proceed to Stillwater by bus. Since adequate bus service is not available in Perry for all trains, transportation will be provided if the Department of Mathematics at the College is notified by December 15. Those traveling by plane will probably have to come from Tulsa or Oklahoma City by bus, though it is possible that air service may be available by the time of the meeting. Those traveling by car will find excellent highways leading into Stillwater from all directions.

Sante Fe trains arrive at Perry at the following times: from the north at 7:38 A.M., 3:50 P.M. and 11:22 P.M.; from the south at 6:05 A.M., 7:50 A.M., 2:20 P.M. and 7:05 P.M.

Frisco trains arrive in Tulsa at the following times: from the north at 5:30 A.M., 7:35 A.M., 10:00 A.M., and 2:15 P.M.; from the south 7:40 P.M. and 9:25 P.M.

Self-Directed Homework

By ROSE KLEIN
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ALL of us know classes in which, the lesson over, the teacher calls out, "Just a minute, class. Homework for tomorrow: page 00, ex. 1–5." Often it is not as rushed as this. Quite calmly, about the middle of the lesson, the teacher announces the stereotyped words, "And now the homework: page 00, ex. 1–5."

What are some objections to the above?

(1) Must *all* pupils have the *same* assignment? Definitely not. If pupil A can do a new type problem expertly at the end of the lesson, and pupil B is still slow at this new work, why should both be given identical assignments? It is definitely unfair to pupil A.
(2) The *teacher* has decided what tasks are to be done. The pupil has no choice in the matter. Let me explain later why this is so unwise a procedure.
(3) The teacher has to inspect, or supervise the inspection of about 40 homework papers in each class. What a waste of valuable time!
(4) The old-type homework assignment leaves many pupils tense and under pressure. Lines must be ruled just so, the papers must be exceptionally neat, etc. Copying is widespread. A conscientious pupil forgoes many opportunities to add to his cultural background, or forgoes opportunities for social contacts which are so important during the teen age. Considering school, homework, music lessons, dancing lessons, dramatic lessons, religious studies, there doesn't seem too much time left for relaxation, for free play.

Here, in the matter of homework assignments, is a wonderful opportunity for pupils to get training in assuming responsibility. We're not dealing with infants to whom mothers continually say, "Do this," "Do that." No wonder some children rebel. Mothers order them around, teachers order them around in school, and even after school indirectly by the usual homework. Under the old method the pupils do what they are *ordered* to do, not what they would *like* to do. It's *work*, not *play*. I propose to make it *play*. Let the pupils plan so that there will be fun in *what they* are going to do, and *how they* are going to do it, and let them give a fairly good reason *why they* are going to do this. A group of girls and boys in one class became so interested in this newer type of work, they met at least once a week in the community center of our school to discuss mathematical topics. It was fun to them, holding initiations suitable for a mathematics club, and doing everything by themselves.

How can we make homework a game? Pupils are filled with wonder, and *curiosity*. They can ask endless questions to which many a parent will testify. Let's use this fact in our homework assignments. Pupils have *hobbies*. Let's use this fact too. Pupils have varied interests, whe her because of background, or future goals. Let's use the idea of *varied interests*.

As an idea pops into some child's head, he may ask a definite question, "How? Why? When did—? Could we do this—?." Make that the basis for the homework assignment. Tell the pupil to look it up and report to the class, or appoint him chairman of a committee to investigate this matter and then to report to the class. In that way, almost everyone has an individual assignment. Perhaps you may even want to challenge the class, "See who will be the first to get Johnny an answer. I wonder if someone will get him an answer by tomorrow."

If a pupil is unimaginative, or has not been sufficiently stimulated to seek knowledge, he doesn't do project work for the time being. Perhaps at the moment, he has too many other things on his mind (Confirmation, music recital, etc.). Leave him alone. Sooner or later, he will be aroused and eager to look up something. However, this does not mean that the person has no homework. There is another part to the daily assignment.

Each pupil must assume responsibility for mastering the day's lesson and for

reviewing previous lessons. All work done in class is kept in a notebook. Everything in this notebook is corrected before the class is dismissed. Therefore, when a pupil wishes to review the work, he has model examples with which he can practise. If he hides the work, just exposing "Given" and "To Find," or the introduction, he can give himself a practise test on either new or old work. If I have reason to believe that a pupil who needs this review is not doing it, I ask him to hand in his practise papers each day until further notice. Others may quickly glance through the notebook and call that a review. His must be in writing, on paper to hand in. If the pupil needs practise beyond what is provided by the classwork, he resorts to the text, choosing his own page, and doing as many examples as he wishes. Upon showing considerable sustained improvement, the pupil is then told he doesn't have to hand in his work in the future.

My objection to ex. 1–5 was that this was *habitually superimposed* by the teacher on an *entire* class. The newer plan provides for a flexible type of homework which can be adjusted to meet the needs of the individual. Some individuals need more than 5 examples, some fewer. Notice that the pupil chooses which type of example he needs to practise, whether this be new or old work, or both. No two homework papers are alike. Any example in the notebook, or later in the textbook, may be chosen, and *any number of these*. How grown-up the pupil feels. He is the boss, choosing his own homework, stopping when he feels like stopping. He decides when the day's practise should be ended. Look at the way his personality expands as he is given free rein and is allowed to *make decisions for himself*. If we consider the pupil's development at this adolescent stage, we note the parent is dropping the reins by giving the child an allowance, letting *him decide* how it is to be spent, and also letting *him decide* on his own wardrobe. Teachers should drop the reins in this matter of *what* homework and *how much*, for the adolescent *is* mature enough to decide these for himself.

You may find a pupil handing in work on topics he knows. This is quite rare, and can be settled by merely pointing out to him that those aren't the topics he is poor in. At the end of every test, and often during classwork, entries are made on a page in the notebook called "Topics I Forgot." This is compulsory after a test. From this list the pupil chooses the topics for his reviews.

Very often I do give a definite assignment, especially on fundamentals, but usually to only a few *individuals*. One pupil may be given drill on division with fractions, another, another page—division with decimals, another may get more problems, say, in commercial discount. You will observe, however, that I *do not* have 40 papers to inspect in *each* class.

Notice, there is intelligent action and purposeful activity *by* the pupils. However, there is also intelligent direction and motivation by the teacher. If a pupil is at all doubtful of his ability to do the grade work, he practises. If you are doubtful of his ability, you ask that this review be in writing, and handed in *every* day (excluding Monday) until further notice. The children do not have complete freedom regardless of the consequences. The pupil shoulders the responsibility, but this is contingent upon the pupil's maintaining work compatible with his ability. The teacher is still in control, though unobtrusively.

Let us go back to the first part of the assignment, which is the purely voluntary work, wherein pupils answer the Hows, Whys, Whens, What ifs—research projects. When a pupil brings in a report, it must be in his own words, or in language on a junior high school level. There must be understanding on the part of the pupil. No one gains from a parrot-like repetition of an article in the encyclopedia or other source. When a pupil brings in a project, for example a sundial he has constructed, this is not just a copy of the model pic-

tured in the book. The pupil must be able to answer in his own words, the question "How does it work?" and be able to convey his knowledge to other members of the class. Another word of caution. These reports should not be the result of aimless browsing, even though browsing has great value. The pupil just doesn't choose any mathematical topic in the encyclopedia. There should be *directed* search arising out of some classroom situation. To illustrate. As a rule, a pupil in a seventh year class would have no occasion to make a report on logarithms, assuming the pupil understands the idea. This would be foreign matter for the class, uncalled for, and totally out of place. On the other hand, a pupil could give a report on cantilever and suspension bridges when the class is studying about triangles. That is apropos, and quite fascinating to the pupils. The pupil who is very much interested volunteers to give such a report because the topic of interesting bridges was mentioned in class, or pertains to the class work.

In conclusion, if a fellow brings in mathematical material which he obtained from his uncle's office, the pupil will peruse that, and learn more about mathematics in real life, than if you said "Homework, ex. 1–5." Pupils have brought in salesman's rate books from insurance companies, income tax charts, sample copies of civil service tests, slide rules, sundials, architect's blueprints of an entire two-story house, booklets on actuarial work and on navigation, even library books on mathematics (Careers for Girls in Science and Engineering). One girl started a scrap book of candy wrappers and labels from canned goods, to show that many commercial companies in America are using the metric system. You'd be surprised at the store of knowledge, and the sources.

Let's give pupils more training in leadership. Give the pupils the responsibility and practise in making decisions.

Earlier in this report I mentioned a group deciding on an initiation suitable for a mathematics club. This is what one boy suggested, and was enthusiastically accepted by the group, that "all members had to wear around their necks a paper chain made from paper about 3 inches square." This, some of you will recognize, is the old problem of taking a small paper, making a small hole in it, and challenging others to make the hole large enough for you to put your foot through. The pupil changed it to, large enough to put your head through.

What are the pupil's reactions to self-directed homework?

Slow pupils:

(1) "I like it (individualized homework) because it is going to help *me*."
(2) "That's a topic I never knew about until now."
(3) "I learned three topics, so far this week. I've really improved a lot."

Normal:

"When I need written work, I do it. Can't hurt me, but can help me." (Like vitamins.)

And the bright pupils:

(1) "We get enough practise in class."
(2) "This way I have more time to learn from people, not just from books. When I go down to my uncle's place, I see where mathematics is used in real life."
(3) "Now I can call some time my own."

As for the teacher:

(1) She doesn't have to answer the question, "Do we have to have homework?" Each pupil concerned knows why.
(2) No text has as yet been written which is challenging enough to bright pupils.
(3) The teacher does not have 40 homework papers from each class. She therefore can devote more time to *planning* work to meet the needs of individuals.
(4) Mathematics is not a series of tricks. Pupils gain the power to organize knowledge (reflective thinking).

What does the guidance counselor have to say? She agrees that

(1) This method keeps the child continually examining his own progress and playfully intrigues the child into further effort.
(2) The teacher knows her pupils and their ability.
(3) Pupils are helped to cross the Rubicon from childhood over adolescence into adult ways.

Anyone, teacher or parent, should welcome an opportune moment for adolescents to make independent decisions.

Critical Thinking—What Is It?*

By ROBERT E. PINGRY
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WHAT IS *critical thinking?* Can we mathematics teachers tell what we mean by the phrase? Do mathematics teachers mean the same by critical thinking as do the teachers of the social studies or the logician or the psychologist?

Kenneth Brown recorded the replies of five hundred classroom teachers concerning the important objectives of the teaching of geometry.[1] The objective receiving the highest rating was, "To develop the habit of clear thinking and precise expression." This is probably a noble objective, but what does it mean? What is the habit of clear thinking? In fairness to Kenneth Brown and the five hundred teachers, it should be stated that the wording of this objective was not theirs. The objective had already been printed on a questionnaire first distributed by Mr. Shibli in 1930 and more recently by Kenneth Brown. It is true, however, that in recent mathematics meetings and in recent articles, phrases concerning *clear thinking* have been used in just as vague a way as the objective printed on this questionnaire.

The title of this paper concerns itself with *critical thinking*. How is *critical* thinking different from *clear* thinking, or is it? If this troubles you, then pity the poor reader who is faced with an avalanche of descriptive phrases of thinking as he reads through the journals and books on teaching, philosophy, and psychology.

The writer has found the following phrases in his reading:

clear thinking
straight thinking
reflective thinking
elaborative thinking
scientific thinking
postulational thinking
autonomous thinking
organic thinking
human thinking
subhuman thinking
logical thinking
rational thinking
factual thinking
inventive, productive, creative, empirical, hypological, and on and on and on . . .

When we mathematics teachers state that one of the objectives of a course in mathematics is to develop the habit of clear thinking and precise expression, aren't we guilty of not being very clear and precise in our own thinking? Possibly J. W. Young was wise when he avoided all of the unusual adjectives to describe thinking and said, " . . . it is clear that the chief end of mathematical study must be to make the pupil think."[2]

THE MANY ASPECTS OF CRITICAL THINKING

It would be an interesting experiment to ask a group of persons to draw up a list of behavior patterns of people who had learned to think critically. Certainly many differences in the concepts of critical thinking would be apparent. In reading on this subject the writer has been aware of at least five different emphases that have been placed upon the concept of critical thinking. They are:

1. Critical thinking as collecting data,

* A summary of a paper presented to the Mathematics Section of the Illinois Education Association, Lake Shore Division, October 20 and 23, 1950.

[1] Kenneth E. Brown, "Why Teach Geometry?," THE MATHEMATICS TEACHER, March, 1950, p. 104.

[2] J. W. Young, *Fundamental Concepts of Algebra and Geometry* (New York: The Macmillan Co., 1936), p. 4.

organizing data, and formulating hypotheses from data.
2. Critical thinking as use of correct principles of logic and understanding the nature of proof.
3. Critical thinking as criticism of thinking.
4. Critical thinking as related to understanding of the psychology of propaganda and advertising techniques.
5. Critical thinking as synonymous with problem solving.

Critical Thinking as Skill in Collecting and Analyzing Data

Some would state that they would consider a student a good critical thinker who was good at collecting and organizing data related to a problem; who could evaluate the appropriateness and the validity of sources and information derived from them; who could distinguish between fact and opinion; who could compare data making use of graphs, tables, and statistics; and who had developed the habit of suspending judgment until sufficient data was present. All of these skills and habits have been used by various people as a basis for measurement of a student's ability to do critical thinking.[3,4]

Critical Thinking as Understanding and Skill in the Use of the Principles of Logic and the Nature of Proof

Others would consider those behavior patterns related to the students' understanding of the principles of logical reasoning and the nature of proof as the behavior patterns desired in our students.[5,6] Such outcomes of learning, among others, would be:

1. The ability to recognize the basic assumptions upon which a conclusion depends.
2. The ability to recognize that certain tacit assumptions have been made and ability to identify the tacit assumptions.
3. The ability to detect *reasoning in a circle* in an argument.
4. The ability to detect *non-sequitur* situations in an argument.
5. The understanding that proof by elimination depends upon knowing that at least one of the list of possibilities must be true.
6. The understanding that crucial words and phrases must be precisely defined.
7. The understanding that if one accepts certain assumptions one must accept the conclusions that are logical consequences of these assumptions.
8. The understanding that some people have an inclination to accept a converse as true just because the original proposition is known to be true.
9. The understanding that a converse need not be true just because the original proposition was true.
10. The understanding that no demonstration really proves anything that is not already implied in the assumptions.
11. The understanding that in a deductive system some words must be left undefined.
12. The understanding of the difference between true conclusions and valid conclusions.

[3] M. F. Rosskopf, "The Present State of Evaluation of Critical Thinking in Algebra and Geometry," THE MATHEMATICS TEACHER, April, 1950, pp. 143–148.

[4] Hilda Taba, "The Evaluation of Critical Thinking," *Teaching Critical Thinking in the Social Studies*, Part Four, Thirteenth Yearbook, National Council for the Social Studies, 1942, pp. 123–175.

[5] Progressive Education Association, *Appraising and Recording Student Progress* (New York: Harper and Brothers, 1942), Chapter II, "Aspects of Thinking."

[6] Harold P. Fawcett, *The Nature of Proof*, Thirteenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications, Teachers College, Columbia University, 1938).

Critical Thinking as Criticism of Thinking

Max Black uses the adjective *critical* in connection with thinking to mean *critical of thinking*. He defines logic as the art and science of criticism of reasoning.[7] He further states:[8]

> The logician searches for *standards* of reasoning. To be in a position to *improve* reasoning means to be in a position to distinguish *good* reasoning from *bad*. A man who judges cattle has some specifications before him (often in a very precise form) of what constitutes a good specimen of the breed he is judging; a thinker who tries to improve his thinking must, likewise, have in mind some standard for discriminating better thinking from worse.

Critical Thinking Related to Understanding of the Psychology of Propaganda and Advertising Techniques

Others would include behavior patterns associated with the students' understanding of the psychological basis for propaganda and advertising techniques. They believe it is not enough for the student to recognize the bad reasoning involved in propaganda and advertising, but the student should also understand some of the psychological bases of why the techniques work so well.[9]

Critical Thinking as Synonymous with Problem Solving

Confusion as to the meaning of the term *critical thinking* also arises when some consider a person good at critical thinking who is able to solve problems. Critical thinking is thought of as a synonym for problem-solving or productivity in thinking. A person who is able to attack an original problem situation and quickly obtain a solution is said to be a good thinker. The person who is able to forge ahead into uncharted areas, make generalizations, be creative, make discoveries, and be inventive is thought by some to be a good thinker. Is productivity in thinking the same as critical thinking?

Wertheimer's very interesting and valuable book has made an important contribution in this area.[10] Wertheimer is skeptical of the traditional logic approach to what he calls productive thinking. He writes:[11]

> If ones tries to describe processes of genuine thinking in terms of formal traditional logic, the results are often unsatisfactory; one has, then, a series of correct operations, but the sense of the process and what was vital, forceful, creative in it seems somehow to have evaporated in the formulations. . . . Training in traditional logic is not to be disparaged: it leads to stringency and rigor in each step, it contributes to critical-mindedness, but it does not, in itself, seem to give rise to productive thinking. In short, there is the danger of being empty and senseless, though exact; and there is always the difficulty with regard to real productiveness.

THE COMPLETE THOUGHT PROCESS

One of the most helpful analyses of the complete thought process is that provided by John Dewey in his book, *How We Think*.[12] He analyzed what he called reflective thinking or problem-solving into the following five steps:

1. Becoming aware of a problem.
   A perplexing situation has arisen. The individual is aware that a problem is present.
2. Defining the problem.
   The rather vague, perplexing situation is now made more sharp and specific. The goal to be achieved is now known.
3. Suggesting hypotheses for the solution of the problem.
   Conjectures, suppositions, guesses are made. "The inference involves a leap, a jump, the propriety of which cannot be absolutely warranted in advance."
4. Reasoning out implications of the suggested hypotheses.

[7] Max Black, *Critical Thinking* (New York: Prentice-Hall, 1946), p. 7.
[8] *Ibid.*
[9] C. R. Miller, *How to Detect and Analyze Propaganda* (New York: Town Hall, 1939).
[10] Max Wertheimer, *Productive Thinking* (New York: Harper and Brothers, 1945).
[11] *Ibid.*, pp. 10–11.
[12] John Dewey, *How We Think* (Boston: D. C. Heath and Company, 1933), pp. 106–115.

Tracing out the consequences of the various hypotheses.

5. Experimental corroboration.
   A test of the hypothesis is made against experience.

Although it is realized that the thought process probably does not follow such a clearly defined sequence of steps, it is believed that the analysis can serve as a very helpful outline. The problem solver probably jumps around among the steps considerably before he is able to solve the problem, but as a general outline to help us understand thinking the analysis is very useful. The many behavior characteristics of students doing critical thinking listed in this paper could be classified under the five steps of the analysis. For example, those behavior characteristics that were listed under the headings *principles of logic* and *the nature of proof* could be classified under point four of Dewey's analysis. Wertheimer's emphasis upon *productive thinking* might be classified under point three—that dealing with the forming of hypotheses.

The Mathematics Teacher's Job

As mathematics teachers, our first responsibility is to the student. We should help him grow and develop in all areas. We are responsible for helping him grow and develop in his ability in the entire area of problem-solving as outlined by Dewey. As a part of this total responsibility we must do a good job of helping some of them gain understandings and skills in mathematics. Because of the great number of attitudes, understandings, and skills associated with the process of critical thinking, the problem of training for critical thinking is too great for the mathematics teacher alone. Especially is this true if mathematics teachers limit their efforts to the tenth year geometry course. We mathematics teachers also have some mathematics to teach—for the students' own good. The problem of critical thinking is the problem of the entire school.

It is believed, however, that because of the nature of mathematics, the mathematics teacher is in a position to help the student in many of the aspects of the complete thought process as outlined by Dewey. The Fifteenth Yearbook of the National Council has provided us with some help in this respect.[13]

Robert Thorndike, for a recent publication,[14] wrote on the subject, "How Children Learn the Principles and Techniques of Problem Solving." He listed some specific suggestions as to what schools and teachers could do to foster the problem-solving abilities of students. Many of his suggestions, although made for teachers of any subject, can be of considerable help to us as mathematics teachers.

Thorndike uses Dewey's outline to group his suggestions. He considers point one, becoming aware of problems, and lists several suggestions under that aspect. He does the same for points two, three, and four.

One of the suggestions Thorndike emphasized was to help the student build up a rich background of experiences. He pointed out that the wider the background of knowledge, skills, and understandings of the student the better are his chances for success in problem-solving. It is especially important to note that the knowledge, skills, and understandings must be built up in the area in which the thinking is to be done. If the student is to think about quantitative problems then he must know some mathematics. If he is to do critical thinking about social problems then he must have a background of knowledge in social problems.

Thorndike also pointed out the impor-

[13] National Council of Teachers of Mathematics, Fifteenth Yearbook, *The Place of Mathematics in Secondary Education* (New York: Bureau of Publications, Teachers College, Columbia University, 1940), pp. 37–41.

[14] Robert L. Thorndike, "How Children Learn the Principles and Techniques of Problem Solving," *Learning and Instruction*, Forty-Ninth Yearbook, Part I, National Society for the Study of Education (Chicago: The University of Chicago Press, 1950), pp. 192–216.

tance of certain attitudes in relation to problem-solving. One important attitude is that of suspending judgment until sufficient evidence is available and present. The mathematics teacher can contribute to the building of this attitude. Abundant opportunities arise in the usual mathematics class for pointing out to students or letting them discover that they have jumped to conclusions too quickly.

In relation to Dewey's fourth step, that of reasoning out the implications of suggested hypotheses, Thorndike stated that the skills required are those of "if-then" thinking, of logical reasoning.[15] Mathematics teachers have a very fine opportunity to help the students with the required skills of critical thinking if they teach the subject in such a way as to emphasize the principles of logic and "if-then" thinking.

The Harvard Committee Report states:[16]

> Mathematics is by no means the only road to an appreciation of abstraction and logical structure. But tactically these ends may be approached most readily through mathematics, particularly with the young. No better example of an abstract logical system for use with adolescents than demonstrative geometry has yet been discovered.

The Fifteenth Yearbook states:[17]

> Especially at the high school level, principles of deductive thinking can most effectively be taught in connection with a well-organized substantial subject such as geometry, which being logical itself and free from personal prejudice, can serve as a yardstick.

[15] *Ibid.*, p. 214.

[16] Harvard Committee Report, *General Education in a Free Society* (Cambridge: Harvard University Press, 1946), p. 162.

[17] National Council of Teachers of Mathematics, *op. cit.*, p. 40.

Summary and Conclusion

1. Critical thinking has a great number of aspects and means many different things to different people. Hence, as a descriptive phrase, *critical thinking* is of little use by itself to describe outcomes of learning. It is necessary and important that the phrase *critical thinking* be defined or supplemented by specific outcomes of learning in terms of actual behavior characteristics and skills desired.

2. High school mathematics teachers can help students develop in many of the aspects of critical thinking. Specifically the mathematics teacher can:

a. help the students have a rich background of experiences in mathematics and in mathematics as related to other fields and to life needs;

b. invent situations that will expose the student to question-provoking experiences;

c. help the students develop attitudes that will lead to thoroughness of study, organization of materials, and suspension of judgment until sufficient data is present;

d. set a climate in the classroom sympathetic to questions and questioning;

e. help the students gain ability in "if-then" thinking and knowledge of some principles of logic; and

f. help the students build a set of standards or norms of good reasoning against which they can compare their own reasoning or the reasoning of others. The mathematics teacher can help the students realize that the method of postulational thinking is not dependent upon the subject matter of mathematics but is applicable to reasoning in non-mathematical areas as well.

Change in Price of Single Copies of The Mathematics Teacher

By action of the Board of Directors of the National Council of Teachers of Mathematics, the cost of all single copies, current number or any back number, of THE MATHEMATICS TEACHER will be 50¢ per copy. This change will go into effect on January 1, 1952.

Motivation in Mathematics: Its Theoretical Basis, Measurement, and Relationships with Other Factors[1]

By E. I. SAWIN
Syracuse University, Syracuse, New York

ıCKGROUND AND PURPOSES OF THE INVESTIGATION

ıHORT time ago an instructor was dis- ıg a topic relating to motivation a class of prospective teachers. To :ate the point that many motives are ɔd, a statement resembling the fol- g was made: "Consider motives re- to the study of mathematics as an ple. Many people develop intense 'es to avoid mathematics. Now *there thing about mathematics itself that cause people to dislike it;* this could only as a result of unfortunate ex- ıces with it." As the words in italics uttered, a chorus of chuckles arose the class. It was apparent from the :mity of the reaction, and from the ısion that followed, that this idea totally strange to the thinking of of the students. A considerable er of them had acquired unfavorable des toward mathematics which were mly established that the idea that udy of mathematics could have been able to them appeared ludicrous. s especially significant in view of the hat the class was composed of a select of college juniors whose classroom iences with mathematics were prob- nore pleasant than for most of their ıates in elementary school and high l.

is incident, unfortunately, illustrates ıtively typical rather than a rare de. Probably every high school ɔr has encountered students who "just hate mathematics." Relatively few students learn to really enjoy mathematical study and to fully appreciate its practical and cultural values.

What are the causes of the development of dislike for mathematics? How can the development of favorable patterns of motives with regard to mathematics be fostered in students? It was toward providing answers to questions such as these that this inquiry was directed.

The primary purposes of the investigation were to draw together, from the literature, the elements of a theory which appears to hold the greatest promise for explaining how motivational patterns develop and to secure evidence which is pertinent to verification of this theory. There were two secondary purposes of the study: (1) to construct a test instrument for measuring certain aspects of motivation with regard to mathematics for use in connection with the primary purposes of the study, and which can be used in other research and in teaching, and (2) to determine the extent to which findings concerning relationships between motivation with regard to mathematics and other factors can be utilized for estimating the motivational status of an individual concerning mathematics.

A SKETCH OF THE THEORY AND STATEMENT OF HYPOTHESES

The theory of the investigation is closely related to the reward-reinforcement theory of learning. The basis for this approach is that motivation with regard to mathematics is considered an attribute of personality and that most attributes of personality are learned. According to this

[1] his is an abstract of a dissertation com- at The University of Chicago in March, Members of the faculty advisory commit- re M. L. Hartung, R. W. Tyler, and B.S. .

theory, the basis for learning is the experiencing of pleasure or satisfaction. That is, in the learning of behavior patterns, those responses are learned which repeatedly occur close in point of time to the experiencing of pleasure or satisfaction. Since this reinforcement through pleasure results in a tendency for the behavior pattern to recur, it follows that a motivational state which initiates and directs this behavior has been established. When the individual meets with frustration instead of satisfaction, aggression often follows. One form that this aggression can take, in situations involving mathematics, is to dislike mathematics.

Most pleasures are experienced through satisfaction of the needs of the individual. Likewise, frustration can be defined as the experiencing of blocks to reaching goals which are set by the needs of the individual. The need concept, therefore, is fundamental to the theory of the investigation. Need is defined as a deficiency or want in the organism which causes tension or striving in the organism. Many needs are products of learning and are therefore partly determined by the environment. Need patterns vary from individual to individual, but it appears that there are a few basic needs which are common to essentially all human beings.

The crux of the theory is represented by the following statement of the major hypothesis: *The motivation of an individual with regard to mathematics is affected positively by experiencing pleasure or satisfaction, and negatively by experiencing unpleasantness or frustration, as an outcome of interaction with a situation having mathematical aspects, the extent of the effect on motivation with regard to mathematics being dependent upon the intensity of the satisfaction or frustration and upon the extent to which mathematics is involved in the interaction.*

The major hypothesis was not tested directly in the investigation. Instead, subhypotheses were deduced from it that were more suitable for testing. The first of these sub-hypotheses is the following: *Any factor which has the effect of increasing the amount of satisfaction, or reducing the amount of frustration, in interaction in a situation involving mathematics, has a positive effect on the motivation of the individual with regard to mathematics, and any factor which has the effect of increasing frustration, or decreasing satisfaction, in interaction in a situation involving mathematics, has a negative effect on the motivation of the individual with regard to mathematics.* From this the following sub-hypotheses were deduced:

1. There is a positive correlation between motivation with regard to mathematics and (a) general mental ability, (b) algebra aptitude, (c) geometry aptitude, and (d) achievement in terms of mathematical processes and mathematical understandings.
2. There is a negative correlation between motivation with regard to mathematics and achievement in subjects not related to mathematics.

This same satisfaction-frustration theory leads to the expectation that there are relationships between motivation with regard to mathematics and other aspects of personality, as measured by a personality test—since basic needs of the individual are controlling factors with regard to many aspects of personality, including motivation with regard to mathematics. It seemed inadvisable, however, to formulate specific hypotheses in this regard, because of limitations of the types of personality tests which could be used in this situation. This aspect of the investigation was, therefore, of a broad, exploratory nature.

The Instrument for Measuring Motivation in Mathematics

The first step in drawing up the instrument was to make an analysis of motivation with regard to mathematics to provide a basis for item construction. This analysis was in terms of content, situation, and

behavior—that is, in terms of the kinds of behavior with regard to the various types of mathematical content in various kinds of situations that are indicative of favorable or unfavorable motivation with regard to mathematics.

The items are all of the stereotyped multiple-choice type in which a situation is described or illustrated and several questions asked concerning what the student does, thinks, or feels in such situations. There are three forms of these stereotyped response choices represented by the following sample items:

Type 1. You probably have solved many problems like the following:

James, Bob, and Jerry wished to save money from their jobs to buy a bicycle costing $37.50. James could save $1.25 per week, Bob could save $2.20 per week, and Jerry could save $1.75 per week. How long would they have to save their money at these rates before they could combine their savings and buy the bicycle?

When you work at problems of this kind,

2. How often do you find it difficult to keep your attention from wandering off to other things?

The student was asked to record his responses to Type 1 items on separate answer sheets according to the following key:

A—Always, or at least 9 times out of 10.
B—About three fourths of the time.
C—About one half of the time.
D—About one fourth of the time.
E—Never, or not more than 1 time out of 10.

Type 2. The student was given a list of topics such as

109. How mathematics is used in air navigation.

He was then asked to record his answers as follows:

A—*Definitely would* like to know more about this topic.
B—Uncertain, but might be *a little interested* in knowing more about this topic.
C—Uncertain, but it is *not likely* that you would be interested in knowing more about this topic.
D—*Definitely would NOT* like to know more about this topic.

Type 3. Imagine yourself in the following situation: One of your classmates is telling you that he (or she) has been working on an idea for a way to extract square root which is better than the one commonly used. He (or she) wants to explain it to you, to have you check it over to see if the reasoning is correct, and to try it out to see how it works.

25. Would you be interested in hearing the idea explained?

Responses to items of this type were recorded according to the following key:

A—YES
B—Uncertain, but probably YES
C—Uncertain, but probably NO
D—NO

There are 142 items of Type 1, 134 items of Type 2, and 43 items of Type 3. It is hoped that further studies of the instrument itself will make possible a reduction in the number of items without decreasing its usefulness.

The instrument was designed so that it could be used with IBM electrically scored answer sheets. It can be scored in terms of a single index of the student's general motivation with regard to mathematics, or in terms of various combinations of part scores. A student's most intense interests and areas of most favorable or unfavorable motivation can be determined by inspection by superimposing specially cut templates over the answer sheets.

Three approaches were used in determining the validity of the instrument:

1. The case for face validity was presented.

2. The relationships between scores on the instrument and other facts about the student which are indicative of motivation with regard to mathematics were studied:

a. Teacher ratings of the motivation with regard to mathematics of the students.

b. Estimates based on rankings by students of subjects liked best.

c. Participation or non-participation in voluntary mathematical activities.

3. The results of testing the subhypotheses were examined in terms of implications for the validity of the instrument.

The results of these studies of validity

are generally favorable to retention of the hypothesis that the instrument is valid, but more conclusive evidence is desirable.

An estimate of the reliability of the instrument was obtained by the split-half method. The correlation between the two halves was found to be .945, using 153 cases. Application of the Spearman-Brown correction formula raised the estimate to .972.

Collection and Analysis of Data

Data for testing the sub-hypotheses were secured by administering tests and questionnaire forms on the factors involved to the ninth grade students in a high school in a suburb of a large mid-western city. Additional data were secured from the school records, scores on tests administered as a part of the regular testing program of the school, and teacher ratings. The types of data of greatest importance to the study are listed in Table 1.

The data were analyzed by computing the product-moment correlation coefficients between the scores on the *Motivation Inventory for Mathematics* and measures of other factors. These correlations are shown in Table 1. For each entry in the table a positive correlation coefficient means that an increase in favorable motivation for mathematics is associated with an increase in ability or adjustment, as the case may be. Several partial correlation coefficients were computed also, so that the relationships between motivation with regard to mathematics and other factors, with mental ability held constant, could be determined. These partial correlation coefficients are shown in Table 2.

Since most of the product-moment correlation coefficients were computed with IBM equipment, with which it is about as easy to compute all of the intercorrelations as it is a few, many intercorrelations not directly related to the purposes of the investigation were obtained. Of particular

Table 1

Product-Moment Correlations Between Scores on the Motivation Inventory for Mathematics and Measures of Other Factors

Measures Correlated with Scores on the Motivation Inventory for Mathematics	Correlation Coefficient	Number of Cases
Scores on *Otis Quick-Scoring Test of Mental Ability*	.281	116
Scores on *Iowa Algebra Aptitude Test*	.341	116
Scores on *Iowa Plane Geometry Aptitude Test*	.287	83
Scores on *Stanford Achievement Test* (Arith.)	.261	85
Scores on *A Test of Basic Mathematical Understandings*, by V. J. Glennon	.514	116
Term grades in Algebra	.252	93
Term grades in General Mathematics	.097	83
Term grades in English	.303	176
Term grades in Social Studies	.316	177
Term grades in Latin	.290	54
Term grades in Home Economics	.354	27
Scores on the *California Test of Personality*		
Self Reliance	.294	116
Personal Worth	.115	116
Personal Freedom	.230	116
Feeling of Belonging	.059	116
Withdrawing Tendencies	.081	116
Nervous Symptoms	.117	116
Total Self-Adjustment	.211	116
Social Standards	.166	116
Social Skills	.150	116
Anti-Social Tendencies	.217	116
Family Relations	.071	116
School Relations	.169	116
Community Relations	.095	116
Total Social Adjustment	.221	116
Total Adjustment	.233	116

TABLE 2

PARTIAL CORRELATION COEFFICIENTS BETWEEN MOTIVATION WITH REGARD TO MATHEMATICS AND OTHER FACTORS, WITH MENTAL ABILITY HELD CONSTANT

Measures Correlated with Motivation with Regard to Mathematics	Coefficient of Partial Correlation
Scores on *A Test of Basic Mathematical Understanding*	.455
Scores on the *Stanford Achievement Test* (Arithmetic)	.101
Term grades in Algebra	.122
Term grades in General Mathematics	−.065
Term grades in English	.166
Term grades in Social Studies	.200
Term grades in Latin	.215
Term grades in Home Economics	.255
Self Reliance score of the *California Test of Personality*	.262
Total Adjustment score of the *California Test of Personality*	.184

interest are some of those showing the relationships between the scores on *A Test of Basic Mathematical Understandings* and the other types of data collected for the investigation. These are shown in Table 3. Reference will also be made to the correlation of .573 between algebra aptitude and term grades in algebra. Two additional findings not shown in these tables follow. The partial correlation between scores in basic mathematical understandings and term grades in algebra, with mental ability held constant, was found to be .540. Similarly, the correlation between basic mathematical understandings and term grades in general mathematics, with mental ability held constant, was found to be .252.

SUMMARY OF FINDINGS

The correlations in Table 1 of .281, .341, and .287 between scores on the *Motivation Inventory for Mathematics* and scores on the *Otis Test of Mental Ability*, the *Iowa Algebra Aptitude Test*, and the *Iowa Plane Geometry Aptitude Test*, respectively, provide evidence in agreement with sub-hypotheses 1 (a), 1 (b), and 1 (c): that there is a positive correlation between motivation with regard to mathematics and measures of general mental ability, algebra aptitude, and geometry aptitude. Each of these correlations is significantly positive at the one per cent level of confidence.

Questions arise, however, concerning sub-hypothesis 1 (d): that there is a positive relationship between achievement in mathematics and motivation with regard to mathematics. The correlation of .514 between scores on the *Motivation Inventory for Mathematics* and Glennon's *Test of Basic Mathematical Understandings* provides evidence clearly favorable to this sub-hypothesis in terms of this kind of achievement. However, the correlations between motivation with regard to mathematics of the type measured by the *Moti-*

TABLE 3

Product-Moment Correlations Between Scores in Basic Mathematical Understandings and Measures of Other Factors

Measures Correlated with Scores on Basic Mathematical Understandings	Coefficient of Correlation	Number of Cases
Scores on *Motivation Inventory for Mathematics*	.514	116
Teacher ratings of students' motivation with regard to mathematics	.626	116
Scores on *Otis Quick-Scoring Test of Mental Ability*	.651	116
Scores on *Iowa Algebra Aptitude Test*	.722	116
Term grades in Algebra	.700	104
Term grades in General Mathematics	.510	86
Total Adjustment score of the *California Test of Personality*	.322	116

vation Inventory for Mathematics, and the types of achievement represented by the *Stanford Achievement Test*, term grades in algebra, and term grades in general mathematics, do not provide favorable evidence, because the correlations are smaller than those for English and Latin. If it were claimed that the *Motivation Inventory for Mathematics* provided a measure of *all* aspects of motivation which affect the manner in which the student applies himself to mathematical study, the findings would warrant serious questioning of this sub-hypothesis. This is not the case, however. To be comprehensive in this regard, the instrument would have to include measures of such factors as the extent to which the student is striving for high marks, the effects of the personal relationships between teacher and student on student effort, parental pressure for good marks, competition between students, and fear of failure in the course and of not graduating. These are important in terms of their effect on achievement, but they are *not* aspects of motivation with regard to mathematics which are the subject of this investigation, and, of course, are not measured by the instrument. Factors of this kind apparently dwarf the effects of factors measured by the *Motivation Inventory for Mathematics* on certain types of *immediate* achievement. It is believed that the effect of the latter type factors would be increasingly apparent over longer periods of time, in terms of success in elected courses in mathematics and creative achievement in mathematics, while the former would, in all likelihood, lose most of their positive effect upon the completion of required study. These considerations make it reasonable to retain for further testing the sub-hypothesis that motivation as measured by the *Motivation Inventory for Mathematics* does affect achievement in mathematics. Additional research involving longer periods of time in which more adequate controls are possible is needed.

With regard to the sub-hypothesis that there is a negative correlation between motivation with regard to mathematics and achievement in non-mathematical subjects, the statistically significant *positive* correlations in the findings indicate that factors such as motivation with regard to school work in general have a greater positive effect on achievement in non-mathematical subjects than the negative effect of the factors involved in the reasoning leading to the formulation of this sub-hypothesis. Further investigation is necessary to determine whether or not such negative effects can be identified with other factors adequately controlled.

The findings revealed no significantly high correlations between motivation with regard to mathematics and aspects of personality represented by part scores on the *California Test of Personality*. The correlation of .233 between scores on the *Motivation Inventory for Mathematics* and the Total Adjustment scores indicates that maladjustment in a student increases slightly the probability that the student will develop unfavorable rather than favorable attitudes toward mathematics.

It is important to note that the correlation of .700 between scores in mathematical understandings and term grades in algebra is slightly higher than the correlation between aptitude scores and term grades in algebra. This indicates that the test in mathematical understandings has considerable value in predicting algebra achievement. It also encourages the belief that teaching arithmetic with understanding pays dividends in terms of algebra achievement.

The correlations between motivation with regard to mathematics and other factors, as shown in Table 1, were not considered sufficiently high to warrant making statistical analyses for estimating indirectly the motivation with regard to mathematics of an individual from knowledge of scores on measures of these other factors.

Implications for Teaching and Curriculum Development

One of the most significant outcomes of the investigation is the instrument for measuring motivation with regard to mathematics. No instrument of its kind, which is comparable to it in comprehensiveness and ease of administration, was previously available. There are a wide variety of uses for the *Motivation Inventory for Mathematics* in educational and vocational guidance, classroom teaching, curriculum development, and educational research.

The theory of this investigation implies that teachers can inhibit the development of unfavorable motivation with regard to mathematics by preventing consistent experiencing of serious frustration by students in their study of mathematics, and can contribute to the development of favorable motivation with regard to mathematics by providing opportunity for satisfaction to result from the efforts of students. This requires taking more action in providing for individual differences than what seems to be the usual practice in our schools. It is impossible for all students to find adequate satisfaction in a classroom geared to the needs of the mythical "average" student, in which essentially the same performance and enthusiasm are expected from all students in uniform activities appropriate for the average student and centered upon uniform subject matter content prepared for the average student.

The extent to which satisfaction is experienced in mathematics classes is determined in large degree by the extent to which the mathematics curriculum is designed to provide for meeting actual needs, interests, and concerns of students. Therefore, further implications of the theory are that the curriculum should be planned in accordance with, among other considerations, the needs, interests, and concerns of students, and that day-to-day planning should be influenced by the students' expression of them.

The fact that knowledge of basic mathematical understandings was found to have a relatively high relationship to both motivation with regard to mathematics and other types of achievement in mathematics is evidence that it is important to teach mathematics with understanding, rather than for mastery of mechanical processes alone, as is common in practice.

The relationship between motivation with regard to mathematics and personal adjustment supports the contention that the responsibilities of the teacher are broader than simply being competent in the teaching of her subject. These findings indicate that it is important for a teacher to have some mastery of the principles of guidance and child development so that the student can be helped to achieve better adjustment and hence relieved of barriers to learning.

Implications for Further Research

Additional research needed on the instrument itself are: additional studies of its validity, simplification of the scoring system, and revision into a shorter form. Other kinds of needed research are: (1) devising an instrument for measuring motivation with regard to mathematics in which the student cannot influence his score by perceiving the "right" answers, (2) determining how motivational patterns with regard to mathematics develop, using approaches such as individual case analysis and experimental studies, as well as the test-statistical approach used in this investigation, (3) investigating what the needs of students are, particularly those which frequently are met, or can be met, through mathematical activity, (4) designing and testing instructional procedures based upon a theory such as the one used in this investigation, (5) making investigations of the effect of teaching arithmetic with understanding on the motivation with regard to mathematics of students, and (6) making use of the *Motivation Inventory for Mathematics*, in in-

Sponsorship of Panel Discussions

Since the First Delegate Assembly held in Chicago in April, 1950 the Committee on Affiliated Groups has sponsored a panel discussion by representatives of the Affiliated Groups at each National Council meeting. These panel discussions have been held at the Madison, Gainesville, Pittsburgh, and Northfield meetings. The participants have been chosen from the geographical area of the meeting, with the exception of the Minnesota (summer 1951) meeting. On the panel at Northfield there were representatives of eight Groups from New Jersey to California. Thirty of the fifty-one Affiliated Groups have been represented in one or more of these panels.

A similar program is planned for the Stillwater meeting during the Christmas holidays. Invitations to serve on a panel have been sent to presidents of the statewide Affiliated Groups in Arizona, Arkansas, Colorado, Kansas, Nebraska, Oklahoma, Texas; the urban area Affiliated Groups in Dallas, Oklahoma City, Tulsa, and Wichita; and the Affiliated Group in Eastern Colorado.

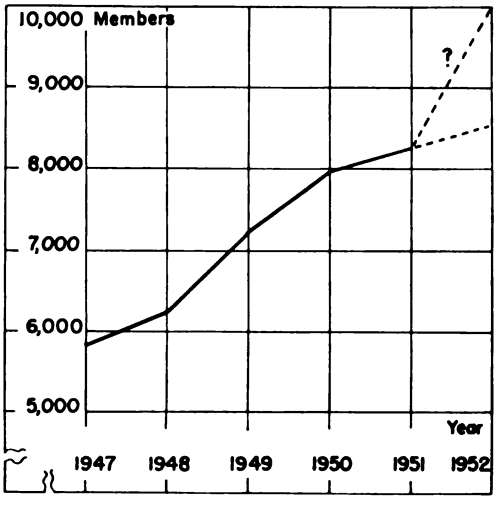
Membership Message

By Mary C. Rogers
Roosevelt Junior High School, Westfield, New Jersey

1951-1952 NCTM MEMBERSHIP SHOULD REACH A 10,000 TOTAL

This is the challenge facing your State Representatives and they, with your assistance and support, are doing a magnificent job meeting the challenge.

NCTM MEMBERSHIP GROWTH
1947–1951



During the year 1950–1951, thirty-four states have maintained their membership status or have shown substantial growth. Fifteen of these states have a record of continuous growth for the past four years.

NCTM membership totals are steadily increasing. Not only is growth currently persistent, but over the years there has been a steadily increasing rate of change. We have found the accompanying table and graph informative and helpful. We submit them to you for your study and consideration.

As has been our custom for the past two years, we shall again this year give special recognition to those schools with outstanding membership records—our 100% and "All but One" schools. We shall publish two listings of these Membership Honor Schools. The first will appear in the February, 1952 issue of The Mathematics Teacher. A second Honor listing, accompanied by a complete analysis of current membership status and periodic growth, is planned for the May, 1952 Teacher.

We need the assistance of each of you in locating these schools. Will you help us? Simply complete the accompanying form and mail it to

Mary C. Rogers
307 Prospect Street
Westfield, New Jersey

If your reports are in our hands by November 30, 1951, we shall guarantee the appearance of qualifying schools in our February, 1952 listing. May we look forward to hearing from a great many of you during the coming weeks.

Please accept our thanks for this assistance. Our best wishes to you for a most successful school year!

MEMBERSHIP REPORT TO THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

School.....
School Address.....
Number of Teachers of Mathematics in Your School.....
Number of National Council Members.....
Report Submitted by.....
Your Address.....
Names of Individual Members.....

MEMBERSHIP GROWTH BY STATES

States	Jan. 1947	Jan. 1948	March 1949	May 1950	May 1951	Average Change per period given	Net increase for four year period
Illinois	475	473	567	730	706	11.3%	48.6%
New York	513	643	644	721	689	8.3%	34.3%
Pennsylvania	412	438	478	520	634	11.5%	53.9%
Texas	200	208	224	360	405	21.2%	102.5%
Ohio	348	331	417	417	388	3.6%	11.5%
California	264	291	298	328	356	7.8%	34.8%
Indiana	137	318	318	318	333	34.2%	143.1%
New Jersey	277	312	325	350	318	3.9%	12.9%
Michigan	241	213	248	268	281	4.5%	16.6%
Massachusetts	229	238	250	266	274	4.7%	19.7%
Wisconsin	179	198	188	202	239	7.0%	33.5%
Virginia	114	123	187	201	244	22.2%	114.0%
Florida	102	130	161	179	220	21.4%	115.7%
Minnesota	138	144	179	204	209	11.3%	51.4%
Kansas	166	150	149	200	174	2.7%	4.8%
Missouri	133	122	146	176	168	6.9%	26.3%
Maryland	112	114	136	165	167	10.9%	40.2%
Iowa	129	107	144	171	163	7.9%	26.4%
Louisiana	80	100	153	142	162	21.2%	102.5%
North Carolina	103	96	158	137	161	15.7%	56.3%
Oklahoma	97	81	113	157	158	15.6%	62.9%
Connecticut	119	114	128	141	134	3.3%	12.6%
Tennessee	88	97	101	124	141	12.7%	60.2%
Washington, D.C.	105	132	120	128	110	2.3%	4.8%
Alabama	79	77	88	114	121	11.9%	53.2%
Georgia	40	67	71	99	113	31.8%	182.5%
Colorado	80	75	107	123	107	9.6%	33.4%
Nebraska	79	78	98	112	110	7.0%	26.6%
Arkansas	64	58	78	99	87	10.0%	35.9%
Mississippi	34	42	49	79	86	25.0%	152.9%
West Virginia	43	52	52	90	93	24.3%	116.3%
Washington	76	67	66	90	79	2.7%	3.9%
Kentucky	69	68	90	93	76	4.0%	10.1%
South Carolina	64	86	70	72	78	6.8%	21.9%
Oregon	77	80	84	65	75	0.4%	−2.6%
Delaware	23	24	24	31	44	18.9%	91.3%
New Mexico	18	42	30	32	43	36.5%	138.9%
Arizona	17	26	41	36	36	24.6%	111.8%
New Hampshire	21	31	27	30	38	18.1%	81.0%
Rhode Island	37	36	42	36	38	1.3%	2.7%
Utah	13	10	14	25	37	35.9%	184.6%
South Dakota	21	25	27	27	27	6.8%	28.6%
Maine	34	28	31	28	28	−4.2%	−17.6%
Montana	20	24	25	24	25	6.1%	25.0%
Wyoming	19	26	19	22	22	6.5%	6.5%
Vermont	16	27	25	21	22	12.6%	37.5%
North Dakota	19	15	18	19	20	2.5%	5.3%
Idaho	4	7	8	6	9	28.6%	125.0%
Nevada	4	10	8	9	6	27.3%	50.0%
Totals	5832	6254	7224	7987	8246	9.1%	41.4%

BOOK SECTION

Edited by JOSEPH J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

HIGH SCHOOL

1. Algebra

A First Course in Algebra, 2nd Edition, by Walter W. Hart, formerly of the University of Wisconsin. Cloth, ix+389 pages, 1951. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. $2.28.

2. Second Course in Algebra

Everyday Algebra, Intermediate Course, by William Betz, Specialist in Mathematics, Rochester, N. Y. and Alfred P. Windt, Hempstead High School, Hempstead, L. I., N. Y. Cloth, x+566 pages, 1951. Ginn and Company, Statler Building, Boston 17, Mass. $2.48.

A Second Course in Algebra, 2nd Ed., Enlarged, by Walter W. Hart, formerly of the University of Wisconsin. Cloth, viii+488 pages, 1951. D. C. Heath, 285 Columbus Avenue, Boston 16, Mass. $2.40.

3. Trigonometry

Trigonometry for Today, by Milton Brooks, and A. Clyde Schock, with Albert I. Oliver, Jr., Consultant. Cloth, ix+203+102 pages, 1951. Harper and Brothers, 49 E. 33rd Street, New York 16, N. Y. $2.96.

COLLEGE

1. 1st Year Mathematics

College Mathematics, A First Course, 2nd Ed., by W. W. Elliott, Duke University; and Edward R. C. Miles, Johns Hopkins University. Cloth, xii+436 pages, 1951. Prentice-Hall, Inc. Publishers, 70 Fifth Avenue, New York 11, N. Y. $4.85.

2. College Algebra

College Algebra, 2nd Edition, by Raymond W. Brink, University of Minnesota. Cloth, xvi+495 pages, 1951. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. $3.75.

Intermediate Algebra, 2nd Edition, by Raymond W. Brink, University of Minnesota. Cloth, xi+295 pages, 1951. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. $3.00.

3. Analytic Geometry and Calculus

Analytic Geometry and Calculus, by William R. Longley, Percey F. Smith, and Wallace A. Wilson, all of Yale University. Cloth, ix+578 pages, 1951. Ginn and Company, Statler Building, Boston 17, Mass. $5.00.

4. Statistics

Mathematics Essential for Elementary Statistics, Revised Edition, by Helen M. Walker, Teachers College, Columbia University. Cloth, xiii+381 pages, 1951. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.75.

MISCELLANEOUS

Mary Everest Boole, by E. M. Cobham. Cloth, xiv+137 pages, 1951. The C. W. Daniel Company, Ltd., Essex, England. Available through the Open Court Publishing Company, La Salle, Illinois; the Concord Book Shop, Inc., 36 S. Michigan Avenue, Chicago 3, Illinois; and from the Lucis Publishing Co., 11 West 42nd Street, 32nd Floor, New York 18, N. Y. $2.00.

REVIEWS

Mathematics, A First Course, Myron F. Rosskopf, Harold D. Aten, and William D. Reeve. New York, McGraw-Hill Book Co., 1951. 476 pp., $2.60.

This book is designed for a first course in high school mathematics—not straight algebra, however, but an attempt to integrate the fundamental concepts of arithmetic, algebra, informal geometry, and numerical trigonometry.

A brief summary of the chapters can perhaps best indicate the content of the course: symbolism of algebra; fundamental operations with numbers; fundamental operations in algebra and very simple equations; geometry from the point of view of "Form, Size, Position," drawing and construction, and measurement (the chapter on measurement has some excellent material on the meaning of measurement and approximate numbers); formulas and their use in geometry; statistics and graphs (the part on statistics is unusual in a high school text—and is good); fundamental relationship; mathematics applied to personal and government finances, business, and aviation; indirect measurement by scale drawings and trigonometry; more simple algebra including positive and negative numbers, systems of equations, problem solving, factoring, power and roots (all with a tendency to avoid complicated cases); logarithms and slide rule.

About the presentation of this material: (1) The print and diagrams are particularly

clear and pleasing to the eye. (2) The explanations are so worded as to be interesting and meaningful to the student. (3) Whenever possible the authors bring out reasons for learning and mastering mathematical techniques. (4) Mathematics applied to everyday living is emphasized.

Some will object that the material in this book overlaps too much into eighth grade and tenth grade mathematics, and that there is not enough algebra. There are many school situations, however, in which this broader integration of basic concepts from various fields of mathematics is desirable. Every teacher, even though he may not want or be able to plan his course along the lines of this book, would find certain chapters (such as the ones on measurement, statistics, fundamental relationships) excellent reference material.—HENRY SWAIN, New Trier Township High School, Winnetka, Illinois.

Elementary Theory of Equations, Samuel Borofsky. New York, The Macmillan Company, 1950. x+302 pp., $4.25.

The traditional text in the theory of equations has had as its chief aim a study of the roots of algebraic equations. Borofsky states that it is the purpose of this volume to introduce the student to some of the concepts of modern algebra as well. The elementary concepts of field and polynomial over a field are used and the author states that it is hoped that the text will serve as a bridge between elementary algebra and some of the more abstract ideas which are a part of modern algebra.

The text is organized in such a manner that the first eight chapters may be considered as a brief but rather complete study of equations of higher order. Ruler and compass constructions are delayed until chapter nine. They are followed by the more abstract concepts of algebraic number fields and symmetric polynomials. A study of determinants and matrices completes the text.

The book is well organized, attractive in appearance, and is written in a clear and concise manner with sufficient problem material to fix the principles introduced.

The inclusion of material not ordinarily found in a text in theory of equations should do much to improve a course which seemed to lack practical considerations and was only too often of little interest to the students. In the opinion of the reviewer, this book is a welcome addition to those commonly used and could do much to revive interest in the course.—HERBERT HANNON, Western Michigan College of Education, Kalamazoo, Michigan.

Differential Equations, James E. Powell and Charles P. Wells. Boston, Ginn and Company, 1950. vi+205 pp., $3.00.

This is an excellent text for a three-semester hour first course in differential equations. It is intended for engineering and science students as well as mathematics majors. A few general methods are emphasized with special methods subordinated to these in an effort to combat the impression frequently left with the student that the solution of differential equations is a matter of learning numerous special and unconnected devices.

The exercises and applications are carefully selected and organized. The reviewer is especially pleased to see for the first time a mathematics text in which the authors deliberately include some problems that the student cannot solve.—R. A. BEAVER, New York State College of Teachers, Albany, New York.

An Introduction to the Theory of Statistics. G. Udny Yule and M. G. Kendall. Fourteenth Edition. New York, Hafner Publishing Co., 1950. xxiv+701 pp. $7.00.

The first edition of this work was published in 1911, the first systematic treatise on the then almost unknown subject of statistics. The theory of probability had reached a very satisfactory form by the end of the ninteenth century, since nearly a century of research had followed the publication in 1812 of Laplace's *Théorie analytique des probabilités*. But the application of this theory to data and the development of what we today call mathematical statistics has been almost entirely the product of the twentieth century. The treatise of Yule thus appeared near the beginning of the modern trend toward statistics. We should not be surprised, therefore, to see that it has reached fourteen editions. The eleventh edition, which appeared in 1937, was published in collaboration with M. G. Kendall, who adapted the older book to the rapidly growing subject of statistics.

The present volume is again a substantial revision. In the words of Kendall "The major additions fall into two groups. Chapters 21–23 expand the former treatment of small-sample theory and give an introduction to the practical problems of sampling. Chapter 25–27 give an account of index numbers and the elementary theory of time series. Chapter 13 on practical problems of correlation has also been re-written. Additions have been made in the remaining chapters to keep the treatment abreast of new discoveries, some of the examples have been modernized and some further exercises added."

This book should be on the shelf of anyone who is seriously studying statistics or who is engaged in applying the methods of this great science to practical data. Let us hope that the book goes into many more editions.—H. T. DAVIS, Northwestern University, Evanston, Illinois.

Contributions to Mathematical Statistics. R. A. Fisher. New York, John Wiley and Sons, Inc., 1950. 672 pp. $7.50.

This work has been prepared in order to make accessible from their many sources of publication the papers of the English statistician Ronaly Aylmer Fisher, more generally known to all statisticians as R. A. Fisher. The volume contains a portrait of Professor Fisher, a bib-

liography of the author, originally published in 1938, an editor's preface by Walter A. Shewhart, a preface by the author, an index prepared by John Tukey, and a facsimile reproduction of forty-three papers.

The reason for the present collection is explained by the editor as follows: "Like so many others, for more than a quarter of a century I have found inspiration and guidance in the original contributions of Professor Fisher. Some of my reprints of his early papers have been almost worn out through constant use, and for several years I have wished that I had a bound volume of his papers, particularly if he could be persuaded to preface each article with a brief note setting it in the proper perspective as of today. After some persuasion, Professor Fisher agreed in the fall of 1946 to undertake the selection of what he now considers to be his most outstanding contribution and to provide brief introductory notes wherever he felt that such information would be of help to students of his work. The present book is the result."

The volume is beautifully printed in large format (11×8½ inches). The pages are not consecutively numbered, but retain the numbers of the pages in the original articles.—H. T. Davis, Northwestern University, Evanston, Illinois

Table of the Bessel Functions $Y_0(z)$ and $Y_1(z)$ for Complex Arguments. Prepared by the Computation Laboratory National Bureau of Standards. New York, Columbia University Press, 1950. xl+427 pages. $7.50.

This fine volume of tables, prepared under the direction of Arnold N. Lowan, is one of the most ambitious projects of the Computation Laboratory, which, originally organized in 1938, has been operated by the National Bureau of Standards since March, 1943. This is the twenty-fourth volume of the series, which is an enduring monument to the computational development of American mathematics. Throughout the preparation of these many volumes Dr. Lowan has been the guiding spirit of the laboratory and the highest credit should be given to him for the excellence and accuracy of the computations.

The present work is devoted to the evaluation of the two Bessel functions of second-kind over a complex domain $z=\rho e^{i\phi}$, where ρ ranges from 0 to 10 at intervals of 0.01 and ϕ ranges from 0° to 90° at intervals at 5°. The computation is carried out to 10 decimal places. Auxiliary tables of the functions $Y_0(z)-(2/\pi)J_0(z)\log_e\rho$ and $Y_1(z)-(2/\pi)J_1(z)\log_e\rho+(2/\pi z)$ and a table of the complex zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$ are also included.

A foreword by R. von Mises explains the importance of these functions in applied work and indicates the relationship of the tables to computation by high-speed calculators. In an extensive introduction the properties of the functions are described and graphs of their contour lines are included. The work is a notable contribution to the many recent tables of the Bessel functions.—H. T. Davis, Northwestern *University*, Evanston, Illinois.

Le Raisonnement Mathématique de l'Adolescent, Louis Johannot. Neuchatel, Switzerland, Delachaux and Niestlé, 1947. 174 pp., $2.00.

This volume is one of several in the series known as Actualités Pédagogiques et Psychologiques, published under the auspices of l'Institute des Sciences de l'Education de l'Université de Genève (Institute J. J. Rousseau). In essence it is an analysis of the psychology of learning in mathematics with reference to individuals from thirteen to eighteen years of age. There is a short preface by the well-known French psychologist and educator Jean Piaget, whose recent volume, *The Psychology of Intelligence*, has just been published in English by Harcourt, Brace (1950).

The Monograph in question is a commendable piece of original research. Part 1, a brief historical survey, is followed by the experimental evidence proper (Chapter 2, 3, 4 and 5). The data consist of a large number of individual pupil-interviews, intimately analyzing their thought processes when engaged in solving problems in arithmetic and algebra. Chapter 2 presents the evidence for the four steps in the development of mathematical reasoning: (1) understanding in terms of concrete manipulations (13 yr.); (2) understanding in terms of graphic representation (12 yr.–14 yr.); (3) understanding intuitively, or in formal arithmetical terms (13 yr.–17 yr.); (4) understanding in formal algebraic terms (17 yr. or over). In chapter 3 we find similar "clinical" evidence of learning difficulties with mathematical operations, notably the concept of inverse operations, as applied to multiplication and division of fractions. Chapter 4 deals in the same manner with the combination and order of operations, and the "psychological significance of algorithms." Chapter 5 deals with "abstract problems,"—generalizations. Here are discussed such considerations as the meaning of an equation and the recognition of relationships between quantities. Chapter 6 gives an overall view of the development of reasoning ability as maturation progresses, including the evolution of perception, sensorimotor intelligence, and intuition, tracing the growth of such concepts as inequality, identity, inverse, and the associative and commutative principles. The concluding section (Chapter 7) suggests certain pedagogical implications, the burden of which may be summed up as follows: comprehension always goes from the concrete to the general; meaning grows out of perception; ideas and generalizations are never "learned" *per se*, but are literally discovered; the learner can only generalize after he understands what he is doing.

A strong case is made for an abundance of laboratory work in teaching mathematics, together with continual emphasis on practical applications. Unless your French is extremely rusty, the book is quite readable, and deserves a place in the library of all who are interested in mathematical education.—William L. Schaaf, Brooklyn College, Brooklyn, New York.

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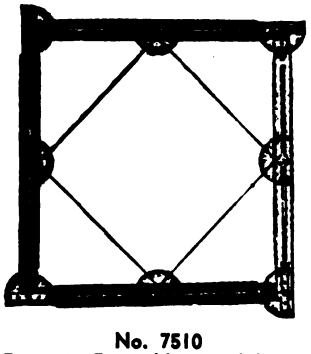

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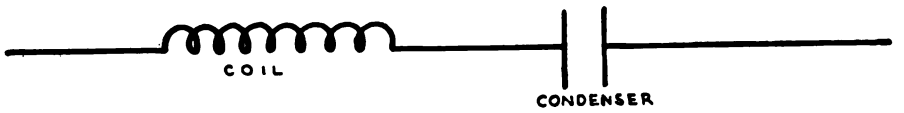
GRAPHS

Application to Circuits



The reactance of a circuit measures the difficulty of passing AC (Alternating Current) through it. The smaller the reactance, the greater the current.

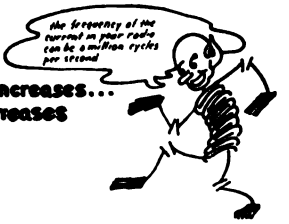
Reactance (X_L) of a coil

$$X_L = 2\pi f L$$

L is the inductance of the coil

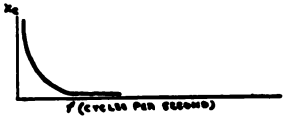
f is the frequency (cycles per second) of the current

As f increases...
X_L increases



Reactance X_c of a condenser

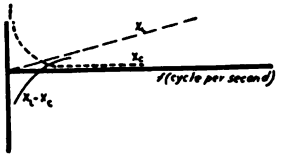
$$X_c = \frac{1}{2\pi f C}$$



As f increases...
X_c approaches zero

At any frequency the reactance of the circuit at the top is the reactance of the coil minus the reactance of the condenser.

$$X_L - X_c = 2\pi f L - \frac{1}{2\pi f C}$$



At this frequency the reactance is zero, and we have "resonance": Current is maximum.

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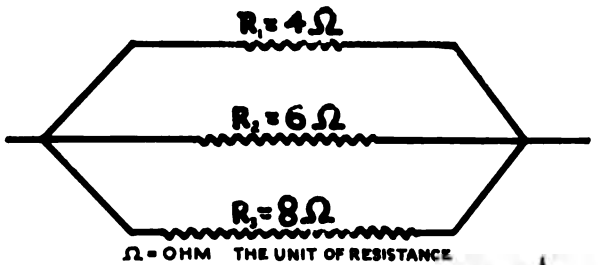
The MATH Problem

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{x}$$

$$\frac{1}{x}=\frac{bc}{abc}+\frac{ac}{abc}+\frac{ab}{abc}$$

$$\frac{1}{x}=\frac{bc+ac+ab}{abc}$$

$$x=\frac{abc}{bc+ac+ab}$$



Ω = OHM THE UNIT OF RESISTANCE

The ELECTRICAL Problem

$$\frac{1}{R_{TOTAL}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$R_{TOTAL}=\frac{R_1 R_2 R_3}{R_2R_3+R_1R_3+R_1R_2}$$

$$=\frac{4\times6\times8}{6\times8+4\times8+4\times6}$$

$$=1.85-\Omega$$

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- **RADIO**

HERE'S HOW FRACTIONAL EQUATIONS AND EXPONENTS ARE EMPLOYED ....

What is the value of X in this expression?

$$\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$
$$\frac{1}{x}=\frac{b}{ab}+\frac{a}{ab}$$
$$\frac{1}{x}=\frac{b+a}{ab}$$
$$x=\frac{ab}{b+a}$$

If $a=2\times10^{-6}$ *and* $b=3\times10^{-6}$ *what does X equal?*

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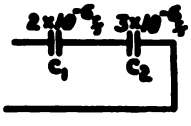

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$
$$C=\frac{C_1\ C_2}{C_1+C_2}$$
$$C=\frac{2\times10^{-6}\cdot3\times10^{-6}}{2\times10^{-6}+3\times10^{-6}}$$
$$C=\frac{6\times10^{-12}}{5\times10^{-6}}$$
$$C=1.2\times10^{-6}\text{ f}$$

C = CAPACITY, MEASURED IN TERMS OF FARADS. A MICROFARAD (μf) IS ONE MILLIONTH OF A FARAD. EXPRESSED IN POWERS OF TEN, THE ENGINEER'S SHORTHAND, ONE MICROFARAD (.000001f) IS 1×10^{-6}f. $2\mu f=2\times10^{-6}$f, AND $3\mu f=3\times10^{-6}$f.

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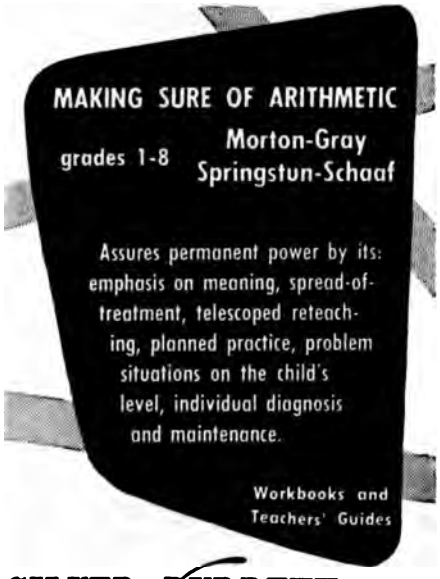
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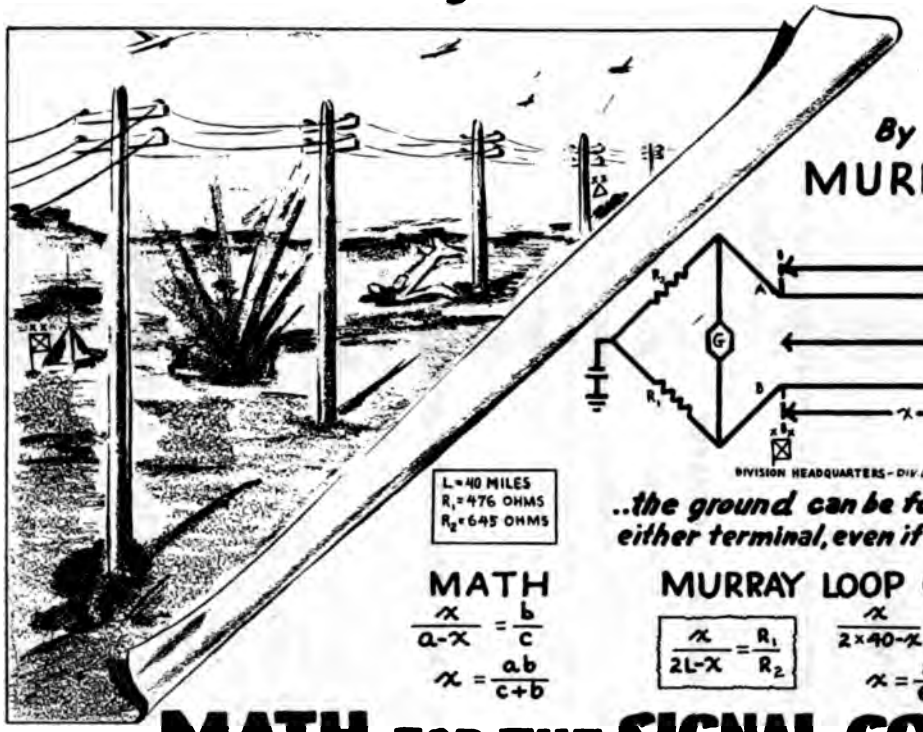
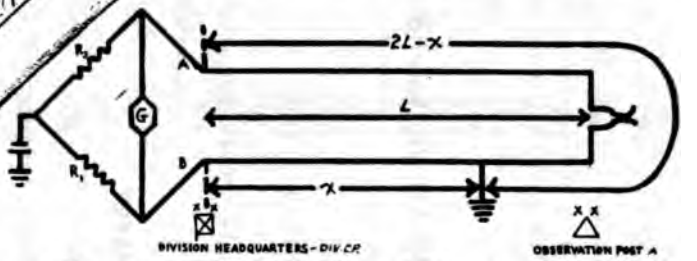


By use of the
MURRAY LOOP...



L = 40 MILES
R_1 = 476 OHMS
R_2 = 645 OHMS

..the ground can be found at once from either terminal, even if separated by miles.

MATH

$$\frac{x}{a-x} = \frac{b}{c}$$

$$x = \frac{ab}{c+b}$$

MURRAY LOOP COMPUTATION

$$\frac{x}{2L-x} = \frac{R_1}{R_2}$$

$$\frac{x}{2\times 40 - x} = \frac{476}{645}$$

$$x = \frac{80\times 476}{645+476} = 34 \text{ Miles}$$

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WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* JOHN A. BROWN
The University of Wisconsin, Madison, Wisconsin

A TABLE summary of the first 15 replies to the questions on Mathematics Enrollments, published in this Department in the November, 1950 number of THE MATHEMATICS TEACHER was given in the April, 1951 number. In that number the questions were repeated with the hope that additional replies might be had. A similar summary of 14 additional replies received by June 15 is given at the end of this report.

The questions included in the three parts of this study are repeated for convenience in reading the summary of replies to part A, Mathematics Enrollments, and to provide one more opportunity for interested teachers to take part in this study.

Final summaries of replies to these parts of the study will be given in the April, 1952 number of THE MATHEMATICS TEACHER.

The questions are:

A. Mathematics Enrollments
1. How many years of sequential mathematics (in grades 9–12) is it possible for a graduate of your school to take?
2. What is the mathematics requirement for graduation from your high school?
3. How long has this requirement been in effect?
4. What per cent of your graduates of 1951 completed
   a. one year of mathematics in grades 9–12?
   b. two years of mathematics in grades 9–12?
   c. three years of mathematics in grades 9–12?
   d. four years of mathematics in grades 9–12?
5. What per cent of your graduates of 1951 entered college this fall?

B. General Mathematics
1. Does your school offer both algebra and general mathematics (or a course which might be called general mathematics) in the ninth grade? If not, does your school offer general mathematics in a later grade?
2. What per cent of your students take a course in general mathematics as their first course in high school mathematics (grades 9–12)?
3. For placement of students in general mathematics do you use:
   a. An algebra aptitude test?
   b. An arithmetic achievement test?
   c. The student's grade in eighth grade mathematics?
   d. An intelligence test score?
   e. Written recommendations of teachers?
   f. Other?
   Please indicate which of these or any other placement procedures you use.
4. Upon completion of one year of general mathematics may your students elect:
   a. A second year course in general mathematics?
   b. Algebra?
   c. Plane Geometry?
   d. Other courses in mathematics?
   e. None?
   If possible, indicate what per cent of these students enroll in each course.
5. Do the colleges which your graduates attend accept general mathematics on the same basis as they accept a course in algebra?

C. Third Year Mathematics
1. What courses do you offer for the third year of sequential mathematics?
   a. A third semester of algebra and a semester of solid geometry
   b. A third semester of algebra and a semester of trigonometry
   c. A second year of algebra
   d. A year of plane geometry
   e. Algebra and another subject in combination different from that in (a) or (b); if (e) is checked, please indicate also
      Number of weeks of algebra
      Number of weeks of solid geometry
      Number of weeks of trigonometry
   f. Other (please describe briefly).
2. Do you have ability grouping in your third year course?
3. Do you have a mathematics club:
   For juniors only?

For seniors only?
For both juniors and seniors?

4. Do you use films and film strips in the third year of sequential mathematics? If so, please list the two you have found most helpful.
5. Do your third year mathematics students use the library as part of the suggested study in this mathematics course?

Please send answers to these questions for your school to John R. Mayor, North Hall, Madison 6, Wisconsin by December 15, 1951.

In replying please give your name, the name and locality of your school and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades, 10–12; junior high school, grades 7–9; etc. The questions may be answered by number without use of a particular form or repetition of the questions. In reports no reference will be made to names of teachers or schools.

Report of Replies in Mathematics Enrollments

The 29 replies to part A which had been received by June 15 came from eighteen states. Twenty-three of the reports are from public schools and six from private schools.

While the number of replies is still much too small to provide an indication of general practices in American high schools it should be interesting for teachers in any school to compare their enrollments with those of 29 other schools, of course, recognizing differences in size and organization. The plan in the earlier report of listing schools in order of the per cent of graduates of 1950 entering college is repeated here.

It should be observed that 27 of the 29 schools offer four years of sequential mathematics. Another special characteristic of this group is that 4 schools require three years mathematics for graduation; 9 require two years of mathematics; 1 requires 1½ years; 9 require one year; 5 have no mathematics requirement; and 1 did not report on requirements. Readers will notice that three of those schools which require three years of mathematics for graduation had 100% of their graduates enter college.

The lengths of time that the present requirements have been in effect provide one of the most surprising aspects of the data in that for those schools reporting, only three indicated a change in requirement in less than ten years while several indicate that their present requirements have been in effect fifty or sixty years. It is likely that in those schools for which no answer to this question is reported there have been no recent changes.

(*Continued on page 501*)

Mathematics Enrollment in 14 Secondary Schools

(The numbers at the tops of the columns are the numbers of the questions above: the answers to questions 1, 2, and 3 are in years; the answers to questions 4a, 4b, 4c, 4d, and 5 are in per cents)

School	Grade Organizations	Enrollment	1	2	3	4a	4b	4c	4d	5
P	9–12	—	4	3	40	100	100	100	50	100
Q	8–12	250	4	3	60	100	100	100	80	100
R	9–12	—	4	1	15	100	90	20	10	70
S	9–12	1600	4	1	50	100	85	51	16	70
T	9–12	327	4	1½	5	100	100	100	78	61
U	7–9, 10–12	1300	4	0	18	81	64	50	11	45
V	9–12	—	4	1	15	96	88	35	9	43
W	9–12	365	4	0	—	95	82	47	15	42
X	9–12	1450	4	0	—	75	65	27	8	40
Y	9–12	340	4	2	—	100	100	14	0	36
Z	9–12	180	4	1	1	100	100	27	10	29
AA	9–12	545	4	1	20	100	50	33	4	20
AB	9–12	480	4	3	25	100	100	100	15	20
AC	9–12	—	3	2	50	100	100	38	0	13

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

32. *Cevians, Medians, and Redians*

In the modern geometry of the triangle, a *cevian* is a line segment whose initial point is a vertex and whose terminal point is on the side opposite the vertex. The terminal point of the cevian divides the side of the triangle, either internally or externally, in a ratio which we may call the *terminal ratio*. For example, in Figure 1, AE, BF and CD constitute a *set of cevians* whose associated terminal ratios are $BE/EC=r$, $CF/FA=s$, and $AD/DB=t$.

Let us call the triangle PQR the *cevic triangle* of triangle ABC. The cevic triangle has its vertices the intersections of the cevians of a set. Furthermore, let triangle DEF, whose vertices are the terminal points, be known as the *menelaic triangle* of triangle ABC.

By analytic geometry—making use of the ratio formula, and the determinant form of the area of a triangle—it is readily shown that

$$(1)\quad \frac{\triangle PQR}{\triangle ABC}=\frac{(1-rst)^2}{(1+r+rs)(1+s+st)(1+t+tr)}$$

$$(2)\quad \frac{\triangle DEF}{\triangle ABC}=\frac{1+rst}{(1+r)(1+s)(1+t)}.$$

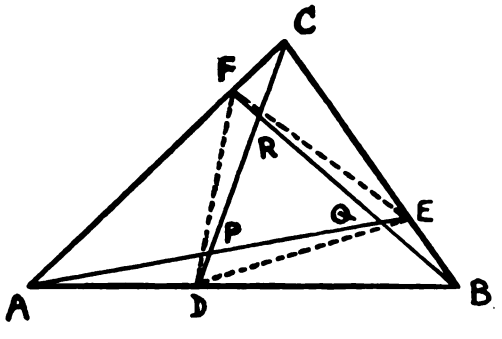

FIG. 1

From (1) and (2), by considering the conditions under which the areas of the cevic and menelaic triangles vanish, we obtain the following well-known theorems.

THEOREM 1. *Ceva's Theorem. In a triangle, if the three cevians of a set are concurrent, the product of their terminal ratios is 1.*

THEOREM 2. *Converse of Ceva's Theorem. If the product of the terminal ratios is 1, the cevians are concurrent.*

THEOREM 3. *Menelaus' Theorem. In a triangle, if the terminal points of a cevian set are collinear, the product of their terminal ratios is -1.*

THEOREM 4. *Converse of Menelaus' Theorem. If the product of the terminal ratios is -1, the terminal points are collinear.*

We note that when $r=s=t$ the cevians become *medians* or *redians*, as discussed in THE MATHEMATICS TEACHER for January 1951 (vol. XLIV, pp. 46–48) and May 1951 (vol. XLIV, pp. 310–311). In this case formulas (1) and (2) above may be reduced to formulas (1) and (2) of page 310 by setting $r=p/q$.

By generalization, we may associate with any polygon its *cevic polygon* and its *menelaic polygon*. In particular, for a regular n-gon of area K, in which the cevians are all redians of ratio r, we may show that

(3) Area of Cevic Polygon

$$=\frac{\left(1+2r\cos\frac{2\pi}{n}\right)^2 K}{(1+r)^2+2r(1+r)\cos\frac{2\pi}{n}+r^2}$$

(4) Area of Menelaic Polygon

$$=\left[1-\frac{4r\sin^2\frac{\pi}{n}}{(1+r)^2}\right]K.$$

It is seen from (3) that the redians of a regular n-gon are concurrent if and only if $r=-\frac{1}{2}\sec 2\pi/n$. Also, by evaluating the limit of the right member of (3) as r increases without limit, it is found that the area S of the polygon bounded by the primary diagonals of a regular n-gon of area K is given by

$$(5)\qquad S=\frac{2\cos^2\frac{2\pi}{n}}{1+\cos\frac{2\pi}{n}}.$$

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33. *More About Factoring*

With regard to *Miscellanea 15. A Method for the Speedy Factoring of the General Quadratic Trinomial* [THE MATHEMATICS TEACHER. Vol. XLII (December 1950), p. 421] may I report that the reverse operation to forming the binomial product $(ax+b)(cx+d)$ to give $acx^2+(ad+bc)x+bd$ has proved a most direct and reliable method in my Grade XI and XII classes.

Forming products and factoring are treated as inverse operations in a single chapter of the approved textbook for these grades of Algebra in Ontario. (The question of the factorability of the general trinomial, and factorizations where surds and complex quantities are needed are for the most part left for Grade XIII treatment.) After numerous and various binomial products are readily formed by inspection and the results given orally by the pupils, trinomials with small coefficients (say less than 15) in the second degree and absolute terms are factored by trial and error. The tediousness and inefficiency of this method is then pointed to for an example such as $24x^2-13x-60$, and so the question is naturally introduced: In a trinomial px^2+qx+r, how can we factor the product pr to give two numbers whose algebraic sum is q?

Once two such numbers are determined they may be taken as the values for ad and bc respectively in $acx^2+(ad+bc)x+bd$. The factoring may then be completed by reversing the steps one would take in multiplication. Thus:

$$\begin{aligned}px^2+qx+r&=acx^2+(ad+bc)x+bd\\&=acx^2+adx+bcx+bd\\&=ax(cx+d)+b(cx+d)\\&=(ax+b)(cx+d)\end{aligned}$$

As a characteristic illustration, let us factor $24x^2-13x-60$. $24\times60=1\cdot2\cdot2\cdot2\cdot3\times2\cdot2\cdot3\cdot5$. We have to re-group these to give a pair which *differ* by 13, the larger of them being the negative one. To do this with 100% accuracy is tedious but instructive when the method is first being used. The possible combinations are:

ad	bc
1	1440
2	720
3	480
4	360
5	288
6	240
8	180
9	160
10	144
12	120
15	96
16	90
18	80
20	72
24	60
30	48
32	45
36	40

The obvious pair is 32 and 45 with 45 negative.

We then have

$$24x^2+45x+32x-60$$
$$=3x(8x-15)+4(8x-15)$$
$$=(8x-15)(3x+4).$$

In examples such as this the pupils quite soon see roughly where to start in this systematic pairing of factors by noticing the size of the first degree term's coefficient.

As a second illustration of this method consider the second "doodler" in *Miscellanea 15:* What must be the missing coefficient in $18x^2 \pm Ax+54$ for it to be factorable? $18\times54=1\cdot2\cdot2\cdot3\cdot3\cdot3\cdot3\cdot3$. Hence, for A to be prime to 18 and 54 as the problem required, its two parts must be $2\cdot2$ and $\cdot3\cdot3\cdot3\cdot3\cdot3$, i.e. $A=\pm247$.

A second proof of the lemma contained in the last sentence of the second paragraph of Case I of *Miscellanea 15* may also be of interest. This lemma stated: *When all three coefficients of the general quadratic trinomial are odd, it is not factorable in the domain of real integers.*

Since ax^2+bx+c has rational binomial factors if and only if its discriminant is a square, $(2p+1)x^2+(2q+1)x+(2r+1)$ where p, q, r, K are integers has binomial rational factors if and only if $(2q+1)^2-4(2p+1)(2r+1)=K^2$. Since the left hand side is odd, so is K^2 which we may then call $(2s+1)^2$. Therefore $(2q+1)^2-(2s+1)^2=4(2p+1)(2r+1)$, i.e. the left hand side is 4 times an odd number. But the difference on the left hand side can be written $4(q+s+1)(q-s)$ and one of these last two brackets must be an even number. Thus the "absurdity" that a multiple of 8 is an odd multiple of 4, completes the "indirect" proof of the lemma.

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34. Some Nedian Details

In joining the discussion on nedians of a triangle[1] the writer wishes to adopt a slightly different definition of the nedian points. Let us say that a nedian point divides a side into two segments with ratio $n:1$. Thus, in figure 1,

$$V_2P_1:P_1V_3=V_3P_2:P_2V_1$$
$$=V_1P_3:P_3V_2=n:1.$$

For purposes of comparison with the $p:q$ of Prof. Anning (The Mathematics Teacher [May 1951] p. 310) and the N of Prof. Satterly (The Mathematics Teacher [January 1951], p. 46) we have

$$n=p:q=(N-1)^{-1}.$$

The segment-to-segment ratio has, in this writer's experience, been more readily applicable than the side-to-segment ratio. Moreover, the basic relationships assume somewhat more simple expressions. The number n need not be integral, nor even rational. Interesting particular cases arise with n irrational.

Each nedian in a complete set of three is divided into three parts. In figure 1, call N_3V_1, N_2N_3 and P_1N_2 the *tip*, *middle*, and *stem* respectively of nedian P_1V_1. Then the equation

whole:*tip*:*middle*:*stem*
$$=(n^2+n+1):(n+1):(n^2-1):1$$

expresses the numerical relationships of these segments. Again in figure 1, notice four sorts of triangles—$V_1V_2V_3$, $V_1P_2N_1$ (3 such), $V_1V_2N_3$ (3 such), $N_1N_2N_3$. Let us refer to these as primary, corner, side, nedian respectively. Their areas have the ratios

primary:*corner*:*side*:*nedian*
$$=(n^2+n+1):(n+1)^{-1}:n:(n-1)^2.$$

These results are easily obtained by elementary geometry (see reference 3 below).

In figure 2, let Q_1 be the intersection of lines V_1N_1 and V_2V_3; similarly Q_2 and Q_3. Then it is easy to show by means of Ceva's theorem that Q_1, Q_2, Q_3 are nedian points such that

[1] For earlier discussions of nedians and related concepts see *Miscellanea 18, 26, 32* in The Mathematics Teacher, vol. 44 (January 1951), pp. 46–48; (May 1951), pp. 310–312 and this issue, pp. 496–497.

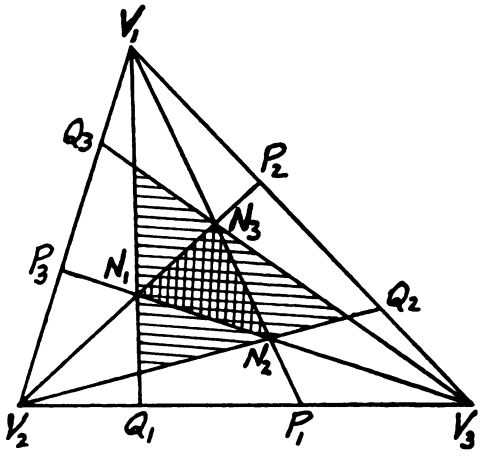

FIG. 2

$$V_3Q_1:Q_1V_2 = V_2Q_3:Q_3V_1 = V_1Q_2:Q_2V_3 = n^2:1.$$

The *tip*:*middle* ratio along the Q nedians is the same as the *stem*:*middle* ratio along the P nedians. The ratio of the areas of the respective nedian triangles is $(n+1)^2 : (n^2-n+1) = \Delta V_1V_2V_3 : \Delta P_1P_2P_3$. Thus the relationship of the two nedian triangles duplicates that of triangles $V_1V_2V_3$ and $P_1P_2P_3$.

Let each side of a triangle be divided into a given odd number (say $2a+1$) of equal parts. The points of division nearest the midpoints are nedian points corresponding to $n=(a+1)/a$. The area of the nedian triangle is then an aliquot part of the area of the primary triangle. The number of times the latter contains the former is $(a+1)^3-a^3$.

Along the nedian the *middle* may be equal to *tip* or *stem*, but *tip* and *stem* can not be equal unless both vanish. The three segments may, however, be in arithmetic, geometric or harmonic sequence. For the geometric progression take $n=(1/2)(1+\sqrt{5})$. The nedian points then divide the respective sides in mean and extreme ratio. On the nedians, also, *tip*:*middle* = *middle*:*stem* $=(1/2)(1+\sqrt{5}):1$. It follows that *tip* = *middle* + *stem*. As to areas in this particular case, the nedian triangle is equal to each corner triangle. Figure 2 has $n=(1/2)(1+\sqrt{5})$. Then $n^2=(1/2)(3+\sqrt{5})$, the ratio for the Q points. For this ratio the nedian triangle is equal to each side triangle.

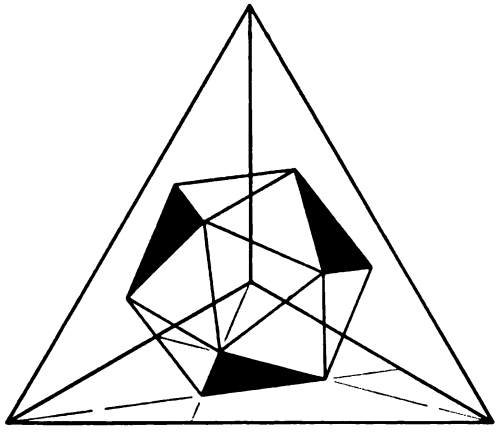

FIG. 3

A regular icosahedron can be inscribed in a regular tetrahedron so that each face of the latter contains a face of the former. To determine the position of a face of the icosahedron on a face of the tetrahedron construct in the latter a set of nedians corresponding to $n=(1/2)(1+\sqrt{5})$. The nedian triangle is the required face of the icosahedron. Figure 3 shows an orthographic projection of two such polyhedra. The icosahedron can be inscribed in the tetrahedron in two ways. The two positions correspond respectively to all "forward" nedians and all "backward" nedians.

In an equilateral triangle the nedians for $n=(1/2)(1+\sqrt{5})$ also furnish the elements necessary for constructing a model of the five interpenetrating regular tetrahedra which circumscribe a single regular icosahedron. Figure 4 shows a pattern for one of the twenty "points" of the model. To construct it divide sides AB and BC of an equilateral triangle ABC in mean and extreme ratio, as in Figure 4, and draw the corresponding nedians. These intersect at D. Draw BD. Extend AD to E, making $DE=DC$. Draw EC. Three faces such as $ABDEC$ surround each "point" of the model. Two adjoining "points" have broken edge $EDBE'$ on one coincident with $E'BDE$ on the other.

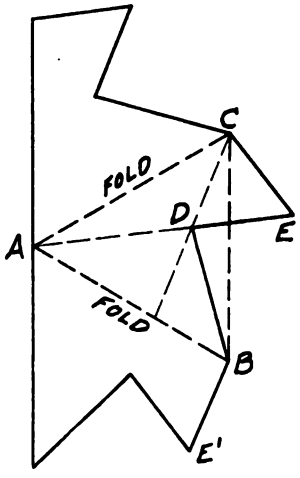

FIG. 4

A classified list of references is offered below.

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9. *School Science and Mathematics*, vol. 39 (Mar. 1939) p. 282, problem 1580.

10. Steinhaus, H., *Mathematical Snapshots.* New York: Oxford University Press, 1950, p. 8.

11. Steinhaus, H., *Mathematical Snapshots.* New York: G. E. Stechert and Co., 1938. pp. 7–8.

RELATED MATERIAL

12. Johnson, R. A., *Modern Geometry.* Boston: Houghton Mifflin Co., 1929, p. 175, p. 284.

13. *American Mathematical Monthly*, vol. 56 (Apr. 1949) pp. 269–270, problem 4260.

14. *School Science and Mathematics*, vol. 38 (Nov. 1938) pp. 935–936, problem 1556.

ADRIAN STRUYK
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35. Le Mathématicien Polycéphale

"Le mathématicien polycéphale" is the title applied to Nicolas Bourbaki, a mathematical personage of rapidly growing fame, by André Delachet in *L'Analyse Mathématique*, a recent pocket-book history of analysis.[2] The footnote to an article by Bourbaki in the *American Mathematical Monthly* states "Professor N. Bourbaki, formerly of the Royal Poldavian Academy, now residing in Nancy, France is the author of a comprehensive treatise of modern mathematics, in course of publication . . ."[3]

Both the adjective "multiheaded" and the "Royal Poldavian Academy" recall the fun that went with a grape-vine notice that Bourbaki was to speak at a section meeting of the American Mathematical Society's joint meeting with the National Council at Christmas-time in Columbus a few years ago. The fun lies in the fact that N. Bourbaki is not one man but a group of mathematicians, chiefly French, who have merged their mathematical personalities to produce in serial form an extremely significant work, *Eléments de Mathématique*, of which twelve volumes, the first dated 1939, the last 1951, have been published so far by Herman & Cie. as a part of the series *Actualités Scientifiques et Industrielles.*

Delachet regards their work as so significant that he devotes the concluding section of the last chapter of his history to "L'Ecole Bourbaki." He lists the chief members of this group as MM. Henri Cartan, Dieudonné, Chabauty, Ehresmann,

[2] André Delachet, *L'Analyse Mathématique.* (Paris: Presses Universitaires de France, 1949. This is booklet number 378 in the series "*Que Sais-Je?*" which was noted in this column in THE MATHEMATICS TEACHER, vol. 42 (Oct. 1949) p. 307.

[3] Nicholas Bourbaki, "The Architecture of Mathematics," *The American Mathematical Monthly*, 57 (April 1950) pp. 221–232. This article is the source of the phrases which we have enclosed in quotation marks. It, in its turn, is a translation by Arnold Dresden of the second of an extremely interesting collection of essays by diverse authors edited by F. Le Lionnais under the title *Les Grands Courants de la Pensée Mathématique.* (Cahiers du Sud, 1948).

Samuel, and Godement with the addition of MM. Weil and Brelot as participants who preserve their autonomy.

There are many interesting aspects of Bourbaki's work. Foremost perhaps is its objective which in its early chapters is to start from the beginning and to give a solid foundation for the whole of modern mathematics. He even says that to read his work from the beginning one would theoretically need to know no mathematics, but he admits that one would need a certain propensity for reasoning mathematically and a certain power of abstraction. This latter is particularly true since the approach is axiomatic and abstract, proceeding from the general to the particular. This is in line with his objective which may be further described as an attempt to show the structure of mathematics in such a way as to display its unity. This is opposed to the feelings of some persons that modern mathematics shows a "progressive splintering" manifest in its recent "exuberant proliferation."

Bourbaki finds a unifying concept not in "deductive reasoning" which is merely a method common to many areas, but in the "*axiomatic method.*" He regards the essential aspect of the axiomatic method to be not its use of syllogisms but its systematic study of the relations existing between different mathematical theories. This it does by means of searching out and analyzing the relatively few basic *structures* which are the common property of many mathematical systems. Examples of these are *algebraic structures* distinguished by a central theme of the laws of composition of abstract elements, *order structures*, *topological structures* dealing with the concepts of neighborhood, limit and continuity.

Recognition of these basic structures, Bourbaki feels, not only effects considerable economy of thought, but also provides a basis for that *intuition* which today more than ever plays a fundamental role in mathematical discovery.

Some of the mechanical aspects of Bourbaki's books are also of interest for their connection with his program. For example, many volumes include a dictionary not only of the terms used in the book but of all equivalent or related terms used in the chief languages. Bourbaki's terms have been selected and defined with care and deliberation. Where it will be particularly useful to readers to have certain axioms, and definitions before their eyes as they read these axioms and terms have been reproduced on a folded sheet which is then pasted in at the back of the book and listed in the table of contents.

Altogether "N. Bourbaki" stands for an extremely interesting and significant cooperative current international mathematical project.

What Is Going On In Your School?

(*Continued from page 495*)

Some interest may be found in the average of the per cents of the graduates of these 29 schools who had completed one, two, three, and four years of high school mathematics. These averages for the 29 schools are: 97% for column 4a, those who completed one year of mathematics in grades 9–12; 86% for column 4b, those who completed two years of mathematics in grades 9–12; 43% for column 4c, those who completed three years of mathematics in grades 9–12; 18% for column 4d, those who completed four years of mathematics in grades 9–12; 47% for column 5, per cents of 1950 graduates who entered college.

These averages may be fairly representative of schools from which about one-half of the graduates enter college but not of schools sending a lower per cent to college.

APPLICATIONS

Edited by SHELDON S. MYERS
Department of Education, Ohio State University, Columbus, Ohio

Your department editor has been doing a great deal of thinking and reading concerning the role of applications in mathematics instruction. Following are several key references which we have found: "The Necessary Redirection of Mathematics, Including its Relation to National Defense," by William Betz in THE MATHEMATICS TEACHER, April, 1942, page 147; "The Use of Applications for Instructional Purposes," by Edwin G. Olds in THE MATHEMATICS TEACHER, February, 1941, page 78; "The Mathematics Most Used in the Sciences of Physics, Chemistry, Engineering and Higher Mathematics," by George H. Nickle in THE MATHEMATICS TEACHER, February, 1942, page 77; *A Study of Problem Material in High School Algebra* by Jesse Powell, Contributions to Education, No. 405, Teachers College, Columbia University, 1929.

In the first reference, we agree with Betz that the organizing principle of mathematics instruction must be the interrelated system of mathematical concepts and processes. Betz emphasized, however, that, with the mathematical system as the center of orientation, experiences significant to children can be designed to provide significance, meaning, or both. Buckingham distinguishes between social significance and mathematical meaning. Applications may contribute to one, or the other, or both.

The study by Powell showed that teachers were unreliable judges in rating problems of interest to children. Pupils rated high verbal algebra problems involving the puzzle element, and boy and girl activities, but rated low problems of finance and those having a work theme. Teachers rated problems of finance or business high and problems with the puzzle element low. Difficulty did not seem to lower pupil ratings.

The foregoing references have been kept in mind in the development of this department. Most of the applications appearing in this department have been used in the classroom by the editor at the indicated grade levels and have proved successful in motivation and learning.

Ar. 11 Gr. 6–9 *Boys Entered in Soap Box Derbies Sometimes Have Real Arithmetic Problems.*

A boy and his racer cannot exceed 250 pounds by soap box derby rules. When a boy and his racer weigh less than 250 pounds, the boy will add weight, usually washers, so that the total will come up to the 250 pound maximum. He does this to take advantage of the pull of gravity. When he and his racer exceed 250 pounds, he must eliminate weight from his racer, since he cannot reduce that fast. Following are five boys with the weights of themselves and their racers. The washers which each boy adds or removes from his racer weigh one-half pound each. What must each boy do so that the combined weight of himself and his racer will qualify under derby rules?

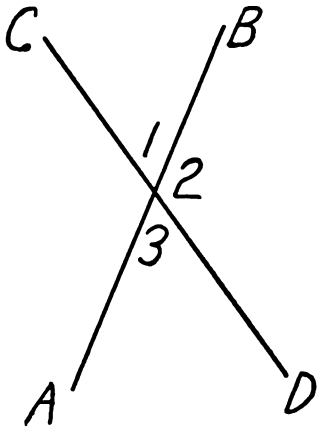
1. John Beekman 119½ lbs.; Blue Star 130½ lbs.
2. Jim Bailey 110¼ lbs.; Comet 134⅛ lbs.
3. Dick Manley 112½ lbs.; Super Jet 140¼ lbs.
4. Skippy Bedford 95 lbs.; Silver Streak 125¾ lbs.
5. Henry Jones 105⅛ lbs.; Red Rocket 126¼ lbs.

P. 4 Gr. 10–12 *A Logical Isomorphism in Plane Geometry*

Recently I examined an M.A. thesis in mathematics entitled *Some Applications of the Principles of Isomorphism and Variation to the Teaching of Geometry* by Ralph Mock, University of New Mexico, 1938. The first part of the thesis deals with an extensive collection of theorems and their proofs. Each theorem has an associated, non-geometric problem whose logical relationships and parallel solution are structurally identical with the geometric theorem but whose context and elements are different. In most cases the isomorphic identity was partial, not complete. The theorems thus illustrated ranged over polygons, including the Pythagorean Theorem, and circles. Here is a simple example of one of the theorems with its associated problem.

Geometric Theorem

If two straight lines intersect, the vertical angles thus formed are equal.



Associated Problem

A merchant at a trading post offers an Indian the following trade:

1. A box of cartridges for a bracelet and ring.
2. A box of cartridges for a bracelet and string of beads.

What is the comparative value of the ring and beads?

Correspondence:

The box of cartridges corresponds to 180°.
The bracelet corresponds to angle 2.
The ring corresponds to angle 1.
The string of beads corresponds to angle 3.

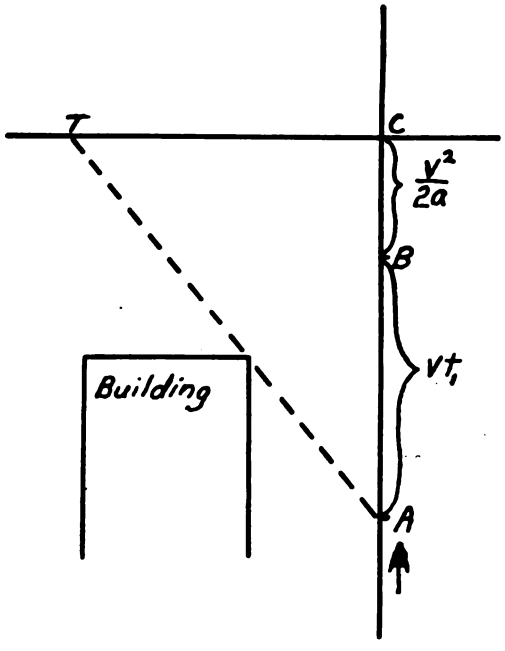
Solution: *Statements*	*Reasons*
1. $\angle 1+\angle 2=180°$	1. The ring and bracelet are worth a box of cartridges.
2. $\angle 3+\angle 2=180°$	2. The string of beads and bracelet are worth a box of cartridges.
3. $\angle 1+\angle 2=\angle 3+\angle 2$	3. The ring and bracelet are worth the string of beads and bracelet.
4. $\angle 1=\angle 3$	4. The ring is worth the string of beads.

The associated problem for the Pythagorean Theorem involved the 1930 populations of nine counties in New Mexico.

The associated problems in this thesis were designed by the author to fit the corresponding geometry theorem. The thesis does not develop the technique for constructing such problems. However, with a little experience and insight, most teachers should be able to make up an indefinite number of associated problems using data and relationships dealing with the school itself. It must be stressed that in these analogues the social significance and context is irrelevant to the basic logical patterns.

C. Al. 3 Gr. 10–12 *A Grade Crossing Quadratic*

Dr. J. B. Coleman, formerly with the South Carolina Highway Department, wrote me last spring that he had had a number of requests for a "Grade Crossing Quadratic" which he had submitted in a paper to the Southeastern Section of the Mathematical Association of America last April. He sent me his brief paper, which had been abstracted in the *American Mathematical Monthly*, December, 1950, page 704. Below is his derivation of the formula.



Our problem is to find the velocity at which an automobile may approach a crossing where visibility is limited, so that upon seeing danger, such as an approaching train, the driver may be able to stop before reaching the crossing.

Suppose the driver, proceeding north with a velocity, v, reaches a point, A, in the figure before seeing that it will be necessary to stop short of the crossing, C.

The total reaction time for perceiving and acting varies with individuals and is generally betweeen .5 and 1 second. Calling the reaction time, t_1, the car will travel a distance of vt_1, or AB in the figure, before the brakes are applied. Perhaps it has never occurred to many drivers that, after the necessity for stopping is seen, the car with a speed of 50 m.p.h. will ordinarily travel a distance of 50 to 75 feet before the brakes can be applied.

After applying the brakes, we postulate uniformly negative acceleration, or deceleration, until the car speed is reduced to zero. With deceleration, a, and initial velocity, v, the distance travelled before coming to rest, s or BC in the figure, is related by the formula of elementary physics, $v^2=2as$. Hence, $BC=s=v^2/2a$. (The value of a will depend upon the character and condition of the road, of the tires, and of the brakes. For dry concrete with good tires and brakes, a may be about $.6g$, where g is the acceleration of gravity of 32 feet per second each second. For wet asphalt, poor tires and brakes, a may be $.2g$, or less.[1]

Calling AC, s_0, then $s_0=AB+BC$. Substituting the values found above, this equation becomes:

$$s_0=vt_1+v^2/2a.$$

Given the conditions of s_0, t_1, and a, the positive root of this equation, solved for v gives the maximum speed allowable under the given conditions. The student will get an appreciation of the formula only by applying it with different values of the conditions. This will serve the purpose of comparison, as well as information. He may check the value of v dimensionally, and will need to exercise care dimensionally as regards feet per second and miles per hour, since the results would ordinarily be given in miles per hour. (In the formula given below in the footnote, t_1 and a have been so modified that r is in miles per hour and d is in feet.)

[1] The Seventeenth Yearbook, *A Source Book of Mathematical Applications*, of The National Council of Teachers of Mathematics, on page 95, assumes an a of 22 feet per second per second and a t_1 of .75 seconds in deriving the formula, $d=.045\ r^2+1.1\ r$, for minimum stopping distance.

The equation might be termed a "live quadratic" in that the coefficients have physical significance. Such coefficients are called "parameters." It should serve a useful purpose if it does nothing more than direct the attention of some of our youthful drivers to the role played by reaction time, in a quantitative way.

C. Al. 4 Gr. 10–12 *An Inductive-deductive Experiment with the Tower of Hanoi Puzzle*

The Tower of Hanoi Puzzle can provide the basis for a neatly controlled inductive-deductive experiment in algebra which has many of the characteristics of scientific method. A description and some of the theory of this puzzle can be found in such books as *Mathematical Recreations* by Maurice Kraitchik, W. W. Norton, Inc., pp. 91–93 and *Riddles in Mathematics* by Eugene Northrop, D. Van Nostrand Co., pp. 25–27. Students can make them easily out of wood in the shop. The puzzle consists of three pegs set in a wood base and five to seven, round, or square, discs with holes in the center and varying gradually in size, so that, when placed over a peg in order of size, they look like a small pyramid. The object of the puzzle is to transfer all the discs from one peg to another by moving one disc at a time and by not placing a larger disc on a smaller one. The idea is to accomplish this with a minimum number of moves. For purposes of trying this application, you can cut the discs as squares from cardboard and imagine the three pegs.

When students attempt this puzzle with 3, then 4, then 5 discs, they realize that the minimum number of moves each time jumps accordingly. It is easy to get them to pose the research problem of finding the functional relationship between number of discs and minimum number of moves. It is interesting to see these young people naturally begin by trial and error to gather experimentally the following table of data:

n	m
number of discs	minimum moves
1	1
2	3
3	7
4	15
5	31
n	?

Soon they restate the problem as follows: What uniform set of operations, if any, can be performed on the left numbers so as to produce the right numbers correspondingly?

Some students begin to take differences between the values of m and discover that as far as they go down the table the first set of differences is a geometric progression with common ratio of 2. They also discover that the differences among these differences (second order differences) are likewise a similar geometric progression and that this is true for the third order differences. They examine the literature and find that if the first order differences are constant, the function is linear; if the second order differences are constant, the function is second degree, and so on. They also find that if successive order differences remain progressions, the function is some kind of exponential function. Before long after this, some discover that the expression $2^n - 1 = m$ correctly relates n and m for the first five cases. They have succeeded in making an inductive generalization from data.

Almost immediately someone will inquire whether this formula will accurately predict the minimum moves for 6 discs. The students then apply the formula deductively for the specific case of 6: $2^6 - 1 = 63$ moves. The cycle is finally completed when the students experimentally verify this prediction with the puzzle.

Usually advanced classes are smaller in number. With puzzles immediately available for groups of two students each, the foregoing experiment can be successfully accomplished in a 50 minute period, providing the students do not have to check too long on the theory of differences.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
135-12 77th Avenue, Flushing 67, N. Y.

SOME years ago much ado was raised concerning the indirect method of proof in Geometry. The readers of THE MATHEMATICS TEACHER may still find great pleasure and derive much profit from reading the chapter on this topic in the FIFTH YEARBOOK OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS written by Professor Clifford B. Upton. In some quarters, however, some doubts have been expressed concerning the efficacy of the indirect method of proof.

This department does not propose to take sides in any controversy concerning the advisability of using or of discarding the indirect method of proof in classroom instruction. However, this department is of the opinion that this method of proof, as well as the processes which are employed in it, need not be confined to geometry exclusively. As a matter of fact, the indirect method may be employed with considerable effect in situations other than geometric. Furthermore, many mathematical recreations utilize the indirect method and with considerable effect, to be sure.

Here are a few problems which could have been probably located in the files of any Government agency which is concerned with the enforcement of the law.

A suspicious character was arrested and the arresting officer found in his possession a bag containing 80 silver dollars. Upon the examination of these coins the arresting officer could not be certain whether they were counterfeit. His tip was that one of the coins was lighter in weight than the remaining 79. He did not have a weighing machine with him. However, he constructed a balanced weighing scale and in four steps he extracted the counterfeit coin. How did he do it?

He divided the coins into three groups. Two groups contained 27 coins each. The third group contained 26 coins.

First, he weighed the two groups with 27 coins each by placing 27 coins on the pans of the balance scale. If the scale was balanced, then the lighter coin was located in the group with the 26 coins. If the scale did not balance, then the lighter coin was located in the group of 27 coins which was lighter.

Once the group in which the lighter coin was located, he put aside the other two groups of coins, and he worked on the smaller group (which could have contained either 26 or 27 coins, as indicated in the preceding paragraph). He subdivided this new group into three new groups of coins. If the group contained 27 coins then each new group contained 9 coins. If the group contained 26 coins, then he had two groups containing 9 coins each and one group containing 8 coins.

He balanced two groups of 9 coins each. This enabled him to determine the group which contained the lighter coin. Thus he finally obtained a group with 9 or 8 coins. Note that his method in this case was the same as the method he employed when he started his experiments.

The last group of 9 or 8 coins was again subdivided into three groups. If there were 9 coins in the group, then each new group contained 3 coins each. If there were 8 coins in the group, then two groups contained 3 coins each, and one group contained 2 coins.

Following the same method he ascertained in which the three new groups the lighter coin was located.

Finally, he was left with either 3 or 2 coins. Applying his method once more he

succeeded in separating the lighter coin.

The readers will note that the method of separating the lighter coin is indirect in nature. This type of a problem could be presented as a puzzler in the ninth grade.

Suppose that there are 12 coins and it is known that one of them is counterfeit. But it is not known whether this counterfeit coin is lighter or heavier than a real coin. The separation of the counterfeit coin may be accomplished in three steps. Only balanced scales without weights are allowed.

The 12 coins are divided into three groups of four coins each. When we place 4 coins on each pan of the balance scale two possible situations may take place.

A. The scale will be balanced.

B. One of the pans of the scale will be heavier.

A. If the scale is balanced, then the counterfeit coin is located in the remaining group of four coins, while the 8 coins which were weighed are all real. Take three coins from the group in which the counterfeit coin is located and place them on one pan of the scale. Balance these three coins with three coins which are known to be real. If the scale is balanced then the remaining fourth coin is the counterfeit. If the scale is not balanced then we know that the counterfeit coin is among the three coins, and we also ascertain whether it is lighter or heavier. The determination which of the three coins is the counterfeit is accomplished according to the method which was described above.

B. If one of the pans of the scale is heavier, we immediately know that the counterfeit coin is among the 8 coins which were weighed. But we do not know whether it is heavier or lighter than a real coin.

Label the coins on the heavier pan by numbers 1, 2, 3, and 4, and the coins on the lighter pan by 5, 6, 7, and 8.

Place coins 1, 2, and 5 on one pan, and the coins 3, 4, and 6 on the other pan. If the scale is balanced, then we know that either coin 7 or coin 8 is counterfeit. The determination of the counterfeit coin is accomplished by comparing (weighing) one of them with a coin which is known to be real.

If the pan with the coins 1, 2, and 5 is heavier, then we know that the coins 3, 4, and 5 are real. This conclusion is based on the fact that prior to the second weighing we exchanged the places of the coins with respect to the coins 1 and 2, and thus the pan on which the coins 1 and 2 are located should be lighter. But this is not observed. Thus, one of the coins 1, 2, or 6 is counterfeit. If the coin 6 is counterfeit, then it is lighter than a real coin. If one of the coins 1 or 2 is counterfeit, then the counterfeit coin is heavier than a real coin. Place coin 1 on one pan and coin 2 on the other pan of the scale. If the scale is balanced, then coin 6 is counterfeit. If the scale is not balanced, then the heavier coin is counterfeit.

If the pan with the coins 3, 4, and 6 is heavier, then, by analogy we conclude that coins 1, 2, and 6 are real, and if coin 5 is counterfeit, then it is lighter than a real coin. If coin 3 or coin 4 is counterfeit, then the counterfeit coin is heavier than a real one. Place coin 3 on one pan of the scale and coin 4 on the other pan of the scale. If the scale is balanced then coin 6 is counterfeit. If the scale does not balance then the heavier coin is counterfeit.

The above problems may be generalized, and this generalization should not offer any undue difficulties to a pupil who studies intermediate algebra.

We have n coins. One of these coins is counterfeit, and it is known that it is lighter than a real coin. Determine the minimal number k of the consecutive weighings on a balance scale (the weighing is done without any weights) which will lead to the determination of the counterfeit coin. All the real coins are of the same weight.

10,052 NCTM Members in 1952?

AIDS TO TEACHING

Edited by

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and

DONOVAN A. JOHNSON
College of Education,
University of Minnesota,
Minneapolis, Minnesota

BOOKLETS

B. 67—Money Management Booklets

B. 68—Better Buymanship Booklets

Household Finance Corporation, 919 North Michigan Avenue, Chicago, Ill.

Booklets; 6″×9″; $.05 each.

Description of B. 67: There are eleven booklets in the Money Management Series of the following topics: Your Budget, Children's Spending, The Health Dollar, The Food Dollar, Your Clothing Dollar, The Shelter Dollar, Home Furnishings, the Recreation Dollar, Your Shopping Dollar, Time Management, and How to Buy Life Insurance. They are colorfully illustrated, immensely practical, and filled with charts, tables and pictures which serve as excellent motivation.

Description of B. 68: There are eighteen booklets in the Better Buymanship Series which discuss the points to watch for when purchasing certain products. The subjects covered are these: Meat, Fish, Poultry and Eggs; Fresh Fruits and Vegetables; Processed Fruits and Vegetables; Dairy Products; Playthings; Cosmetics; Soap and Other Detergents; Shoes; Hosiery; Fabrics; Furs; Household Textiles; Floor Coverings; Furniture; Dinnerware; Kitchen Utensils; Vacuum Cleaners; and Home Heating.

Appraisal of B. 67 and B. 68: This is the best set of inexpensive booklets on consumer education available at a price low enough so that sets can be given to individual pupils. The Money Management set should be in the classroom in quantity so that each pupil can have one of each in the set. Among these it may be well to buy enough each year so that pupils can have their own personal copies of the one called "Your Budget." This has blank pages which can be filled in as exercises while the class discusses the budget. The other volumes in the Money Management set contribute useful information, but they can be turned in to the teacher at the end of the year and used over again.

One complete set of the Better Buymanship Series for each class would be sufficient for reference purposes.

B. 69—Facts You Should Know about Borrowing

B. 70—Facts You Should Know about Securities

B. 71—Facts You Should Know about Security and Commodity Exchanges

B. 72—Facts You Should Know about Saving Money

B. 73—Facts You Should Know about Investment Companies

B. 74—Facts You Should Know about Investment Banking

B. 75—Facts You Should Know about the Consumer Education Study

B. 76—Facts You Should Know about Commercial Banks and Trust Companies

B. 77—Facts You Should Know about Budgeting

B. 78—Facts You Should Know about Buying or Building a House

National Better Business Bureau, Inc., Chrysler Building, New York 17, N. Y.

Booklets; $5\frac{1}{4}'' \times 8''$; Generally 8 pages each; $.05 each.

Description of B. 69 through B. 78: The titles above give the general nature of the pamphlets. They are well written and approach each problem from an elementary standpoint suitable for secondary mathematics classes. Typical problems are solved which illustrate the topics.

Appraisal of B. 69 through B. 78: These inexpensive, authoritative booklets should be used in all classes which study the consumer aspects of mathematics. Some junior high school classes will find the vocabulary a little advanced, but the teacher can use the booklet herself from which to draw information to pass on to the class. Sets should be available to loan to the pupils at all levels. The usefulness in schools would be greatly increased, however, if the material were made more attractive. There is a need for pictures, drawings and charts to enliven the appearance of the pages. Examples for pupils to do to follow up the reading would also be useful, but it is not too difficult to devise these—either the teacher or pupils can do so.

B. 79—Teaching Aids for Financial Security Education

B. 80—The Story of Life Insurance

B. 81—Careers for You in Life Insurance

B. 82—Life Insurance Fact Book

Institute of Life Insurance, 488 Madison Avenue, New York, N. Y.

Booklets; Free.

Description of B. 79: This is a general catalog which includes the other booklets mentioned above as well as other aids to teaching. There are 15 booklets on insurance and money management, 3 filmstrips, 4 wall charts, 2 comic books, and 5 sound films (16 mm.).

Description of B. 80: This unit on life insurance for the intermediate and upper grades is $8\frac{1}{2}'' \times 11\frac{1}{2}''$ and contains eight pages which are reprinted from the November, 1950, issue of *The Grade Teacher*. There is a short history of life insurance and a description of the present practices. Two pages of correlated activities are grouped under the headings of language arts, social studies, science and health, arithmetic, graphs and charts, arts and crafts, and cumulating activity. A short bibliography is included.

Description of B. 81: On 72 pages, $6'' \times 9''$, is a well-illustrated summary of the various positions in life insurance work. Among others are included the actuarial, medical, clerical, legal, and sales departments.

Description of B. 82: This fact book is filled with tables of figures concerning life insurance companies and their policies. It is an endless source of material for teachers and pupils to construct problems. The short mortality tables are especially useful to illustrate the facts upon which premiums are based.

Appraisal of B. 79 through B. 82: These are very useful aids for the sections of courses concerning insurance. They are so attractive that they will lead to excesses in stressing this topic.

The ability to find material on a subject in mathematics within a welter of similar information is a skill often neglected in mathematics classes. The usual problems in school contain enough information, but not too much. Is that reality?

CHARTS

C. 26—Automobile Stopping Distances

General Motors Corporation, Educational Service, Department of Public Relations, Detroit 2, Michigan.

Chart; $22'' \times 34''$; Black and white; Free

Description of C. 26: The front of the chart shows graphically the relative distances which a car moves during the driver's reaction time, and while stopping on dry pavement, wet pavement and ice. Each of these facts is shown for 20, 30, 40,

50, 60, 70 and 80 miles per hour. The back of the chart discusses factors which affect stopping time.

Appraisal of C 26: This is an excellent example of a statistical graph and always causes comments and interest in a class. From the obvious mathematical conclusion that the stopping distance is not a linear function of the speed, one could go on in more advanced classes to finding a formula which will fit the data given. In elementary classes the chart is useful to read graphs and compare lengths.

FILMS

F. 61—Areas

Knowledge Builders Classroom Films, 625 Madison Avenue, New York 22, N. Y. Educational Collaborations, R. C. Jurgensen.

16 mm. film; Black and white; Sound; 400 feet, 12 minutes; $40.

Description: The concept of area is illustrated by use of a square whose dimensions are one unit each. Several rectangles are presented and their areas found by counting the number of units of area. From these illustrations the formula for the area of a rectangle is assumed. The area formulas of the parallelogram, triangle, trapezoid, regular polygon and circle are then derived in order without formal proof. A parallelogram is compared to the rectangle by use of congruent right triangles, and it is stated that it can be proved that the area of a parallelogram also can be found by using the formula, $A = bh$. Altitude is illustrated and defined. A triangle is formed by the diagonal of a parallelogram and its area is shown to be half that of the parallelogram. Triangles having the same base and equal altitudes are shown to be equal in area by moving the vertex of the triangle so that it remains equidistant from the base. By dividing a trapezoid into two triangles and a regular hexagon into six congruent triangles two more formulas are derived. As the number of sides of the regular polygon is increased indefinitely, *the apothem* approaches the radius and the perimeter approaches the circumference of a circle. $2\pi r$ is substituted for circumference and r for apothem in $A = \frac{1}{2}ap$, and $A = \pi r^2$ is derived. A summary of the six figures, by use of diagram and formulas, concludes the film.

Appraisal: The concept of area is presented clearly without formal demonstration. Because a plan of proof is given for each of the six figures, this film must be shown at the beginning of the study of the unit on areas. Pupils then should be ready to study the formal proofs. All the plane geometry classes in our school saw the film, and all the teachers were pleased with the pupil reactions and results. Although the area formulas for the regular polygon and the circle are not studied at this time in our classes, their inclusion presented an interesting application of the triangle. The presentation and sound track are very clear. Every class in plane geometry can profit by seeing this film. The only objectional explanation in the film is the use of the word "cancellation" for divide in the derivation of the $A = \pi r^2$ formula. (Reviewed by Herbert Freed, Senior High School, Atlantic City, N. J.)

F. 62—Fixed Gauges

Producer: Division of Visual Aids for War Training, U. S. Office of Education, Washington 25, D. C.

Film obtainable from: United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

16 mm. film; Black and white; Sound; 18 minutes; 1 reel; $28.58; study guide available.

Description: This film shows in detail: (1) types and uses of snap gages; (2) types and uses of ring gages including taper ring gages and thread gages; (3) types and uses of plug gages including straight, taper, and screw plug gages; and (4) design and use of a flush pin gage.

The film opens by showing that every drawing contains the required information about dimensions, and how the allowable tolerances are stated. Many types of

ɔision gages are shown including: es, ring gages, plug gages, taper es, taper ring gages, and their ses.

ɾpes and uses of snap gages are These are fixed gages designed g outside measurement. The use o button and not-go button is d. The correct way to gage a ece is shown, also how to gage ides. How adjustable limit gages et to a precise dimension is ex-

ɾpes and uses of ring gages are vn. This includes the not-go ring taper ring gage, the thread ring roll thread snap gage, and other ypes. Demonstrations of their ɪrly brought out.

ɾpes and uses of plug gages are There are two principal classes, plug gages and taper plug gages. vn that the accuracy of any fixed lassified according to the type of which it is designed. That the gages are widely used for check- ɪ dimensions and that the screw es are used for checking internal ɾeads is illustrated.

e from the gage inspection or gage department is shown where all ɪd other measuring tools are and adjusted. A precision gage being used with an electrolimit ɔheck the accuracy of a flat plug

isal: For the evaluation of this e were seven teachers of mathe- resent. They considered the entire ɡood for the introduction of new to augment explanations, to ɪ common experience, to motivate ɪse in review. Four of the evalua- eved that the film was just fair ɑ developing skills. It was agreed film could be certainly utilized ɜ 9th to the 12th grade, but only mathematics. The entire group ɐd the speed of development of ɪd duration of scenes as being e and that there were no inaccura- cies in the mathematical content. Four of the teachers saw the film as partially holding the interest of students while the remainder saw it as completely holding the interest. The teaching methods were partially conducive to five of the evaluators while the remainder found them to be completely so. The photography was good, and three of the previewers believed the sound to be good and three that it was fair. Four checked the effectiveness of characterizations and background objects as good, but there was one check each for very good, fair and poor. Five thought that the presentation would encourage further pupils activities in mathematics and in other subjects. Two of the previewers scored that the captions were not of appropriate length while all remarked that they were pertinent. The greater number considered the dialogue as effective. All agreed that there was coordination between sound and picture and that the arrangement of topics was in a logical sequence. The evaluators were four to three that the amount of material was reasonable for comprehension in one showing. Four felt that the content was not able to be just as effectively and efficiently presented in some other way. Five saw the film as supplementing rather than replacing the teacher. Finally, five of the teachers would definitely use the film. (Reviewed by A. diLuna, R. Fleet and K. Hathaway)

F. 63—The Steel Rule

Producer: Division of Visual Aids from War Training, U. S. Office of Education, Washington 25, D. C.

Film obtainable from: United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

16 mm. film; Black and white; Sound; 14 minutes; 1 reel; $25.71.

Description: Discusses in considerable detail the steel rule, emphasizing the variations of the steel rule, the type of scales found on them, their proper use, and the correct procedures in transferring

measurement by means of calipers and dividers.

Appraisal: Eight teachers were together for the evaluation of this film. Considering the entire film they found it as being good when used to introduce new material, to develop skills, or for review. Three of the teachers evaluated the picture as good for both augmenting explanations and providing a common experience, but one fourth also checked "fair" in these two purposes. When considering using the film for motivation, all categories were checked with three each in "good" and "fair." The grade level was placed between the 8th and 12th grade with marked predominance in the 9th. The film is especially useful in shop mathematics with some value in arithmetic. The speed of development of ideas is moderate and there are no inaccuracies in the mathematical content. Five previewers saw the film as partially interesting to pupils and three found it to be completely so. The same figures were also reported when they evaluated the teaching methods in the film as conducive to learning. Six scored the photography as good, but two scored it very good and 1 as poor. One half thought that the sound was good and the remainder were split on very good and fair. The effectiveness of characterizations and background objects was good to five and poor to one. The film presentation will encourage further pupil activity in mathematics as was penciled by three-fourths of the evaluators, and all agreed that it would in other subjects. All were in unison in saying that the captions were both of appropriate length and pertinent. The dialogue was definitely effective. The entire group voiced that the arrangement of topics was in a logical sequence and that there was coordination between sound and picture. Of the evaluators, five definitely believed that the content of the film could not be just as effectively and efficiently presented in some other way. Seven of the teachers did think that the film did attempt to supplement rather than replace *the teacher.* Finally, six would use the film if given the opportunity. (Reviewed by A. diLuna, R. Fleet and K. Hathaway)

F. 64—An Introduction to Vectors

Producer: Division of Visual Aids for War Training, U. S. Office of Education, Washington 25, D. C.

Film obtainable from: United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

16 mm. film; Black and white; Sound; one reel; 22 minutes; study guide available; $34.31.

Description: A number of key points are covered in this film such as: (1) explanation of the meaning of scalar and vector quantities; (2) explanation of how to add scalars and vectors; (3) explanation of the various methods of vector composition and vector resolution; (4) explanation of the relationship between vector composition and vector resolution; (5) clarification of the relationship between the analytical and graphical methods of vector composition and vector resolution; and (6) shows how vectors may be employed to solve engineering problems.

The film opens with a discussion of scalar and vector quantities. It is shown that the miles flown by an airplane may be represented by lines proportional in length to the number of miles covered. These quantities which have magnitude only are called scalar quantities and are added by simple arithmetic. Scalars have but one property, magnitude or amount. A plane is then shown flying from one city to another, and an explanation follows that in this instance two values are indicated, magnitude and direction. When quantities of this type have magnitude and direction. they are called vector quantities.

A graphic illustration of vector quantities is given. Line of action is defined and the fact that direction of the quantity indicates a vector.

The addition of vectors by the triangular method is shown. Two examples are given to illustrate this. In the first exam-

ple a plane is flying from Charleston to Pittsburgh, and it is shown how the combined action of the two displacements results in the plane's net displacement or the resultant. In the second example, a plane is flying east with a velocity of two hundred miles per hour and at the same time a wind is blowing from the north with a velocity of sixty-five miles per hour. The plane's displacement by vector addition is determined. These examples are illustrated by animation.

Another way is shown by which the resultant may be obtained. This is the parallelogram method. The last method shown is the analytical method. This method is discussed by use of a right triangle.

It is explained that vector quantities of any type can be combined by the same methods. An example given is a force of forty pounds acting in a certain direction applied to an object, and another force of thirty pounds acting in a different direction. The vectors in this problem are added first by the parallelogram method and then by the triangle method. It is shown that either method may be used, for the resultant is the same in both cases.

A discussion follows on the order in which the vectors are arranged in the triangle method and parallelogram method. In the triangle method of vector composition it is shown that the order in which the vectors are arranged, tail to head, makes no difference in the resultant, but the resultant always joins the tail of the first to the head of the second. In the parallelogram method the vectors are always joined together tail to tail, and regardless of their size or direction the resultant is always the vector drawn diagonally from the common point of origin.

What components are; how the pairs of components may be obtained; and the fact that the only components considered are those at right angles to each other is discussed. The plus and minus properties of the x and y components are illustrated by animation.

The x and y axes may be in positions other than the horizontal and vertical, but regardless of their position the method remains the same. This is shown in the example of a ball weighing one thousand pounds resting in a ninety degree angle plate, one side of which makes an angle of thirty degrees with the horizontal. The force which the ball exerts on each face of the block is determined by vector resolution which is carried out graphically.

It is shown that very often several quantities may act together. An example of four men pulling on a heavy object in certain directions may be represented by a vector, and the total effective pull on the object can then be determined. Both the triangle and parallelogram method are used to work this problem.

The film ends with a short review of the fundamental issues taken up.

Appraisal: Ten teachers of mathematics reviewed this film. Five thought the film was excellent for introducing new material while two thought it was poor for this purpose. Six said the film was good for developing skills. Five checked the film as excellent in providing a common experience and in reviewing. The grade level was placed between the 11th grade and the college level. 47% favored the 12th grade while 42% the college level. The film was considered useful in trigonometry and advanced mathematics. Two of the evaluators thought the speed of development of ideas and duration of scenes was too fast, while one found some inaccuracies in the film. 60% said the film would hold the interest of the students and that the teaching methods in the film were conducive to learning. 50% said the photography and sound were very good. All of the teachers said they would use this film in their classroom and the general comment was that the film represents an unusually effectively arranged presentation. (Reviewed by Anthony DiLuna, Raymond Fleet, Kenneth Hathaway)

FILMSTRIPS

FS. 90 to FS. 94—Introduction to Fractions

Jam Handy Company, 2900 East Grand Boulevard, Detroit, Michigan.

35 mm. Filmstrip; Black and white; Silent; $4.50 each, $19.50 per set.

Description: FS. 90, Part I, is entitled "The Fractional Parts of a Whole" and consists of 28 frames. The first four frames show "John and Susan" cutting an apple in half. The next frame shows a half-red, half-blue pencil. Then a circle on the blackboard with a diameter drawn in is shown. The division into thirds and quarters is also shown. Two frames are taken up showing one-third and one-fourth respectively written on a blackboard.

Description: FS. 91, Part II, is entitled "Fractional Parts of Groups" and consists of twenty-four frames. This filmstrip begins with "Johnny" having some marbles and then dividing them, first, into two parts, then into three parts, etc. Frame nine is concerned with counting or addition only. Several frames show only numbers such as $\frac{1}{3}$ of $12=4$, $3\overline{)12}^{4}$

Description: FS. 92, Part III, is entitled "Fractional Parts of a Whole and Groups" and consists of twenty-two frames. This filmstrip shows rabbit hutches, doll boxes, a container for a shell collection, and cupboards all divided into fractional parts. The students are then shown the number of things to be placed in the containers and asked what the desired fractional part will be. There are five frames in which only numbers are shown.

Description: FS. 93, Part IV, is entitled "Non-unit Fractions of the Whole and Groups" and consists of twenty-six frames. This filmstrip shows a garden divided into four parts, each fourth belonging to a different person. Then the question of how much of the garden three of the children own, two of the children own, etc., is asked. Again one frame is used in showing "$\frac{3}{4}$" written on the blackboard. The rest of the filmstrip shows such things as flowers and raspberries, and asks the children for the specific numbers of fractional parts of each example.

Description: FS. 94, Part V, is entitled "Comparing Fractions" and consists of twenty-five frames. The first five frames deal with halving candy bars, cutting a pie into thirds, etc. The sixth frame shows a kite with the four parts colored and asks whether each part is equal to one-quarter. From here on the filmstrip deals mostly with paper, different fractions of which are colored.

Appraisal of FS. 90 through FS. 94: The photography is excellent, the colors are attractive and the captions easy to read. Third and fourth grade children would probably enjoy the variety of colors and the change the filmstrips might provide, if they are accustomed to blackboard work entirely. However, there is little presented in the filmstrips that could not be shown in the classroom to better advantage. The numbers pictured by themselves could just as easily be shown on the classroom blackboard. (Reviewed by Dale A. DuVall, Mathematics Instructor, Boston University, General College.)

INSTRUMENTS

I. 33—Sighting Level

Scott and Anderson, Inc., 952 Dorchester Avenue, Boston 25, Mass.

Surveying instrument; $1'' \times 1'' \times 5''$; $2.00 (FOB Boston).

Description: This level contains no lenses, but has a peep-hole at one end and a glass scale with cross-hair and scale at the other. A bubble level is fastened to the top and is reflected in a mirror so that it can be observed through the eye-piece at the same time that the instrument is sighted. By centering the bubble on the hair-line, one can hold the instrument horizontal. Combined with a measuring rod this level can be used to measure heights above and below the level of the observer.

Appraisal: It is remarkable how much mathematics can be derived from a simple level of this type, and how extraordinarily

accurate a simple, inexpensive device can be. Although home-made levels which the pupils build are effective in theory, this compact, professional-looking level is often more fun for them to use. Seldom is it justified to buy such equipment as this in quantity for a whole class. But if schools could only begin to buy single instruments of various types and thus build a collection of many different kinds, there would soon be a livelier interest and fuller understanding of the applications of mathematics.

I. 34—Slide Rule and Mathematical Tables

Standard Transformer Corp., 1500 N. Halsted Street, Chicago 22, Ill.

Cardboard slide rule and tables; 4″×10″; $.05 each.

Description: This rule consists of a cardboard case with cut-out sections to allow readings, a cardboard slide to fit in the case and a sheet of plastic which also slides within the case and has hairlines engraved upon it. One side of the device has *A*, *D* and *K* scales on the case and *B*, *CI* and *C* scales on the slide; the other side has *A* and *D* scales on the case and *S*, *L* and *T* scales on the slide. In addition there are formulas, decimal equivalents, and slide rule settings printed on the case. A unique feature is the sets of four-place logarithm tables, and natural trigonometric function tables (sine, tangent, cotangent, and cosine) which appear through a slot in the case as the slide is moved. The former require interpolation for the fourth figure of the number which enters the table; the latter is given by degrees from 0 to 90. A two-page set of directions is enclosed.

Appraisal: This ingenious and sturdy little device will be very useful in the classroom. There is a tendency for the slide to bind in the case, and this sometimes makes accurate settings difficult. Having only one side provided with a hairline restricts the usefulness somewhat. Also, the figures in the logarithm and trigonometric tables are extremely small and therefore somewhat hard to read. It should be noted, however, that the appearance of only one line in the table through the opening in the case is apt to decrease the possibility of reading the wrong entry from the table. This device is well worth adding to the mathematics collection for observation and for manipulation.

MODELS

M. 24—Computation Unit

M. H. Ahrendt, National Council of Teachers of Mathematics, 1201 Sixteenth St. N. W., Washington 6, D. C.

Science kit; 4½″×6¾″; In envelope; $.50 each, 3 for $1.00

Description: This new kit of the mathematics series which is planned by the National Council of Teachers of Mathematics in cooperation with Science Service contains a booklet of instructions (5″×6″, 12 pages), a sheet to cut out and mount of Napier's Rods (8½″×11″), a sheet to cut out with addition-subtraction scales and slide-rule scales (5½″×12″), and a plastic slide rule (1″×6½″). The rule contains *A* and *D* scales on the base and *B*, *CI* and *C* scales on the slide. When the slide is withdrawn it discloses inch and metric scales on the edges. The booklet describes 28 "experiments": 6 on the addition-subtration rule, 7 on the slide-rule, 4 on logarithms, 5 on square root, and 6 on the Napier's Rods.

Appraisal: Since small plastic slide rules are so expensive this kit is probably marketed at a fair price. The small rule is useful, however, only to learn the theory of the slide-rule, since it is so short and the accuracy of the scale is to be doubted. The idea of cutting out scales and mounting them on cardboard is excellent. Certainly every school should have one of these kits; from the paper cut-outs the pupils can make similar ones, then the school can purchase better slide rules to add to the collection.

(*Continued on page 517*)

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
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THE PROBLEM OF PURSUIT

AMONG THE problems of frequent occurrence in textbooks in arithmetic and algebra are those which deal with the length of time needed for one traveler to overtake another when the velocity of each is known and when the difference in the starting time of each is given. It is possible that these had their origin in the use of pace-scales in the ancient world but this is a matter of conjecture. It is common observation, however, that if a person walks at a uniform rate for a given time, the distance can readily be computed. For the purpose of making a problem, uniform rates were assigned in somewhat unlikely situations and time intervals were stated even when the author of the problem was unable to determine the interval with the desired precision. For examples of this, consider the problem of the hare and the hound. The first of these examples is from the collection attributed to Alcuin (*c.* 775) and the second is from the work of the Italian Cardan (1539).

> A field is 150 ft. long. A dog stands at one corner and a hare at the other. The hound goes nine feet in each leap, the hare seven. In how many feet and in how many leaps will the hound catch the hare?

> A dog is chasing a hare. The hare is 60 leaps ahead of the dog and three leaps of the dog equal five of those of the hare. The dog takes three leaps in 20 seconds and the hare 5 in 21 seconds. When does the dog catch the hare?

The problem of pursuit where one man catches up with another permits the use of a large amount of human interest material as is seen in the following examples:

> Reuben sets out from his city in the morning of the first day of the new moon to go to meet his brother Simon in Simon's town. On the selfsame day, Simon also leaves his town to go to see Reuben in his city. The distance between the two places is 100 miles. Reuben goes 19 miles a day. Simon goes 17 miles a day. We ask when they will meet.—Rabbi ben Ezra (*c.* 1140)

> Two citizens of Oppenheym called Heynrichson and Conts von Treber, decide to go to Rome together. Heynrichson was old and could travel no more than ten miles a day. Conts was young and strong and could go 13 miles a day. Accordingly, Heynrichson started out from Oppenheym nine days before Conts left, so he was 90 miles away when Conts started. The question is, in how many days will Conts overtake Heynrichson?—Köbel (1514)

> A certain slave fled from Milan to Naples going 1/10 of the whole journey each day. At the beginning of the third day his master sent a slave after him and this slave went 1/7 of the whole journey each day. I do not know how far it is from Milan to Naples, but I wish to know when the second slave overtook the first.—Cardan (1539)

> Suppose *A* leaves Newburyport at 6 o'clock on Monday morning, and travels toward Providence at the rate of 4 miles per hour without intermission: and that at 3 in the afternoon *B* sets out from Providence for Newburyport and travels constantly at the rate of 7 miles per hour; Now, suppose the distance between the two towns to be 90 miles; whereabout on the road will they meet?—Nicolas Pike (1788)

> A man who had stolen a horse rode away on its back. When he had gone 37 Chinese miles, the owner discovered the theft and pursued the thief for 145 miles. He then turned back being unable to overtake him. When he turned back the thief was riding 23 miles ahead of him; if he had continued in his pursuit without coming back, in how many miles would he have overtaken him?—Chang Ch'iu-chien (*c.* 575)

The variants of the problem include all sorts of conveyances. The speed may vary as is shown in the following example which occurred in several works of the fifteenth and sixteenth century in Italy.

> There are two birds in a tree. One flies toward the east going 1 mile the first day, 2 the second, 3 the third and so on. The second flies toward the west going 1 mile the first day, 8 the second, 27 the third and so on according to the cubes of the numbers. The distance around the earth is believed to be 44,310 miles. In how many days did the two birds meet?—Cardan (1539)

From this situation, the problem took on an astronomical application.

> If Saturn revolves around the earth in 30 years and if Jupiter goes around it in 12 years, how many years will there be in the interval between conjugations?—Cardan (1539)

The situation was applied also to the question of when the hands of a clock are together. A Chinese problem has a simpler case where the problem is not complicated by astronomical terminology—

> There are three sisters of whom the eldest comes home once in every five days, the middle one once in every four days, and the youngest one once in every three days. In how many days will all three meet together?

The problem of the couriers, the problem of pursuit, the problem of the planets, whatever it is called, keeps reappearing in many different forms, but when a class has been working with one of these as an exercise in the analysis of a problem situation, introduce the Chinese problem as given above and then follow it with this problem which is taken from W. W. Sawyer's *Mathematician's Delight* published by the Penguin Books Ltd., London, 1943.

> Albert Smith and his wife Betty are both in the forces. Albert is off duty every ninth evening; His wife is off duty every sixth evening. Albert is off duty this evening: Betty is off duty tomorrow evening. When (if ever) will they be off duty the same evening?

Aids to Teaching

(*Continued from page 515*)

RESOURCES

R. 2—Our Banks and What They Mean to Us

Dr. Kenneth P. Kidd, College of Education, University of Florida, Gainesville, Fla.

Tentative Resource Unit; 8½″×11″, 47 pages, mimeographed; $.35.

Description: Twenty-two pages of this booklet are devoted to background material in the form of text which explains the organization and services of a bank and how to use them. Then there are five pages of a glossary of terms and the remainder is a source of teaching suggestions. The teaching suggestions list the goals of the unit, materials (books, films, periodical articles, and pamphlets and posters), a discussion of dramatization, bank tours, and the use of displays and posters, and other miscellaneous suggestions.

Appraisal: Since this unit is in a tentative form there are places which merely state that a certain form will appear there in a later edition; this is easily supplemented by local illustrations, but it gives the booklet an unfinished appearance. It is unfortunate that so many references must be made to a local source, but then the Florida references will suggest similar addresses in other communities. The arrangement of the material, the type of material chosen and the text explanation are quite satisfactory and should suggest to other groups how to build similar units. It is good to have samples of this kind as examples of the meaning of "resource unit."

It has been called to our attention by the National Forum, Inc., that our review of their social studies charts (C. 20) should have said that the charts can now be purchased by individual selection; 5 to 9 charts—$.60 each, 10 to 29—$.55, and 30 or more charts—$.50. We are glad to make this correction.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York, N. Y.

The Question: What should constitute the program of tenth year mathematics?

The Study: Hlavaty, Julius H. *Changing Philosophy and Content in Tenth Year Mathematics.* Teachers College, Columbia University. 1950.

FOR SOME time there have been serious doubts that plane demonstrative geometry constitutes the best mathematical education for the tenth year. Dr. Hlavaty sought "to examine the situation that exists in the teaching of demonstrative geometry, to investigate the influences that have brought it about, and to explore the directions in which there seem to be promising answers to the problems raised by it." (P. 2.)

An examination of the history of geometry in English secondary education was found necessary because early American educational history is a story of transmission of ideas from the Continent. In analyzing trends in this country the investigator concentrated on three periods, namely, 1890–1923, 1923–1939 and 1939–1948.

To study "the situation that exists" Dr. Hlavaty supplemented his analysis of significant reports and addresses by a searching diagnosis of eighteen American geometry textbooks published in recent years.

"To explore the directions in which there seem to be promising answers" the investigator integrated his previous findings with the recommendations and practices of outstanding leaders in mathematical education and general education.

Findings

The English experience revealed over the years a decrease in the emphasis on deduction and an accompanying increase in the use of experimentation and inductive procedures, a reduction in memorization of proofs, the acceptance of hypothetical constructions, the grouping of propositions into topics or units, greater attention to generalizations of results, and more use of the analytic method of proof. In more recent years English schools have been having difficulty in deciding the place of demonstrative geometry in an education for all adolescents.

During the period, 1890–1923, in the United States "originals" were not used much while verbatim recitation of proofs was still the accepted practice of the majority of teachers. This period was terminated by the appearance of the famous 1923 report[1] which concluded that geometric "facts" should be acquired in the junior high school, that demonstrative geometry should be an elective and that the aims of geometry should include the development of spatial imagination, the mastery of the great propositions and their applications, the understanding of deductive proof, the acquisition of habits of precise statement and logical organization of ideas and the growth of logical memory. It was apparent that the automatic transfer of mental characteristics was still considered to be within the realm of possibility.

During the period 1923–1939 there was a decline in the percentage of high school students electing geometry but an increase in the number. The Third Report of the Committee on Geometry[2] recommended in 1935 that transfer materials of a non-mathematical sort be provided.

In the 1939–1948 interval, the Joint

Commission Report of the National Council of Teachers of Mathematics and The Mathematical Association of America[3] as well as the Progressive Education Association's "Mathematics in General Education"[4] were published. The former recommended only minor curricular modifications for the tenth year while the latter, in its chapter on proof, reflected Fawcett's thinking[5] that geometry must be so taught that improvement in reflective thinking in non-mathematical situations would also be achieved.

For the period, 1930–1948, Dr. Hlavaty discovered an emphasis on practical geometry, attention to the nature of proof in non-mathematical situations in addition to geometric ones, a radical break with the Euclidean approach in the work of Birkhoff and Beatley,[6] Swenson's advocacy of integrated mathematics,[7] and the coming of the general mathematics movement. He also found that geometry was more frequently being considered "as an elementary illustration of the scientific method by encouraging experimentation and clarifying the complementary roles of deduction and experiment" (p. 91), an emphasis on three dimensions as well as two, a tendency to use instruments other than the straight-edge and compass and a modification of the sequence of topics "to permit a more gradual approach to formal proof." (P. 91.)

His analysis of geometry textbooks from 1930–1948 revealed, among other trends, a decrease in the number of propositions, an increase in the number of assumptions, originals and practical applications, more attention to reasoning in non-mathematical situations and the appearance of materials from analytic geometry.

Finally, Dr. Hlavaty proposed for the tenth year a program. "The method traditionally associated with demonstrative geometry is a major objective of the course but it is to be developed with arithmetical, algebraic, non-mathematical as well as geometric materials." (P. 112.) Considerable emphasis would be placed on applications. Deduction would be only one of the aspects of thinking considered. The experimental-inductive procedures of scientific thinking would be examined along with probability and statistics. Of the eight units proposed for the course only one would be devoted entirely to traditional plane geometry; only one or two sequences, e.g., congruence, areas, circles, would be treated in this unit. The remaining units would involve coordinate geometry, trigonometry with simple proofs, geometry of the global earth, statistics and probability, thinking in non-mathematical situations and arithmetic and algebraic proofs and rationalizations.

It is quite apparent that such a course would be of value to a much greater percentage of tenth year students than demonstrative geometry alone.

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Fourth Annual Institute for Teachers of Mathematics
August 21-27, 1952, The Phillips Exeter Academy, Exeter, N. H.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are requested to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning **Devices for a Mathematics Laboratory** to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

THE SUBDIVIDED CUBIC YARD—A STUDENT'S ACCOUNT OF ITS CONSTRUCTION

Few students have adequate conceptions of the actual size of the cubic units, to say nothing of the square units and linear units. Without this knowledge, problems involving the use of such units are devoid of meaning.

The model illustrated in the accompanying picture shows simultaneously the inch, the square inch, the cubic inch, the foot, the square foot, the cubic foot, the yard, the square yard, and the cubic yard. By examining the model students can gain a clear idea of both the actual and relative sizes of the common units of mensuration. Also they will be able to tell by inspection or simple calculation how many smaller units are contained in each

Pictured holding the subdivided cubic yard which they produced are Walter Schoenfeld and James Gavin, students at Cleveland High School, St. Louis, Missouri.

larger unit. The model in the picture was produced by the two students shown holding it; their names are James Gavin and Walter Schoenfeld. The two boys constructed the subdivided cubic yard in connection with their work in solid geometry at Cleveland High School, St. Louis, Missouri. The following résumé, written by James Gavin, is an actual account of how the boys constructed the model.

Construction of the Subdivided Cubic Yard

Our work on the construction of the subdivided cubic yard began in my garage on the evening of March 2, 1950. For the framework of the model we decided to use scrap poplar wood. Using a power saw, we ripped out twelve pieces $\frac{1}{2}'' \times \frac{3}{4}'' \times 37''$ for the framework of the cubic yard. The extra inch was allowed for later trimming so as to have accurate dimensions. We also sawed out 108 feet of sticks $\frac{3}{8}''$ square for the framework of the cubic feet. To make the cubic inches we bought 450 feet of chalk-line and dyed it yellow to make it show up better.

In order to make the model rigid we made eight corner braces from sheet aluminum. A model of the brackets was first made from paper. Then the aluminum brackets were cut according to this paper pattern. (See Fig. 1.) Using $\frac{1}{8}''$ stove bolts $\frac{3}{4}''$ long, we assembled the framework of the cubic yard with little difficulty.

Next we proceeded to construct the 27 cubic feet. First step in the process was to complete the framework of the faces of the cubic yard. For building material we used some of the $\frac{3}{8}''$ square sticks which we had previously cut. Again we cut the sticks slightly longer than three feet so they could be trimmed to the proper length when fitted into the framework of the cubic yard. Also we cut notches $\frac{3}{8}''$ wide and half-way through on each of these sticks at one-foot intervals. This enabled us to have flat surfaces at the places where the sticks crossed each other. The framework for the 54 square feet of surface area was assembled with fish glue and wire

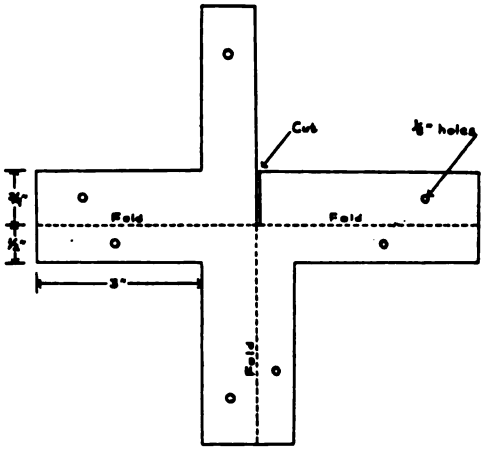

FIG. 1. Diagram of corner braces.

brads. To further strengthen the model, we made 12 "L" shaped aluminum brackets to fasten the newly added sticks to the edges of the cubic yard wherever they came together. In constructing the inside framework of the cubic yard we first formed two parallel planes with eight 37″ sticks—four for each plane. Then we fitted 12 one foot sticks into place, fastened them with brads and glue, and the outline of the 27 cubic feet was complete.

The most difficult part of our project was the construction of the 1728 cubic inches in one of the cubic feet. We considered using wire for this job but finally decided to use string. The strings dividing the surface of the cubic foot into 864 square inches were put in place first. A paper stapler was used to fasten the strings to the wood, and a dab of aeroplane cement was daubed on the vertices of all the little surface squares in order to keep their shape intact while the inner strings were put in place. Then, with the aid of a long needle fashioned from wire, we wove foot long strings between corresponding intersections on opposite sides of the cubic foot. In order to join the ends of loose strings to those that were already taut, aeroplane cement was again used to eliminate the need for tying knots. This part of the project took the longest time. After finishing the weaving we sprayed the net-

work of string with shellac to prevent it from shrinking and also to stiffen it.

Finally we painted the model orange and blue, our school colors. The cubic yard was painted orange, and the cubic feet framework, blue. The network of string forming the 1728 cubic inches, as previously stated, was dyed yellow. On April 27, the model was finished. A total of approximately 72 hours had been spent in making it.

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A TRIANGLE AREA DEVICE

The area device illustrated in Fig. 2 was developed by a tenth grade girl in plane geometry class. The reader will enjoy the unique little domestic touches which are part of its construction features.

To build the device first procure a piece of board 8″×14″. Sand one side of it, rule it into ½″ squares, and apply a coat of shellac so that the lines will not smear when the device is used.

Next dig the family button box out of the sewing basket and select five buttons—four small white pearl tailor buttons and one metal shank button. Obtain a thin stout piece of wire 14″ long and about 16″ of colored elastic. Pass both the wire and the elastic through the hole in the back side of the shank button, and bend each end of the wire into a small loop. The materials are now ready to be assembled.

Place the iron wire on the board so that it will coincide with the top horizontal ruled line. Slip a tailor button under each of the two wire loops, and secure both the wire and the buttons to the board with small household nails. The two tailor buttons must be between the board and the wire so that the metal shank button will have clearance when it is moved along the wire. Nail the two remaining tailor buttons loosely to the board on the same horizontal line near the bottom edge of the board. They should be about 6″ apart. Finally tie the two loose ends of the elastic together and slip the completed loop around and beneath the two bottom buttons.

By sliding the shank button along the iron wire any number of triangles with a common base and equal altitudes can be outlined with the elastic. The area of any one of them can be found approximately in terms of the square units marked off on the board by counting the number of squares

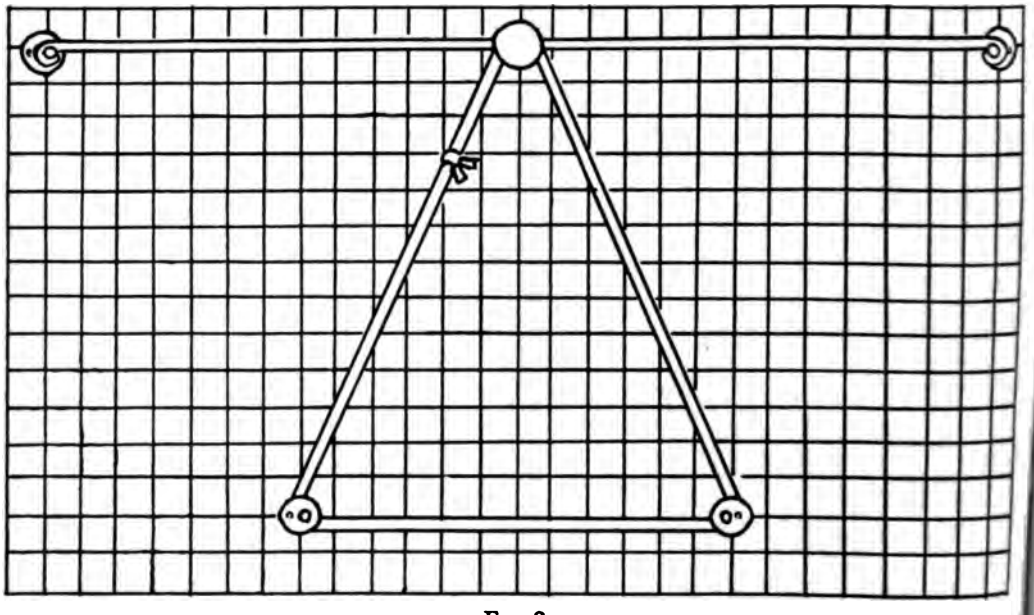

FIG. 2

included within its sides. When determining area in this manner every square should be counted as one full unit of area if more than half of it lies within the boundaries of the triangle. If more than half of a square lies outside the triangle it should not be counted at all.

By using this device students can discover informally that all triangles which can be formed with the device are equal, or what is the same thing, that all triangles with the same base and equal altitudes are equal. The device is of value because it provides a means by which students can arrive at a generalization. If the device is used in junior high school general mathematics classes this is usually sufficient. In the plane geometry class the device can be used to help students formulate the statement of the generalization which may then be proved deductively.

Another generalization which can be arrived at in an analogous manner through the use of the device may be stated as follows: The locus of the vertices of all the equal triangles that can be constructed on a common base is a line parallel to the base and at the distance of the altitude of any one of the triangles from the base.

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GRAPHING DEVICE

The device suggested here can be used in both elementary and second course algebra classes to help give meaning to linear equations in two variables. It's amazingly simple to produce—so simple in fact that every student in class can with a little direction devise one of his own. The description given in this article is based on a device produced by an eleventh grade second course algebra student.

Materials needed include one piece of plywood $\frac{3}{8}'' \times 15'' \times 15''$, two pieces of welding rod 15″ long, two wooden beads which are large enough to slide over the welding rods, four small blocks of soft wood, four metal staples, one piece of elastic 15″ long, twenty-six round key tags, and two thumb tacks.

To build the device first mark the plywood board into $\frac{1}{2}''$ squares with a heavy black pencil. Make the center vertical and horizontal lines more prominent than the others. If a really finished product is desired the plywood should first be painted white, then marked into squares, and the completed grid coated with shellac. Next fasten the ends of the elastic to the wooden beads; the elastic may be passed through the bead holes and knotted securely on the outside as is shown in the diagram. (See Fig. 3.) Place one of the little soft wooden blocks at each corner of the board, slip one bead over each of the welding rods and staple the rods down over the blocks along the vertical edges of the board. One admonition should be followed with respect to selecting the blocks; they must be thick enough so that the beads can slide freely along the welding rods without scraping against the board. To make the device useful with as many different equations as possible letter each of the twenty-six key tags with a different letter of the alphabet.

The device described above is probably best suited for use by individual students. A workable demonstration device can be constructed by increasing the dimensions proportionally and substituting curtain rods for the welding rods. For this larger device ordinary metal curtain rod wall brackets will work out much better in fastening the rods to the board than the wooden block and staple arrangement used with the welding rods. If the beads selected for the smaller device are inadequate for the demonstration device, Tinker Toy wheels with the center holes drilled larger will work out quite nicely as a substitute.

In elementary algebra classes the device can be used in a variety of situations involving relations between two variables. As soon as students have learned how to plot points, and are satisfied that two points determine a straight line, experimentation with a sequence of equations

like those listed below should help them understand the significance of the constant in those equations.

$$y = x$$
$$y = 2x$$
$$y = 4x$$
$$y = -2x$$

How much meaning students eventually attach to situations of that kind will of course depend on the skill of the teacher. By thumb-tacking appropriate letters on the two axes the teacher can assist students in making real transfer to practical situations. Consider the following:

$$C = \frac{22}{7} d$$

$$C = 2\left(\frac{22}{7}\right) r$$

$$I = .05t$$

$$P = 4s.$$

Once students discover the scheme it will be a challenge to their insight to ask them to write down relations for two variables for different positions of the elastic line.

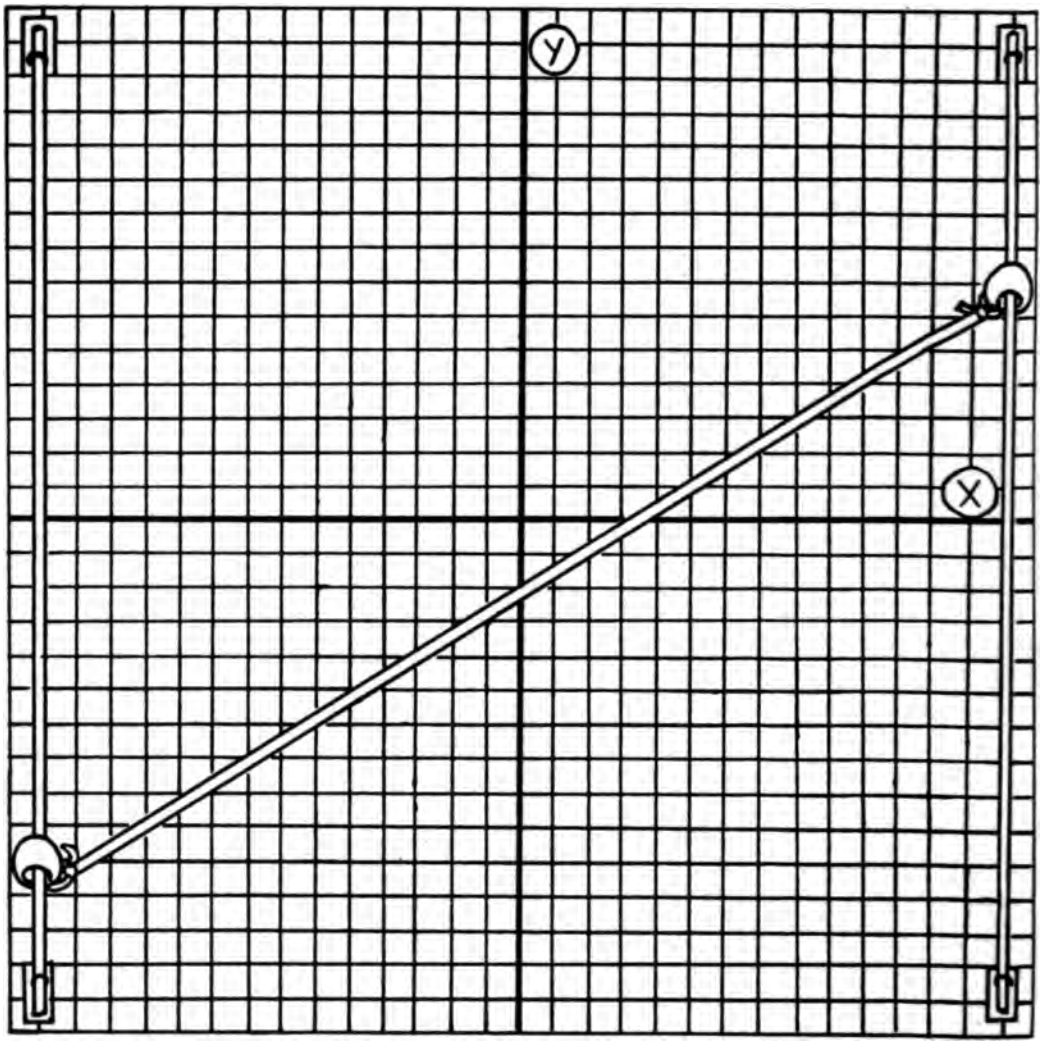

FIG. 3

Another suggested sequence of equations that should prove fruitful in terms of pupil understanding is the following:

$$x+y=5$$
$$2x+2y=8$$
$$2L+2W=6$$

$5n+p=10$ (nickels and pennies relation).

Consider now some of the possibilities that might be tried in a second course algebra class. The notion of what constitutes a solution of an equation in two variables can be made meaningful by experience with a variety of different lines. Such concepts as slope, intercept, and slope-intercept form can be made meaningful by a rapid consideration of a large variety of lines in different positions. In connection with the last named concept a nice application can be developed by considering the following relations:

$$y=9/5x+32$$
$$F=9/5C+32.$$

In order to handle problems which involve pairs of simultaneous, inconsistent, and dependent equations a second elastic line is needed. A convenient way of holding this second line in place is by using map pins or plastic handled bulletin board pins.

The illustrations given thus far are only suggestive of the uses which can be made of this device. Lack of space precludes further enumeration; however, a teacher who has the device in hand will undoubtedly be able to work out many uses by himself.

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COMMENT ON "AN ELLIPTICAL BILLIARD TABLE"

Some years ago, I built a device equivalent to that described on page 405 of the December, 1950 issue of THE MATHEMATICS TEACHER. I found that while it was fairly satisfactory it seemed to have some shortcomings. The main difficulty seemed to me to be associated with imperfect rebounding.

The "Elliptical Whispering Gallery" depends not only on the tangent property of the ellipse which insures that the sound is reflected from one focus to the other, but also on the well known constant sum of focal radii because of which the sound reaches the other focus by all paths simultaneously. The device mentioned does not seem to illustrate this second property except in connection with its manufacture.

A contrivance which I have found to be in some ways more satisfactory is constructed in an essentially similar manner except that the base is of transparent plastic and the joints are carefully sealed. This is then mounted as the top of a lightproof box with a small source of light at the center of the bottom of the box. If an inch or so of water is placed in the elliptical top, shadows of ripples on the surface of the water may be projected upon the ceiling of a darkened room.

If, after the water has become still, a pebble is dropped into it at one focus, clearly defined circular ripples spread from it and after reflection converge circularly to the other focus.

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Hofstra College
Hempstead, New York

Warning of "The grave shortage of scientists and engineers which we face in this country at the present time," Dr. James R. Killian, Jr., President of the Massachusetts Institute of Technology, stated that this shortage will become still more acute in the next few years. "Last year," Dr. Killian said, "the engineering schools of the country graduated about 50,000 young engineers. This, of course, was an exceptionally high number, reflecting the wave of G.I.'s which had been going through the colleges. Based upon present enrollments, estimates have been made which indicate that in 1954 we will turn out of our engineering schools from 12,000 to 16,000 engineering students. This compares to an output from our engineering schools just before World War II of some 20,000 engineers a year.

REFERENCES FOR MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

Mathematical Plays and Programs

IN THESE days of group activities and similar undertakings, teachers in search of material appropriate for their particular needs may find the following notes helpful.

A. PLAYS, SKITS AND PAGEANTS

"A Mathematics Playlet." MATHEMATICS TEACHER, 1945, 38: 309–313.
A costume play depicting the use of mathematics in business, science, music, art, etc.

Anning, Norman. "Socrates Teaches Mathematics." *School Science and Mathematics*, 1923, 23: 581–584.
A short play in one act; depicts the expansion of $(x+y)$ and $(x-y)$ by the use of blocks.

Brownell, Ella. "Mathesis." MATHEMATICS TEACHER, 1927, 20: 459–465.
Costumes and properties simple; incidental music; fifteen or more characters. Appropriate for junior high school.

Cohen, Dena. "Problem Play: a Musical Play in One Act and Three Scenes." MATHEMATICS TEACHER, 1936, 29: 78–83.
In which King Math of Numberland helps Alice to solve a problem in fractions; for junior high school. Detailed directions for costumes and scenery.

Cowley, Elizabeth. "A Mathematical Fantasy." *School Science and Mathematics*, 1933, 33: 535–536.
Brief skit (10 minutes) showing that numbers are indispensable in the modern world; fourteen players, no properties.

Crawford, Alma E. "A Little Journey to the Land of Mathematics." MATHEMATICS TEACHER, 1924, 17: 336–342.
A play in which two high school pupils get a vision of the value of mathematics in chemistry, drafting, bookkeeping, etc.; calls for twenty-six characters; suitable for ninth grade or junior high school.

Denbow, Carl. "Traders and Trappers." MATHEMATICS TEACHER, 1941, 34: 61–65.
A short play dealing with the principle of place value and the number concept in arithmetic; suitable for junior high school.

Fletcher, R. "A Quarter After Ten." *Texas Outlook*, 1931, 15: 31–35.
A mathematical play in one act.

Forrest, Louise. "Euclid, Agrarian Arbitrator." MATHEMATICS TEACHER, 1932, 25: 22–26.
Sketch based upon the concept of congruent triangles; requires four main characters, simple scenic properties. Time, about twenty minutes; for tenth year geometry classes.

Fort, Tomlinson. "Mechanics—A Dramatic Skit." *American Mathematical Monthly*, 1931, 38: 42–46.
Sophisticated skit depicting the relationships between mathematics and modern physical science; nine characters; no properties. For college students.

Gowan, J. C. "Mathematics Skit." MATHEMATICS TEACHER, 1943, 36: 169–170.
A very short play showing how mathematics is used in everyday life.

Graff, Margaret. "Euclid Dramatized." *School Science and Mathematics*, 1921, 21: 381–382.
A brief dramatization of some of the stories of early mathematicians.

Hall, Alton L. *The New Assistant*. The Metric Association, 1938. (10¢+postage.)
Deals with the spread of the metric system throughout the world; about fifteen characters, ten speaking parts. Pamphlet may be obtained from J. T. Johnson, 177 N. Grove Ave., Oak Park, Ill.

Harding, P. H., *et al.* "A Mathematical Victory." *School Science and Mathematics*, 1917, 17: 475–482.
A play in two acts for more mature students.

Hatton, Caroline and Smith, Doris. "Falling in Love with Plain Geometry." MATHEMATICS TEACHER, 1927, 20: 389–402.
A comedy; list of properties and directions for dances and musical numbers; sixteen characters. Excellent for senior high school.

Karapetoff, Vladimir. "Agha and Math." *Scripta Mathematica*, 1946, 12: 153–159.
A fantasy on "the way the logarithms might have been discovered even though they weren't." Readily adapted for dialogue, with a narrator. For high school and college.

Leps, J. M. "The Radical Dream: A Playlet for Puppets." *School Science and Mathematics*, 1933, 33: 279–287.
A play in two acts for algebra students; eleven characters.

Leskow, Olive. "The Cubic." MATHEMATICS TEACHER, 1943, 36: 312–316.

Depicts the contributions of Cardan, Tartaglia and Ferrari; eight characters.

Lloyd, Daniel. "In Quest of Truth." *School Science and Mathematics*, 1951, 51: 275–285.

A historical pageant, suitable for high school, college, or adult audiences. In two parts; playing time for each part, 20 minutes. Part I, suitable for junior high school, can be given independently.

McSorley, Kathryn. "Mock Trial of B versus A." *School Science and Mathematics*, 1918, 18: 611–621.

A clever dramatization based upon Stephen Leacock's celebrated story "A, B, C."

Meals, Louise. "X on the Spot." MATHEMATICS TEACHER, 1939, 32: 296–300.

A short play on the practical values of learning mathematics; six characters, minimum properties. Junior or senior high school.

Millay, Kathleen, and Free, Lucile. "Flatlanders." *American Mathematical Monthly*, 1919, 26: 264–267.

A brief play for four players; simple properties. Based on Abbott's "Flatland." See also *School Science and Mathematics*, 1914, 14: 583–587, as well as *The Mathematical Gazette*, 1914, 7: 228–231.

Miller, Florence B. "An Idea That Paid." MATHEMATICS TEACHER, 1932, 25: 470–479.

A play in three acts dealing with the subjects of profit and loss, partnerships, corporations, and stocks.

Miller, Florence B. "A Near Tragedy." MATHEMATICS TEACHER, 1929, 22: 472–481.

A play in three scenes showing the effects of having no mathematics; suitable for junior high school.

Miller, Florence B. "Out of the Past." MATHEMATICS TEACHER, 1937, 30: 366–370.

A two-act play written by a ninth-grade class, dealing with the development of our number system and the use of mechanical devices for computation.

Novinger, F. F., *et al.* "I had the Craziest Dream." MATHEMATICS TEACHER, 1947, 40: 20–23.

A play in three scenes dealing with practical applications of mathematics; fifteen characters, for junior high school.

Parkyn, Hannah. "When Wishes Come True." MATHEMATICS TEACHER, 1939, 32: 16–24.

A play about algebra and geometry; thirty-three players; detailed directions for costumes; for senior high schools.

Paterson, Edith. "Everyman's Visit to the Land of the Mathematicians." MATHEMATICS TEACHER, 1938, 31: 7–18.

Fairly elaborate in scope, dealing with mathematics from ancient to modern times. Includes notes on scenery, properties and costumes.

Pitcher, Wilimina. "Alice in Dozenland." MATHEMATICS TEACHER, 1934, 27: 390–396.

A play suggesting the nature of the duodecimal system; three scenes, five characters, exceedingly simple properties.

Pitcher, Wilimina. "The Mathematics Club Meets." MATHEMATICS TEACHER, 1931, 24: 197–207.

A historical play on methods of computation among the Egyptians, Romans, Italians; Russian peasant method of multiplication; sixteen players; modest settings and costumes; for senior high schools.

Potwine, E. B. "Bobby Learns His Numerals." MATHEMATICS TEACHER, 1943, 36: 29–33.

Depicts the history of numbers; eleven characters; presentation time, twenty minutes; calls for preparation of fifteen fairly elaborate charts. Suitable for junior/senior high school.

Raftery, Gerald. "The Eternal Triangle." MATHEMATICS TEACHER, 1933, 26: 85–92.

Simple playlet dealing with indirect measurement; three characters. Suitable for junior high school or ninth grade.

Reid, Noma. "Archimedes: A Mathematical Genius." *School Science and Mathematics*, 1941, 41: 211–219.

Outline of an interesting costume play describing some of the contributions of Archimedes to science and mathematics. Lines to be supplied by the players.

Russell, Celia. "More Than One Mystery." MATHEMATICS TEACHER, 1933, 26: 477–481.

Light skit, based on puzzle problems; seven players, virtually no properties; songs and dances. Appropriate for junior high school.

Schlauch, Helen. "Point-College." *School Science and Mathematics*, 1931, 31: 448–454.

A collegiate mathematical farce concerning Analytics College, in which the student body is composed of animated points; fifteen players.

Schlierholz, Tillie. "A Number Play in Three Acts." MATHEMATICS TEACHER, 1924, 17: 154–169.

A dramatization of Prof. D. E. Smith's *Number Stories of Long Ago*. Suitable for junior high school.

Scott, Emma. "Geometry Humanized." MATHEMATICS TEACHER, 1928, 21: 92–102.

A play for eighteen characters; suitable for geometry classes. Dialogue clever and brisk; properties and costumes simple but effective.

Shea, John. "Mr. Chips Teaches Geometry." *School Science and Mathematics*, 1940, 40: 720–726.

Short humorous skit on locus concept; nine characters, no properties.

Skerrett, Josephine. "A Mathematical Nightmare." MATHEMATICS TEACHER, 1929, 22: 413–417.

A play written by a high school pupil, showing what would happen if there were no mathematics; ten players; for junior high school.

Slaught, H. E. "The Evolution of Numbers: An Historical Drama in Two Acts." MATHEMATICS TEACHER, 1928, 21: 305–315; *American Mathematical Monthly*, 1928, 35: 146–151.

Play based on the history of the number

system of algebra; instructive as well as entertaining. Calls for an audience of some mathematical maturity.

Smith, Alice. "The Case of Matthew Mattix." MATHEMATICS TEACHER, 1933, 26: 286–291.

A comedy in two scenes; requires seventeen speaking parts.

Smith, Alice. "Snow White and the Seven Dwarves." MATHEMATICS TEACHER, 1944, 37: 27–30.

A light skit in three short acts; calls for eleven players. Appropriate for junior or senior high school.

Snyder, Ruth. "If." MATHEMATICS TEACHER, 1929, 22: 482–486.

A play in two acts showing the importance of mathematics; nine players.

Sperks, Marie. "The Determination of Taxes in the Community." *School Science and Mathematics*, 1942, 42: 454–462.

Can be tied in readily with a unit on taxation and with related projects. Requires ten speaking parts; suitable for junior high school.

Stark, Marion. "Modern Mathematics Looks Up Its Ancestors." *American Mathematical Monthly*, 1936, 43: 299–304.

Costume play on the history of mathematics; blank verse; ten characters. Stimulating as well as instructive.

Ward, J. A. "A Wax Works Show." MATHEMATICS TEACHER, 1941, 34: 266–268.

A brief comedy skit on Newton, Einstein and Archimedes.

Whelan, A. Marie. "It Can't Happen Here." *American Mathematical Monthly*, 1938, 45: 617–628.

A clever and amusing musical farce, based on the theme: Why study mathematics in college? In two acts and five scenes; six principal players, and chorus.

Whitaker, Helen. "The Mathematical Quest." *School Science and Mathematics*, 1920, 20: 457–459; MATHEMATICS TEACHER, 1925, 18: 356–358.

A short mathematics pageant, in verse; one act. Requires fifty characters, but only two speaking parts. Suitable for senior high school.

Woodard, Mary. "A Study in Human Stupidity." MATHEMATICS TEACHER, 1945, 38: 362–367.

A biographical play on the life of Evariste Galois; short, but compelling; musical accompaniment.

Woolery, Freda. "Much Ado About Mathematics." MATHEMATICS TEACHER, 1945, 38: 23–35.

Portrays the life of a person who has been forbidden to use mathematics; five scenes and an epilogue. Suitable for junior school.

B. PROGRAMS FOR ASSEMBLIES AND MATHEMATICS CLUBS

Adler, Irving. "Fun with Mathematic Assembly Program." MATHEMATICS T ER, 1949, 42: 153–155.

Adler, Irving. "Theory and Practice." M MATICS TEACHER, 1948, 41: 218–220.

Bogen, Isidore. "Mathematics in Life." *Points*, 1949, 31: 73–79.

Braverman, Benjamin. "The Quiz in a M matics Assembly Program." *High* 1947, 29: 64–69.

Fisher, Genevieve. "A Mathematical Ass Program." *Bulletin, Kansas Associat Mathematics Teachers*, 1945, 20: 5–7.

Gulden, M. "Mathematics Club Prog MATHEMATICS TEACHER, 1924, 17: 350

Hatcher, Frances. "A Living Theorem; a Day Program." *School Science and matics*, 1916, 16: 39–40.

Hoag, R. "Sources of Program Materi Some Types of Program Work Which Be Undertaken by High School Mathe Clubs." MATHEMATICS TEACHER, 193 492–502.

Klotz, C. E. "Mathematics Clubs for Schools." *School Activities*, 1949, 21: 59

Lehmann, Paul. "A Math Circus; an As Program." *School Science and Mathe* 1947, 47: 560–563.

Parsons, G. "Work of a Junior Mathe Association." *Mathematical Gazette*, 19 65–72.

Person, R. "Junior High School Mathe Clubs." MATHEMATICS TEACHER, 19 228–229.

Phillips, K. "Junior High School Mathe Club." *High School Journal*, 1930, 13:

Porterfield, Jacob. "Fun for the Mathe Club." MATHEMATICS TEACHER, 19 354–357.

Reed, Z. "High School Mathematics (MATHEMATICS TEACHER, 1925, 18: 34

Sullivan, O. A. "The High School Mathe Club." MATHEMATICS TEACHER, 19 275–276.

"Thirty Topics for a Mathematics Pro *School Science and Mathematics*, 19 170–171.

Vaughn, Adah. "Professor Whiz and Hi in Math. Magic." *School Science and matics*, 1939, 39: 540–545.

Weiss, M. "Math Club—Streamlined. *Points*, 1939, 21: 74–77.

Wheeler, A. "Mathematics Club Pro MATHEMATICS TEACHER, 1923, 16: 38

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THE MATHEMATICS TEACHER

Volume XLIV Number 8

The Teaching of Mathematics in the Argentine Republic

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THIS ARTICLE is addressed to the North merican colleagues who feel curious to ow what, how, and to whom mathemat- s is taught in the Argentine Republic.

Last year we visited their hospitable untry with the same curiosity, attend- g more than fifty classes in high schools, lleges and universities, to see with our n eyes how this subject was taught after ar that had clearly shown the necessity every citizen to know more mathemat- s. With this experience, and a descrip- n of the aims and characteristics of our athematics curriculum, we wish to show me differences existing in the teaching of at subject in these two American coun- ies.

WHAT DO WE TEACH IN OUR ELEMENTARY SCHOOLS?

Although education is divided into ele- entary, secondary, and university levels, e same as in the United States, the ele- entary teaching is the only one obliga- ry in Argentina for girls and boys 6 to years of age. It has a duration of seven ars and is directed towards giving chil- en that basic knowledge which is indis- pensable for practical life. We shall limit our writing to the mathematics program.

In the elementary schools, mathematics is taught daily during the seven years. The following subjects are developed:

(a) Oral and written numeration, and the four fundamental operations with whole numbers, decimal and common fractions.
(b) The metric system of weights and measures.
(c) Solving problems with the contents of (a) and (b).
(d) Simple and compound rule of three.
(e) Problems of interest, discount and proportion.
(f) A knowledge of plane and solid figures; drawing of perpendicular and parallel lines with ruler, square and compasses. Construction of triangles, parallelograms, regular polygons, etc.
(g) Calculation of the area of a triangle, parallelogram, trapezoid, regular polygons, etc.
(h) Length of the circumference, and area of the circle.

(i) Area and volume of such solids as the cube, right parallelepiped, prism, cylinder, pyramid, cone and sphere.

The teaching of these topics is presented intuitively, avoiding formal definitions and proofs, but having the pupils solve the problems in a reasonable way. The totality of this mathematics knowledge is similar to what we believe is known in the United States as an elementary social or general mathematics.

Some Characteristics of Our Teaching

Teaching in our country has tended, little by little, to become uniform for all schools. The national government permits, in an absolutely free character, the establishment of secondary and university teaching throughout the country. The provinces (which are the divisions corresponding to the states of the Union) finance a program of free elementary education in their own territories. Beyond this, the national government supports the schools of the federal capital (Buenos Aires City), the national territories, as well as a great number of Schools of Campaign established in the provinces.

As a result of this national support, the elementary teaching plans are practically the same in the whole Republic. Provincial and private programs agree with those of national schools, principally because it is required that the last course of the elementary school must agree with the national official program in order that pupils be able to enter the secondary schools.

In the secondary schools the uniformity is now absolute, because the private secondary schools must adjust themselves to the official program in which teachers check their pupils by national examinations. This rigid system also differs from the North American system in that all students in each program of studies have to pursue the same subjects to get their diplomas. The system of credits does not exist in Argentina. The application of this uniform criterion for graduation is supported by the following arguments:

(a) It is desired that all the students have the same minimum of preparation.
(b) The student generally is not capable of making an intelligent choice of subjects, the content of which he does not know.
(c) The principal aim of the secondary teaching is to educate, and a student cannot escape prescribed study merely for reasons of dislike or antipathy developed in initial study, or for simple prejudices. (We believe that the second world war proved, in the United States, that the soldiers and industrial workers must know more mathematics than that which had been taught to them.)

Argentina's culture, small or large, is of European roots (especially French and Italian), and in these countries mathematics has always been one of the classical foundations of all levels of education. Our secondary schools aspire to give a general knowledge that allows the young students to continue university study, if they wish to, but not without some knowledge of the elementary notions of the other subjects that contribute to general culture.

We have had the satisfaction of noting that in all the universities we visited, including Northwestern, Harvard, Columbia, Chicago and even the Massachusetts Institute of Technology, which is predominantly technological, the teaching of mathematics had a cultural nature, and many courses of abstract mathematics were offered. The compulsory courses in "foundations of mathematics" for all students of the University of Chicago is another comforting example that mathematics has a classic place in the education of youth.

What Do We Teach in the Secondary Schools?

There are four fundamental types of secondary schools. They prepare:

(a) Graduates with high school diploma ("bachilleres").
(b) Teachers for elementary schools (graduated in normal schools).
(c) Commercial experts (graduated in commercial schools).
(d) Industrial technologists.

The "bachilleres" attend schools called "Colegios Nacionales" (literary national colleges) corresponding to an American high school for boys, and "Liceos de Señoritas" (Lyceum for girls). These schools have two divisions, basic and superior (something like junior and senior) of three and two years respectively. In our country, with some few exceptions, pupils of each secondary school are of the same sex.

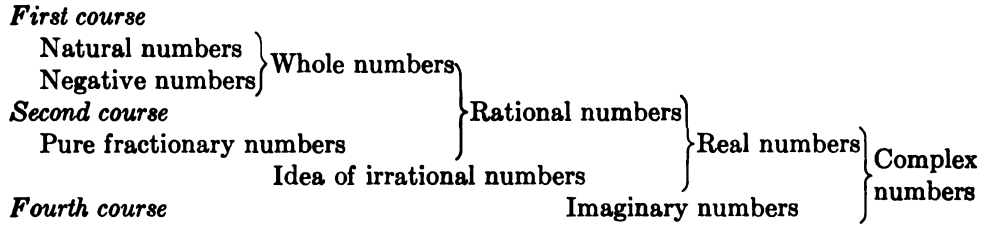
In all five years of secondary school, mathematics study is compulsory for all students. There must be at least four periods of 45 minutes each week during the whole scholastic year which is from thirty to thirty-two weeks, each week having six school days. For national colleges and lyceums, the study is distributed in the following way:

First year: Arithmetic, first part; plane geometry, first part.
Second year: Arithmetic, second part; plane geometry, second part.
Third year: Algebra, first part; plane geometry, third part.
Fourth year: Algebra, second part; solid geometry.
Fifth year: Plane and spherical trigonometry; cosmography.

Arithmetic and geometry are always taught by the same teacher in each course in alternative classes.

Since 1926 the mathematics programs follow the logical development shown below, assuming those mathematical properties for which a logical proof is too difficult for the students to understand. Program instructions say: "The teacher can omit a demonstration rather than give a deficient proof."

Here is the logical order of the arithmetic program.



Some details concerning the first course arithmetic program for the national colleges and lyceums are:

(a) It is assumed that the students know the set of natural numbers, and have an idea of the decimal system of notation. (From the beginning, the literal representation of numbers is used to teach by symbols the operations with numbers and their properties.) Graphical representation and the Roman system of notation are also taught.
(b) Addition of natural numbers; the symbolical representation, and examples of its properties.
(c) Subtraction of natural numbers; the symbolical representation and its properties. Proof of the law of monotony is given in order to initiate the students to proofs by "reductio ad absurdum." (Indirect method of proof.)
(d) Algebraic sums (combined additions and subtraction of natural

numbers). Use and removal of parentheses. Transposition of terms in an equation.

(e) Multiplication of natural numbers: definitions and properties. Here for the first time examples of the direct method of proof are given in establishing the distributive law.

(f) Division of natural numbers: definitions and properties. Some proofs applied to the study of subtraction.

(g) Transposition of factors and divisors in an equality.

(h) Powers of natural numbers: definitions and properties. (The students' only previous knowledge of this subject has been with square and cube.)

(i) Square root: definitions, examples and rules to extract the square root of a number greater than 100.

(j) Prime and compound numbers. H.C.F. and L.C.M. Rules to find them mentally or by using factors.

(k) Negative numbers: the necessity of their creation in order to make the subtraction of natural numbers always possible. Practical examples (profit, loss, etc.). One whole number is equal to, less than, or greater than another.

(l) Addition, subtraction, multiplication, division, powers and roots of whole numbers.

In Argentina the teaching of geometry is inspired by the works and experiences of the Italian geometers of the first two decades of this century, and especially those of the great geometer Federigo Enriques. We believe that our programs of geometry, which have been used with successive revisions since 1926, are a real success from the points of view of content and of mathematical rigor, both considered with respect to the age of the students, and teaching principles. In this program students quickly form their geometric vocabulary and learn to employ a simple and exact language. By simple experiences they discover the properties of the figures. Then the students are guided by the teacher to the discovery of the rigorous proof of these properties.

We have always found that students learn geometry easily, because it agrees with their spatial intuition (even in solid geometry), while in the United States there is reluctance to teach geometry, because, as we heard it said very often, it is beyond the level of the mentality of a great percentage of pupils. It is very simple, as everybody knows, to make the students discover properties of plane figures by measuring, folding paper models, superposing cardboard figures, etc., and properties of solids by observing relations between walls and edges in the classroom. This opinion on learning geometry is supported by our experience of twenty-eight years of teaching, and it depends, perhaps, on the way we teach, that is, recurring to all previous experiences, permitting the student to discover properties by himself, and then guiding him, with opportune questions, to the proof of these properties.

We never permit a student to study or learn a lesson for the first time by the use of the textbook. First the teacher teaches the lesson to him in the classroom. At the next class meeting there is collaborate activity on the lesson by both the teacher and the students. It is in this second lesson that the pupil must review, preparing to show his learning on the blackboard. During this review the pupil may use his textbook.

How Do We Teach?

The method of teaching mathematics in the secondary schools is optional with each teacher. However, as a rule it is demanded that the good teacher must teach in each class with the active participation of all students, the lesson for the next class. It is assumed that the majority of students have not reached a state of learning where they can assimilate, by themselves, the lesson in the textbook. Even for solving exercises and problems assigned for homework,

the teacher must teach, during the class, with the active participation of all the students, one or two examples which will serve as a guide for doing his homework. For young pupils who must learn five different lessons every day, there is a real need for this method of teaching.

In our visit to the United States, three quite common practices attracted our attention. The classes always began by giving the assignment for the next class; the students were permitted to have their textbooks open during the class; there was an excessive use of tests. We stress that, in Argentina, the lesson for the next class is that one learned in the former class.

It is not obligatory for a student to have a textbook. If he has one he is forbidden to keep it open on his bench, because his duty is to go to school knowing his lesson and with the homework done in his copybook. A teacher permitting open books in his classes would be criticized even by his own students. According to our customs the student can only bring exercises written in a paper or a special book of exercises.

Periodically we give written examinations and in these cases it is prescribed that the questions must be on past lessons that are quoted in the book of topics that each professor has written during his former classes. In these examinations the teacher can also propose exercises resembling those solved in class, or a theorem that the pupil must demonstrate with all the details of a perfect mathematical proof.

What are the methods used? That depends, generally, on the schools from where the teachers have graduated.

According to official regulations for certifying teachers in the secondary schools, mathematics can be taught by graduates from the mathematics department of the National Institute of the Secondary Professorship, graduates in science in a normal school for teachers, and graduates in faculties of sciences.

We shall report only on the methods prescribed by the National Institute of Secondary Professorship, because we are among its graduates and we think it has given best results. The first Institute was created in Buenos Aires City in 1904, and its direction and teaching was confined to professors who had specialized in each specific area through study in Germany. The Institute of the City of Buenos Aires, actually consists of 12 departments, one of which is the mathematics department. It has a professor of mathematical methodology and teaching practice, whose duty it is to develop "the art of teaching." The method recommended, which we have always practiced during our teaching career, and which we recommend to our students in the normal school for teachers "Mariano Acosta," is the "heuristic" or "genetic" or the rediscovering by the pupils.

A classic work of methodology in Argentina is "The Teaching of Mathematics in the Elementary and Secondary Schools" by Professor J. W. A. Young, former Emeritus Professor of the University of Chicago. We agree with this distinguished teacher, who visited Europe in 1900 to study the teaching in the secondary schools there, that the heuristic method is the best, not to say the unique one that should be employed in teaching mathematics in the secondary schools.

In fact, if the statements of mathematics propositions are never imposed, nor previously studied in a textbook, the students' experience in observing appropriate examples and abstracting through answers to a chain of questions awakens confidence in themselves and a tendency to correct thought.

This method gives the teacher a daily check of the interest and intelligence of each of his students, and, what is more important, tests his efficiency as a teacher. This method also enables the teacher to fill in the blanks in the preparation of each pupil, putting the whole class at the same level in a short time.

We had the pleasure of observing very good classes taught in this way in two high

schools of science for bright pupils in New York City. We appreciated the activity and gaiety of these classes, in contrast to the silent classes where tests were given or exercises were solved or discussed.

The National Institute of the Secondary Professorship was initiated by foreign teachers, and the heuristic method was established. In 1926 the official program was revised in accordance with the inspiration of the teaching of mathematics in Europe, and gives the subjects to be taught in the secondary schools of the Argentine Republic. This revision met with great success, thanks to the capacity and devoted service of a distinguished Argentinian teacher, Professor Florencio D. Jaime, teacher of mathematics methodology and projective and descriptive geometry in the National Institute of the Secondary Professorship, author of the mathematics program of 1926, and retired as General Inspector of Secondary Teaching a few years ago.

Under his direction, the heuristic method, which until this time had been taught in the Institute in a formal manner, was improved because Professor Jaime insisted that the teacher must not employ one method exclusively to instruct his students. Following his teaching, we now distribute the 45 minutes of a class in the following way:

(a) 10 minutes to review the former lesson.
(b) 25 or 30 minutes to teach the new theme.
(c) 5 or 10 minutes to a general review of this new theme.

We now employ the heuristic method only to teach the new topic, in which there is developed the statement of definitions, the discovery and demonstration of a theorem, and the solutions of exercises or problems.

Under the heuristic method the student will learn and express his knowledge only with the teacher as a guide. This deficiency is now corrected by the use of the expository method in part (a). That is, in order to get the student to express himself, to put order into his ideas, and to give him the chance to become qualified, we now devote the first 10 minutes in all classes to a review of the theme of the last class, which, as we said before, is the obligatory lesson.

As a student recites, the other students of the class are judging the quality of his recitation, and correcting him, under the teacher's direction, if that student makes mistakes. In such a situation the teacher will by his own teaching demonstrate the art of giving a good lesson, demanding accurate drawings, judiciously employing the blackboard, correcting the mistakes of language and of logic if the other students have not perceived it, and he will ask all the necessary questions to determine that the student really understands the material about which he is speaking.

At other times, the teacher also employs the recitation to recall the several properties learned in former courses, especially at the end of the scholastic year when a general review of the course is made before the final examinations. When a student persists in an incorrect conclusion, supporting his deductions with drawings, (as for instance in a triangle that looks isosceles when there is no reason for it to be isosceles) the socratic method can be used to help the student discover his error.

It is thus evident that the group of teachers graduated from the National Institute of the Secondary Professorship, use different methods than those we observed in many classes in the United States.

Through daily employment of this method of collaboration of pupil and teacher, which Professor Young refers to as "talk and chalk," we have developed a high percentage of good pupils in mathematics; an absolute control upon each of them; a natural discipline; and no loss of time or coverage of the program.

But what is the role played by the textbook? Merely to have clearly printed that which is developed by the teacher on the blackboard, and to give sets of exercises,

r that the students may prepare sson and homework after a first ion received in class. Some teach- er not to use textbooks, making lents take notes. But these notes ierally incomplete and defective, teacher must correct them, and the class the student usually is reoccupied in taking notes than in ig the theme of the lesson.

Whom Do We Teach?

each all students, girls and boys, of or more, who have passed the seven of the elementary school, and, in n, the entrance examination to the ary school. This examination was ced in 1936 in order to solve, in a atic and equitable way, the prob- t arises when the number of candi- greater than the vacancies in the

questions on these examinations the subjects of Spanish (our official ge) and mathematics, considered arned in the previous training in an tary school. We do not try to meas- ir aptitude, because in Argentina sumed that any student who has the elementary school work is pre- to follow the secondary program. l we arrived in the United States not thought of the problem of the students, that is, those who do not he capacity to learn mathematics, lly solid geometry. We had not t of it because we do not face this n in Argentina.

lways find some "bright" students classes, and the heuristic method forth many "mathematics inven- For us, "bright" students are those tuit, infer, and retain easily, who methods to solve problems and give strations. Even though sometimes ve or even false, the inventions re- ie students' creative urge and their lity.

do not say that we have no "bad" ts who very often do not know their , but generally it is the students' fault, because they do not study at home, because they do not solve exercises, or plainly because they do not review their lessons, and do not pay attention in class.

Each class has students who are not well trained in former courses or in the elementary school, other students who have forgotten nearly all they were taught, and even some students who have failed in mathematics in former courses. These students interfere with the purpose of carrying on the course in the manner described before, but the heuristic method allows the teacher to pay more attention to these students, answering their questions more frequently than those of the others in the class.

The parents are also requested to watch over the students during the study hours at home, and very often these deficient students make remarkable progress. In Argentina the student spends more time with his parents than in the United States. Because classes are held in a single session they have lunch at home, and they return to school for the other session only twice a week for a couple of hours, for physical culture or handwork.

The secondary teaching is not compulsory. It is directed to the average ability of students. The boys and girls without interest or of a very low level of intelligence quickly fall behind the others and they end by dropping their studies.

The "bright" students usually jump over intervening courses by taking special examinations, written and oral, as "free students." The examinations are given a short time before the beginning and after the end of the scholastic year. These examinations can also serve as make-up tests for students who failed, and for those with incomplete study due to illness or other reasons.

Summary

In short we can say:

1. Many of those who teach mathematics in Argentina are specialized teachers for the teaching in secondary schools.
2. We teach a new subject by the heu-

ristic method, but we also use the recitation, examination, and socratic methods when necessary.

3. In the scholastic curriculum, mathematics occupies the position of a formative subject, serving principally as a model of logical and deductive reasoning, without neglecting the practical uses in daily life.

4. We always teach arithmetic and geometry, algebra and geometry, or trigonometry and cosmography in the same course.

5. The teaching of arithmetic is done in a genetic way: natural numbers, negative numbers, whole numbers, rational, irrational, real and complex numbers using literal representation from the very start.

6. The instruction in geometry follows the Italian school.

7. After the second or third month of the class in the first course, we begin to give simple but rigorous demonstrations in arithmetic and geometry, making the pupils learn the reasons of each step and know how to express themselves properly on the blackboard or in written lessons.

8. Solid geometry, which is taught in the fourth course for the bachelor's degree and the school teachers' degree, and in the third course of commercial school, never gives us trouble.

9. Teaching in secondary schools is not obligatory; following the bachelorship those pupils who desire, may go to the university, others may wish to have only the general culture secured through the instruction.

10. The secondary teaching is uniform in all the territory of the Argentine Republic and it is supported directly by the national government.

11. The student cannot only choose the type of school he wishes to attend, but he can also transfer to others by taking complementary examinations.

12. We do not give special classes for discussion because discussion comes naturally from the teacher's questions and from the students' answers, doubts, or expressed opinions.

13. General mathematics education is implicitly contained in the elementary school, and especially in the arithmetic program of the second course, where the calculations of areas and volumes are applied to solve problems of daily life.

14. With the study of the cosmography, the pupil has the opportunity to apply his knowledge of mathematics, physics, and chemistry learned in former courses.

15. The application of the heuristic method gives the pupil a feeling of elaboration of his own knowledge, and the pleasure of knowing that he really learns at school even without a textbook.

These are the main points of view regarding instruction in mathematics in Argentina, and they are in some cases different from those we observed in our short visit to the United States. The visit of North American teachers to Argentina during their summer vacations, which coincides with the season in which our programs are in full operation, would provide an opportunity to inform opinions about our defects and virtues, and it would contribute to the formation of bonds among the members of the teaching family of both Americas, which until today have long been missing.

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Concerning the Teaching of Second Track Mathematics

A Report Prepared by the Standing Committee on Mathematics of New York City*

I. Introduction

The problem of adapting our courses in mathematics to groups of varied abilities, interests, and needs has been with us for a long time. Our most recent attempt in this area has been the experiment on the Ninth Year Second Track Mathematics.

The report of the Committee on Ninth Year Second Track Mathematics was issued in June 1948. This report reviewed and stated the problem, discussed the question of the selection of the students for the course, analyzed the characteristics of the students, made some general suggestions on methods of teaching, outlined the aims and objectives of the course, and presented a tentative order of topics (with time allotments) to be covered in 28 weeks.

A workshop of teachers from the Junior High Schools, the Senior High Schools, and the Vocational High Schools then worked on this report during the Fall 1948 term. The workshop produced an exceptionally valuable set of "Teacher Notes" to accompany the syllabus contained in the original report. Objectives, procedures, sample exercises and suggested projects for individual units of work, plus an additional section on mathematical recreations, comprise these "Teacher Notes."

The Spring 1949 term was used as a try-out period for the Second Track course. Eight Junior High Schools, ten Senior High Schools and four Vocational High Schools taught the syllabus to approximately 4,000 pupils. Additional instructional materials were prepared and issued to the teachers by Mr. Harry Sitomer, who was assigned to coordinate the work of all the schools. This syllabus will now be introduced generally in all the schools.

The purpose of this report is to underline for all teachers of mathematics some general questions of approach and attitude toward the whole problem, as well as to list a number of specific and concrete suggestions on classroom procedure.

II. Characteristics of the Second Track Pupil

The guiding philosophy for second track mathematics, the formulation of aims and objectives for this course, the selection of subject matter, and the methods of teaching must all be determined by the type of pupil for whom a second track course is necessary.

It will be found that these pupils are frightened and inhibited by the ordinary aspects of mathematics. They have had, in many instances, rather unhappy careers in school. The problem for the teacher is to rebuild the self-respect of these pupils, to imbue them with more favorable attitudes toward school and to convince them of the worthwhileness and practical value of mathematics.

While in many respects the slow learners are much like the average or superior groups, the qualities which really differentiate them have to do with the higher mental processes. The slow pupil, as a rule, is weak in forming associations between words and ideas; he cannot generalize as readily as the average or superior pupil; his memory is relatively poor; he has little aptitude for dealing with abstract

* The members of the committee were: Irving Cohen, Antonia Higginson, Julius H. Hlavaty, Michael M. Katzoff, Daniel Malament, Maurice Nadler, Harry Sitomer, Alan Wayne, Catherine Wilson, and Theresa Molloy, Chairman.

material and learns most readily through a concrete inductive presentation.

Slow pupils have a shorter attention span than normal pupils; they cannot concentrate so long on a single topic. Their interest may be quite as high in the initial presentation of a topic but may wane almost at once. While they need many exposures to a mathematical principle for any kind of mastery, they react unfavorably to routine repetition and ordinary drill. Since they do not make generalizations as readily as the average or superior groups, there is, consequently, less transfer to related or similar fields. These pupils must therefore be given a direct appreciation of the role of mathematics in understanding, controlling, and improving our environment. Finally, they, perhaps more than any of the others, need to acquire a sense of accomplishment and to gain the satisfactions of success.

III. The Elements of a Guiding Philosophy

The problem of the increasing number of failing pupils in mathematics has been with us since the high schools were opened to all the youth. Two major attempts to solve the problem have been made. The first constituted a denial of the existence of the problem with the statement to the pupil: "If you can't succeed in the mathematics we offer, then elect some other subject." The second consists of "watering down" the curriculum with the hope that the removal of syllabus difficulties will enable some of these pupils to receive passing grades. In this attempt, parallel courses in plane geometry have been prepared for students who are not expected to take the Regents' examinations.

The first attempt to solve the problem disregards our obligation as teachers to prepare youth for life in a technical society in which the applications of science and mathematics are assuming larger and more significant proportions, and in which a citizen is expected to make sound civic judgments based on a knowledge of facts—a knowledge which in part is given by science and mathematics. To disregard the need for a familiarity with the methods, concepts, and contributions of mathematics, is to handicap any individual in his role as a citizen and to permit the perpetuation of ignorance in a large section of the population. No democracy can evolve to its destiny as a nation of free and secure individuals, while a large section of the population is doomed to a second class education.

The second attempt also fails. This may be seen in terms of objectives and needs. These needs may be summarized as follows: the revival of a lost interest in mathematics; the removing of a depressing sense of failure; the restoration of self-respect; the acqusition of the fundamental concepts of mathematics which are necessary for the adequate functioning of an individual of our democratic society—and the acquisition of the concomitant arithmetic skills. How does a watered down syllabus meet these needs? The history of our present (Regents) syllabuses indicates that they were made to prepare students for college studies in the sciences and mathematics. Thus, problem-solving techniques are emphasized. Topics which are vital for college mathematics receive extended treatment on a technical level, e.g. equations, graphs, formulas, irrationals. We also find such topics as complex fractions, logarithms, the binomial theorem, and loci—all helpful in preparing the student for the calculus. In geometry, tradition has protected such topics as geometric constructions, area transformations, some phases of angle measurement, and a long list of theorems which seem to point to no obvious goal. What happens, say in geometry, when we "water down" the syllabus? We omit the difficult theorems, but continue to demand that pupils solve problems which depend on them. We thus introduce a new category of "assumed theorems,"

and this contradicts the very concept of a logical system—presumably a major objective in the teaching of plane geometry. By merely attempting to simplify, we render impossible the achievement of our original objectives. Similarly, in algebra, we teach processes which are not applied in their ultimate technical purposes. Thus we add algebraic fractions in watered down syllabuses, with no real purpose or application.

When objectives are clear, then we may set up standards. If college preparation is the objective, then achievement in problem solving will determine standards. But if appreciations are the major objectives, then those standards no longer apply. Teachers who bemoan the "lowering of standards" are correct in their implied criticism of the watering down of syllabuses, but err when they apply the same complaint to a syllabus with different objectives.

It would seem from the foregoing considerations that a syllabus for our group of students must be built from a fresh beginning, with the needs of these pupils in constant view. In this manner must a teacher of the second track view his assignment. It is a serious error to teach these pupils using the traditional examination as criteria for lesson development and drill. We have been doing that for more than 15 years, with a resultant progressive deterioration in learning situations, pupil-teacher relationships, pupil attitudes toward mathematics and the school, and in "results obtained." It is vital for successful teaching of these pupils that the watering down of old syllabuses be abandoned. And it is equally vital that our preconceived notions of standards be replaced by a new set of standards which will reflect more valid objectives. We should be able to say, after teaching these pupils, that their attitude toward mathematics is less antagonistic and more appreciative; that they understand, within their limits of comprehension, the vital role which mathematics plays in our civilization; that their number concepts are functional and their arithmetic competence improved; that they have seen and enjoyed some of the recreational possibilities of mathematics; that they have gained some understanding of the general methods of mathematics; and finally, that they have learned some of the basic mathematical concepts, in terms of which a large part of our everyday thinking is done.

IV. What Kind of Teacher for the Second Track?

Since it is estimated that thirty percent of our student body belong in second track classes, the policy of assigning these classes to one or two teachers specially skilled in handling slow learners is no longer feasible. More and more teachers of mathematics will be confronted with the problem.

The question then becomes: "What characteristics and attitude of the good teachers of mathematics assume critical importance in teaching second track pupils?" And the answer lies in the areas in which second track and first track pupils show appreciable differences.

Such differences are found both in the objectives of the course and in the characteristics, capabilities, and needs of the pupils. Many of these pupils are maladjusted. Their need is for sympathy, understanding, and the hope of success. They need a teacher who is more interested in their better adjustment and relative growth than in the attainment of rigid artificial standards. If the teacher is to give the pupils this feeling of success, "to rebuild the pupil's self respect and imbue him with more favorable attitudes," he must have a strong personal conviction of the value of this learning experience for the pupil. The pupil's weakness in forming associations, his inability to generalize, his poor memory, his short attention span—all these challenge the teacher's resourcefulness, patience, and adaptability. The teacher must be willing to

evaluate achievement and the improvement of work habits with reference to progress made and not to objective measures of the attainment of a body of facts and skills. The pupil's inability to transfer his knowledge to related fields calls for a teacher who possesses a rich background in mathematical applications.

These pupils are restless and find it difficult to observe the customary routines of an orderly classroom. They fail to bring their tools, dawdle, answer out of turn, distract their neighbors and are guilty of many similar infractions. The teacher needs to keep constantly in mind that these misdemeanors are not necessarily manifestations of the extent of their maladjustment. The teacher will have made a significant contribution to the adjustment of these pupils if she has helped them to form better work habits and better attitudes.

All children need patience, kindliness, and affection. The second track pupil is lost without them.

V. Some Down-to-Earth Suggestions

While objectives, syllabuses, and materials of instruction are of vital importance, what takes place in the classroom will finally determine the success or failure of the course. The conventional procedures of the traditional classroom must be modified for the second track pupils in accordance with their special characteristic abilities, and interests.

A. Methods of Instruction and Recitation

1. Slow pupils have a shorter attention span than normal pupils. Hence, the method of recitation should be varied with periods and from period to period. The teacher should provide a variety of individual tasks, supervised study, group work, work at the blackboard, work in individual notebooks, individual and group projects, and contests. Avoid attempting to conduct a full period of the traditional question and answer type of recitation. A means of modifying the question and answer type of recitation, when used, is to require the pupils to write down answers to questions in their notebooks, or at the blackboard.
2. This type of pupil has little aptitude for dealing with abstractions and learns more easily through a concrete presentation. Hence the teacher should provide for a certain amount of experimentation and laboratory work. Materials such as oaktag, paper, and scissors interest many of these pupils. It is desirable that students shall provide their own rulers, protractors, and compasses. However, the teacher will be well advised to have a supply of spares. A system of merits may tend to induce pupils to bring their own tools. Good order in a laboratory does not necessarily mean absolute silence.
3. Another method of providing concrete presentation is through the use of concrete materials such as models, diagrams, pictures, films, and slides. Have students bring materials from home as part of a homework assignment.
4. Slow pupils have difficulty in making generalizations. Hence there should be more use of motivations, dramatizations, and various visual aids to stimulate this process. Furthermore, this lack of power of generalization results in less transfer of their knowledge to related fields. Therefore, the teacher should work with concrete situations, and apply the mathematics to practical situations, as much as possible.
5. These pupils are less resourceful mentally and seek physical outlets. They should never be left in doubt as to what is expected of them. Careful, detailed instructions for the carrying out of their tasks should be given them. A certain amount of physical movement should be provided periodically in the class, such as going to the board.

A notebook kept by each pupil for classwork provides some student activity. Medial and terminal summaries (in graphic and tabular form where possible) can be written on the blackboard and in the notebooks.

6. The slow pupil has verbal difficulties. Hence, oral reading of problems and explanations should frequently supplement silent reading. Be sure all words used are understood. Teach the use of the textbook.
7. These pupils need a sense of success. The pace of a lesson, therefore, should be such that the group can absorb the materials. Attempting to do too much or to adhere too rigidly to a set lesson plan may seriously compromise the teacher's hope of success with these pupils. Allow ample time for generalizations and conclusions. By distribution of questions and by careful inspection of work, check constantly that the pupils are learning the lesson. By using a student secretary at the board at times, the teacher may be able to get around the classroom at the same time to check individual work.
8. As slow pupils have relatively poor memories, there should be frequent reviews and drills. Use varied settings and avoid pure repetition. Practice tests graded by the pupils themselves may be helpful.
9. Flexibility in teaching procedures is of the utmost importance. The teacher must be resourceful in changing types of activities during any period should there be a loss of interest or other indication that the procedure used is not meeting with success.

3. Classroom Management
   1. The details of classroom management should tend to become routinized. The pupils should have an understanding of just what is expected of them. Instructions should always be simple and detailed.
   2. The assignment should be written in a specially set-apart place on the blackboard, or should be on a mimeographed sheet giving the assignments for some time ahead.
   3. Some simple task or assignment can be given at the beginning of the period, to be begun while the class is getting settled.
   4. The assignment of definite responsibilities to individual pupils, such as erasing blackboards, distributing or collecting materials, will help establish good routines and give the pupils a sense of participation.
   5. To arouse interest and also to give the pupils a sense of achievement, attractive exhibits should be periodically posted on the bulletin board. Written work of pupils and construction work by them should be exhibited.

C. Home Work
   1. Some home work should be expected of the pupils. This can impress on them that the course has its standards, just as academic courses do, and can provide them with independent activity.
   2. Specific instructions should be given as to the form and quality of the homework. The homework should be simple, and within the ability of the pupils.
   3. Because of the brief span of attention that these pupils have, the homework should be comparatively short, to take no more than 20 or 30 minutes. The types of homework assignments should be varied so as to include the collection of illustrative materials, verbal answers, newspaper clippings, etc, as well as the usual solution of problems or exercises.
   4. Home work may be in individual units daily, or in more comprehen-

(Continued on page 548)

The Jewel and Its Setting

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IT IS both educationally sound, and intellectually refreshing to back off periodically and look at the process of education. We human beings are continually exercising one of our human frailties by losing perspective. Education and the teaching of mathematics, of course, continue whether we lose perspective and direction or not. Parents pay taxes, teachers teach and draw salaries, administrators administer, and students learn a lot of things. But all are convinced that education is "a rum go" if all those interested in it miss the forest for trees.

When a person buys a diamond ring he examines both the stone and the setting. A properly cut stone in the right setting! This is a fair analogy to the situation in education. One examines the various disciplines in the secondary school curriculum in terms of the job of secondary education.

There are three points of reference one must keep in mind when considering the "oughts" of the program (content and method) of secondary education: 1. Those social processes that are basic to group living, processes which if neglected can result only in societal retrogression, disintegration, and decay; 2. The deveopmental needs of youth which become manifested as youth come of age in our confused, industrial culture; and 3. The values which uniquely characterize what we call "the democratic way of life." A sound educational program neglects none of these whether it uses the particular names given above or not. Let us look briefly at each of these, for the role of every subject in the curriculum (and this includes mathematics) should be determined in part by these points of reference.

THE BASIC SOCIAL PROCESSES

When one has centrifuged the relevant research of sociologists, historians, philosophers, there remain something eleven basic social processes. It is difficult to identify them. Look at l man spends his waking hours. Cons the processes that pervade, in one f or another, all ways of life. Think of processes which when not adequa carried out result in social ulcers juvenile delinquency, crime, unemp ment, epidemics, class cleavages, divor wars, and mass migrations of people.

These eleven basic social proce are:

1. Producing goods and developing serv
2. Buying and consuming goods and serv
3. Making a home and rearing the your
4. Protecting physical and mental hea
5. Spending leisure time
6. Engaging in social and civic activiti
7. Educating both old and young
8. Conserving natural and human resou
9. Expressing aesthetic and religious pulses
10. Communicating with others
11. Deciding on the "oughts" of living

The on-coming crop of youth must so educated as to be intellectually a and emotionally disposed to solve problems associated with these proces Any other alternative means our cult like those of the Incas, Babyloni Romans, and Carthaginians, will marked "F" and consigned to the wa basket of time.

THE DEVELOPMENTAL NEEDS OF ADOLESCENTS

For these needs, we listen to sociologi psychiatrists and clinical psychologi parents and the adolescents themsel Being the kind of human "critters" ado cents are and living in the confused, ind trial society in which they do, they h certain psychological-social needs. It is hard to reach a consensus on the followi

1. The psychic hungers, e.g., feelings of

curity, of success, of being accepted by and belonging to a group, and of having some freedom in one's choices and actions
2. Adjustment to a changing body
3. Wholesome relationships with age-mates of both sexes and with adults
4. Gradual assumption of adult status

Of this we are sure—if these needs are not adequately met, our schools sow wheat and reap chaff, and several of the basic social processes mentioned above are impaired.

The Values Which Define the Democratic Way of Life

The basic social processes exist in every society, but how they are carried out depends on the "oughts" accepted by the society. These processes enable one to make a choice among alternative courses of action; to do certain things and refrain from doing certain others. These "oughts" emerge by implication from the postulates to which we are committed. They are:

1. All men are of supreme and hence of equivalent moral worth.
2. Given the facts, the common man can be trusted to arrive at intelligent solutions to the problems associated with the basic social processes listed above.
3. The welfare of all is promoted as each individual accepts this as his responsibility, and works cooperatively with others in the solution of the problems of living.

To these three sets of tasks, the subjects of the curriculum must face up. Of mathematics we specifically ask: What does it have to offer? By nature it is a language—a basis of communication when man thinks in terms of quantitative, form and space relations. By nature it is a method of thinking as well as a body of organized implications. How does the jewel fit the setting?

Mathematics and the Basic Social Processes

Every significant social problem has a quantitative aspect to it. As a matter of fact, we do not begin to get excited about most problems save as we think in terms of quantitative data. The A-bomb scares us so because only one—and an old-fashioned one—killed somewhere near 100,000 people.

For people to cooperate in the solution of problems associated with social processes, they must be able to understand each other. No communication; no cooperation. Hence they must be able to read, speak, and write the language of mathematics. And one does not have to wait until students become adults before this need is real.

It seems equally clear that people should have a command of those mathematical operations and processes which must be understood in order to make computations with exact and approximate numbers. This is clearly a responsibility of mathematics, and, from what experimental evidence we have, cannot be taught incidentally. As Fawcett has pointed out,[1] such a mastery probably cannot be attained if mathematics courses are absorbed by a core or common learnings course, as worthwhile as such a course might be in the setting described at the outset. Most general educators recognize this and provide in their "picture-in-the-head" of the curriculum elective courses in mathematics outside the core.

There is a third implication. Youth and adults alike should understand the technique of solving problems, especially the technique which involves several people working together. Subsumed under this are such skills as stating problems which offer a solution; formulating fruitful hypotheses; gathering, organizing, interpreting, and evaluating data; drawing inferences; examining the inferences of others; and maintaining good human relations when several people work together.

To be sure, it cannot be said that such skills can be taught only or even chiefly in mathematics classes. They are "stuff for the troops" in every subject. The point is that mathematics teachers can tailor the subject to provide an understanding

[1] H. P. Fawcett, "Mathematics and the Core Curriculum," The Mathematics Teacher, XLII (Jan., 1949), 6–13.

of, and practice in using these skills. This will probably mean somewhat of a shift in emphasis and in the kinds of experiences provided students in the typical mathematics classroom. This can be done and still provide a proper balance between this and teaching students the operations of arithmetic and algebra.

There is a need to study significant problems related to the basic social processes discussed above. Such a study should focus on those problems in which quantitative relationships or the facts and relations of form and space predominate. There are problems of the finances of earning and spending one's money; of getting one's money's worth; of various forms of credit, investment, and protection against risks; of interpreting tables, graphs, and other expressions of functional relations; of deciding whether the town needs a new school or hospital; of polling public opinion on some civic or school issue; of how the government raises and spends money; of measuring the run-off water from nearby fields; and so on.

Some mathematics teachers—judging from what they do and say—would as soon be seen leaving a burlesque show as be found discussing social problems or issues in a mathematics class. Yet if we are to help students clarify social goals and decide about the "oughts" of living we cannot leave this entirely to the social studies teacher.

Values are best clarified in the context of real problems. A teacher whose class is studying the cost of national health insurance as against "every man for himself" misses the boat if the "ought" of such a plan is not considered. Certainly this involves subjective and non-quantitative data! But it also involves thinking—intentional, critical, and reflective thinking. There will be stated and unstated assumptions, accurate and inaccurate data, sound and unsound reasoning, good and bad authorities, and hence opportunities to examine all of these. The mathematics teacher ought to be able to help students in both spheres: to clarify their ideas concerning the "oughts" of social living, and to sharpen their ability to think as logically and consistently in this matter as in geometry for example.

There is another clear implication. To aid in the solution of our problems of living we need specialists who will provide the leadership and supply the "know how." Particularly, we need engineers, statisticians, doctors, top management personnel, teachers, research workers, and a host of others. Many of these will need all the mathematics time will allow them to get. In training the future specialized personnel, the role of sequential mathematics courses in high school is a vital one.

Mathematics and the Needs of Adolescents

Mathematics does not offer a bare larder for meeting this set of needs. It has been pointed out how concerned youngsters are about whether their physical development is proceeding normally. An answer to this question is through comparisons and measurements which the students can make. An understanding of concepts like "average" and "dispersion" helps them answer their questions. The interpretations they make from tables and graphs supplied by a resourceful teacher will provide further help. But without adequate mathematical understanding and instruction in interpreting data, they will not get the help they need.

Let us look at the psychic hungers of human beings, adolescents included. One is the need for a feeling of security. Now a feeling of security, in part, is derived from feelings of being able to cope adequately with a situation. Suppose a person is relatively incompetent in ordinary arithmetic. Percentage is very nebulous. Fractions give him an uneasy feeling. Even a simple formula is an enigma. Such a person will find it difficult to maintain his ego-ideal when faced with situations demanding mathematical proficiency. Doubting his competency, he may become

fearful and lacking (in such situations) in the self-confidence that feeds a feeling of security.

In the past some educators have indicted mathematics for destroying feelings of self-confidence and security because of the failure of students to succeed in mastering the subject. This indictment is a fair one, but can be easily met by smaller classes, proper teaching, etc. But many educators, parents, and students have failed to realize that the proposed remedy—*eliminate mathematics*,—is certain to engender feelings of insecurity. One cannot dodge the fact that a person must be mathematically literate to live in the world today and in the world he will face in the future. Building secure children means equipping them with the tools it takes to get along in this life, a life becoming increasingly mathematical.

Our belief in the dignity of every child makes us believe every child should experience success in some way. There is a story for mathematics in this. We are not clear what mentality is, but we know that some children are born "long" in what it takes to succeed in abstract thinking. They are the ones who "eat up mathematics." If for no other reason than just because a student is successful and gets a "kick" out of manipulating mathematical concepts and relations, he should be permitted to have such experiences. Suffer some children to dig deeply into mathematics, for such is the joy of living and the source of future pleasure in leisure time activities.

In meeting the students' needs for acceptance by their group and assuming increasing responsibility for managing their own affairs, the mathematics teacher is no different from any other teacher. If he considers these needs important, he will plan his teaching so as to include activities involving small groups which plan their work largely by themselves. He may plan with the class how a unit should be studied, how much time should be devoted to the study, and how their work should be evaluated. In short, he does not make all the decisions; he teaches students to make decisions.

To draw the isolates into the group, the teacher can plan activities in which these folks probably will do well. Identification of their special abilities such as those associated with woodwork, making posters, willingness to collect various kinds of data, etc. may lead to their being sought after by the other students. If acceptance is withheld because of undesirable personality traits such as being unclean or insincere, a personal talk in many cases can straighten out the situation. In this sense the mathematics teacher functions as a friendly counselor as does every teacher at times.

Mathematics and Democratic Values

Our final consideration of the place of mathematics in the setting of secondary education should be its contribution to underwriting an acceptance of some of the postulates of democratic living. This aspect of mathematics teaching is often missed by teachers who spend ninety-five per cent of their time drilling their students on manipulations. A study of the lives of some of the famous mathematicians, some Jewish, some Russian, some eccentric, many poor, ought, if properly handled, develop the empathy which is basic to tolerance as well as engender a reverence for the potentialities of the human mind.

Intolerance is seated, as we know, in ignorance. It may be just as deeply seated where there is a belief in absolute truth. After all, if one has the answers, of what value is there to being tolerant with anyone who persists in questioning them? It is possible to portray the relativity of truth through a study of the arbitrary nature of units of measure and symbolism, or through a study of Non-Euclidean geometry. Such studies are within the maturity level of high school students. I have seen dogmatism and smugness in some social

beliefs of students disappear (at least overtly) when the principle was extended from mathematics to the realm of social beliefs.

We say democracy rises or falls with the ability of the common man to think—creatively, independently, and critically. The contribution of mathematics and particularly of geometry to developing such understanding and skill is well known. We do not want for practicable suggestions; we want more for more teachers who will use the method of thinking in geometry as the center of attention. We should be turning out more students who appreciate the significance of words and their impact on thinking, the importance of clearly defined terms whether defined formally or operationally, the role of assumptions in thinking, inductive as well as deductive proof, the weight of evidence, and the dependence of conclusions on assumptions and the rigor of the reasoning. While the mandate to teach students how to think is clear, more energy is expended in high schools in teaching them what to think.

And Finally

It is obvious that the sparkle of the jewel of mathematics is intrinsic. It needs only to be cut so as to bring out the sparkle. In less figurative language, the educative experiences mathematics has to offer are a function both of the "what to teach" and the "how to teach it." The key is to keep an eye on the tasks of secondary education, and let the sparkle emanate from this setting.

Teaching Second Track Mathematics

(*Continued from page 541*)

sive units for longer periods of time. While homework should be the rule rather than the exception, there should be at least one day in the week on which no homework is required.

5. To help establish a proper respect for good workmanship, the homework should be checked. This will also tend to emphasize the importance of regular performance of tasks. The homework should be marked periodically, and the mark should be included in the term mark.
6. A good technique is to require that all homework be kept in a notebook which should be checked regularly by the teacher.

D. Evaluation

1. Written tests should be given frequently. There should be both short and long tests. They should be simple, for success on these tests will help bolster self-confidence. They will also reveal what remedial work is needed.
2. Specific instructions for taking the test should be given. Expected arrangement of the work should be discussed.
3. Questions should be clearly stated.
4. Material tested should be comparatively recent, as these pupils have short memories.
5. Frequently, notebooks and textbooks should be permitted as reference materials. Formulas to be used should be supplied in many instances.
6. Allow sufficient time for completion. Make adjustments if the test has proved too long.
7. To encourage these pupils, many of whom have previously experienced failure, emphasize progress shown. Post excellent papers. Students can be encouraged to keep their own progress charts or graphs.
8. Final grades should be based upon effort and industry as well as actual achievement. In general they should be a measure of the pupil's progress in relation to himself. Complete failure should be a relatively rare occurrence.

Let's Teach Angle Trisection

By B. E. MESERVE
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EVERY YEAR several people announce that they have discovered a method for trisecting angles in the belief that they have solved the classical trisection problem. Many of the methods discovered are correct in the sense that they may be used to trisect an arbitrary angle; many of the methods have been discovered so frequently that they are practically classics themselves. The part of the problem that these discoverers apparently do not understand is that the classical problem requires that the trisection be done using only compasses and a straightedge. The straightedge may be used only to draw straight lines. In solving the classical construction problems one is not allowed to use the length or width of the straightedge, and similarly one is not allowed to use marks upon it. When these classical restrictions upon the trisection problem are disregarded, many of the constructions for the trisection of angles can be performed using only the methods now taught in secondary schools. The restriction that there should be no marks upon the straightedge is disregarded in one of the simplest and oldest constructions of the trisection of angles. This construction is attributed to Archimedes. It requires only compasses and a straightedge with two marks on it. A ruler will serve very well.

Given any angle $\sphericalangle ABC$ (Figure 1) construct a circle of radius r about the vertex B. Let D be the intersection of the circle and the side BC. Extend the side AB through B. On the straightedge mark off the length $r=BD$. Now keep the straightedge on D and slide one mark along AB extended until the other mark contacts the circle at some point F. Draw DF and let it intersect AB at E. Then $\sphericalangle BEF = \frac{1}{3}\sphericalangle ABC$. This fact may be proved as follows: Draw BF and designate the angles as in Figure 1. Then $EF=FB=BD=r$, $\sphericalangle 1=\sphericalangle 2$, $\sphericalangle 3=\sphericalangle 4$. Since $\sphericalangle 3$ is an exterior angle of triangle BEF and $\sphericalangle 5$ is an exterior angle of triangle BED, we have $\sphericalangle 3=\sphericalangle 1+\sphericalangle 2=\sphericalangle 1+\sphericalangle 1$, $\sphericalangle 5=\sphericalangle 1+\sphericalangle 4=\sphericalangle 1+\sphericalangle 3=\sphericalangle 1+\sphericalangle 1+\sphericalangle 1$ and thus $\sphericalangle ABC=3\sphericalangle BEF$. This is not a solution of the classical trisection problem since marks are used on the straightedge.

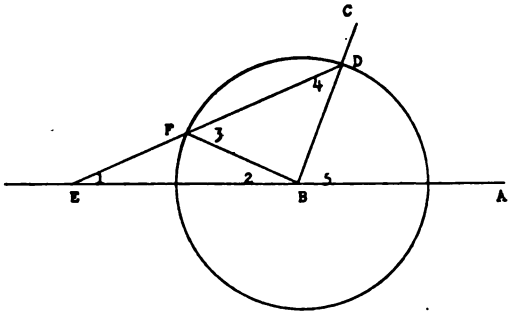

FIG. 1

There are also numerous mechanical angle trisectors that disregard all the classical restrictions upon the trisection problem. One of the best known was designed by a London barrister, Alfred Bray Kempe (Plate 1). It is based upon similar contraparallelograms and is described by Joseph Hilsenrath in THE MATHEMATICS TEACHER, October 1937, pages 282–283. Another trisector was designed by James

PLATE 1

Joseph Sylvester (Figure 2). Since this type of mechanism could be used to divide an angle into any number of equal parts, it is called an "isoklinostat." The origin of Sylvester's basic idea for this linkage may be indicated by the following quotation from the title of the paper in which it was first published—"On a Lady's Fan. . . ".

Mathematical literature contains a great deal of material on the trisection of angles.

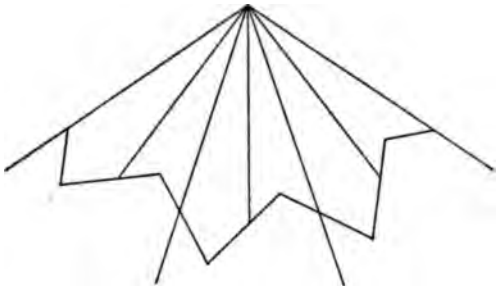

FIG. 2

Yates' book, *The Trisection Problem*[1] contains descriptions of seven curves and fifteen mechanical devices, including the two mentioned above, that may be used in trisecting angles. Periodicals frequently contain material that could be used in class on this topic. Several methods of trisecting angles are discussed in *Civil Engineering*, March 1948, pages 50–51. A method developed by a high school student may be found in THE MATHEMATICS TEACHER, October 1950, pages 278–279. More technical discussions involving the impossibility of the classical problem may be found in many college textbooks in the theory of equations.

We have now observed that (1) the trisection problem has become a game with many people who are not adequately informed of the nature of the classical problem; (2) the construction of mechanical trisectors and the trisection of any angle may be performed in many secondary school classes without introducing any new methods or ideas; and (3) there is a great deal of literature on the subject for any interested teacher or student. It is the thesis of this paper that proper instruction in the nature and types of possible solution of the trisection problem are a part of the social responsibility of the teachers of mathematics. Let us now examine the problem in detail and determine its nature.

The trisections with straightedge and compasses of certain special angles such as 90° and 180° are regularly taught in most plane geometry courses. The classical trisection problem requires a method that may be applied to *any* given angle. This problem will therefore be impossible if there exists a single angle that can not be trisected in the prescribed manner. Before specifying one particular angle that can not be trisected using the classical methods, let us consider briefly the limitations of these methods from an algebraic viewpoint.

Using a straightedge we may draw a line through any two given points. Using a pair of compasses we may draw a circle with any given point as center and any given distance as radius. Using both the straightedge and the compasses we may find the intersection of two lines, the intersections of a line and a circle, and the intersections of two circles whenever such exist. From an algebraic viewpoint any line may be considered as the graph of a linear equation, any circle as the graph of a quadratic equation. The determination of the above intersections then corresponds to the simultaneous solution of two linear equations, a linear and a quadratic equation, two quadratic equations respectively. Using these processes we may add, subtract, multiply, and divide the numbers represented by any given segments, i.e., we may perform the four rational operations graphically. We may also extract the square root of a number represented as a segment. However, these five operations are the only ones that can be performed using straightedge and compasses in the sense that an operation can be performed

[1] Robert C. Yates, *The Trisection Problem* Published by the author, (West Point, N. Y., 1942).

in this classical manner if and only if it can be accomplished by a finite number of the above five operations.

Given any unit of distance we may construct a segment of length d if and only if the number d may be represented in terms of the unit 1 using the four rational operations and the extraction of square roots a finite number of times. Thus all rational numbers and expressions such as $\sqrt{10-2\sqrt{5}}$ may be constructed using straightedge and compasses as soon as a unit is known. The above expression represents the length of a side of a regular pentagon inscribed in a circle of radius two. Such numbers and distances are called constructible. With respect to a given coordinate system a point may be constructed according to the classical methods if and only if its coordinates are constructible. A line may be constructed if and only if it may be expressed by a linear equation with constructible coefficients. In general, *a geometric figure is constructible using straightedge and compasses if and only if the coordinates of its points (vertices, etc.) can be constructed from those of the given figure using a finite number of rational operations and extractions of real square roots.* It is not sufficient to be able to construct a line as close to the desired line as one wishes. One must be able to draw the desired line.

The usual proof that there exists at least one angle that can not be trisected using the classical methods makes use of a few trigonometric identities and an angle of 120°. Given an angle of 120° we seek an angle of 40°. The construction of an angle of 40° is equivalent to the construction of a right triangle with hypotenuse unity and base cos 40°. Thus an angle of 120° can be trisected if and only if a segment of length cos 40° can be constructed. From $\cos 120° = -\frac{1}{2}$ and the trigonometric identity

$$4\cos^3 x - 3\cos x = \cos 3x$$

we may obtain

$$4\cos^3 40° - 3\cos 40° + \tfrac{1}{2} = 0.$$

Taking $y = \cos 40°$ we have

$$4y^3 - 3y + \tfrac{1}{2} = 0.$$

Multiplication by 2 and the substitution $z = 2y$ now gives

$$z^3 - 3z + 1 = 0.$$

The only possible rational roots of this equation must be integers since the leading coefficient is unity and must divide the constant term. Thus the only possible rational roots are $+1$ and -1. Since neither of these integers is a root, the equation has no rational roots. If the equation had a quadratic surd root $a+b\sqrt{2}$, it would also have the conjugate quadratic surd $a-b\sqrt{2}$ as a root, and the remaining (third) root would be rational. Since any constructible number may be expressed using a finite number of rational operations and extractions of real square roots, every non-rational constructible number is one of a set of an even number of conjugates (including itself). Thus any equation with integral coefficients having a constructible surd root must have an even number of surd roots. In particular any cubic equation with integral coefficients having at least one constructible surd root must have exactly two surd roots and one rational root. Thus the fact that the above equation has no rational root implies that it has no constructible root. Since z is not constructible and $z = 2\cos 40°$, we have proved that cos 40° is not constructible and the angle of 120° can not be trisected using straightedge and compasses. Thus the classical trisection problem is impossible.

Two other classical construction problems may be quickly disposed of on the basis of the above discussion. It is not possible using straightedge and compasses to construct a cube with a volume double that of a given cube (Delian problem) since this requires the construction of a segment of length x where $x^3 = 2$ or $x = \sqrt[3]{2}$ taking the side of the given cube as unity nor is it possible to construct a square

with area equal to that of a given circle.

Models of mechanical angle trisectors will catch the eye of almost any secondary school student. Archimedes' construction for the trisection of any given angle less than 90° may be given at almost any time in secondary school. The proof of this construction may be considered as soon as the students recognize that an exterior angle of a triangle is equal to the sum of the opposite interior angles. Since an angle of 30° may be constructed, the students may be shown how Archimedes' construction may be used to trisect any given angle by first subtracting an angle that is a multiple of 90°. Even the above demonstration that the classical trisection problem can not be solved may be considered after the student has studied a few elementary trigonometric identities and advanced algebra in secondary school. The whole question of the trisection of angles may also serve as a basis for a very enjoyable meeting of a mathematics club.

We have now seen that not every angle may be trisected by the classical methods and therefore the classical problem of trisecting an angle, i.e., trisecting an angle using only straightedge and compasses, is impossible. We have also seen that there exist simple methods for trisecting angles using common instruments, such as a ruler or even a protractor, that are forbidden under the classical restrictions of the problem. The frequent "solutions" published in newspapers or even in copyrighted pamphlets published at considerable expense to the discoverer are a constant reminder of the failure of the public schools and colleges in the past to inform the students properly regarding this very interesting problem. The popularity of the problem may in part be due to the fact that it is often stated to be impossible without a full explanation of the reason for this verdict or the classical restrictions that make it necessary. Although this cloud of confusion can not be dispersed suddenly by any path of action, it could be gradually cleared away by the acceptance of all mathematics teachers of a responsibility to see that the students are correctly informed as to the nature and types of possible solution of this most interesting problem, the trisection problem. Let's teach angle trisection.

Editor's Note

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Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers*

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THIS paper will attempt to show the mathematical training prescribed by eighty-five institutions of higher learning which prepare elementary teachers. These institutions are all accredited by the American Association of Colleges for Teacher Education. Geographically speaking, they are located in forty-five states and include state colleges, state teachers colleges, state universities, and privately endowed institutions.

Catalogs available as of June 19, 1950 formed the basis for this study and consequently all of the material referred to is of a recent nature.

We shall investigate the following: mathematics required or recommended for admission to the freshman class of institutions which train elementary teachers; courses in mathematics content required for elementary teachers in the one-, two-, three-, four-, and five-year courses of study; courses in mathematics methods required for elementary teachers in the one-, two-, three-, four-, and five-year curricula; and, for purposes of comparison, requirements in art, English, and geography content in the one-, two-, three-, four-, and five-year programs of study for elementary teachers.

Let us examine now the college entrance requirements in mathematics for those preparing to teach in the elementary school. We refer to the unconditional admission of students on the basis of high school units required. We do not refer to admission through examination or to the admission of mature students. We assume that mathematics is not required for admission to the colleges whose bulletins were considered unless it is specifically referred to as an entrance requirement.

Of the eighty-five catalogs studied, three gave insufficient data concerning admission. Of the remaining eight-two, twenty, or approximately one-fourth, specify mathematics as a requirement for admission. For these twenty schools which require mathematics for entrance the mean is 1.20 high school units.

A few of the catalogs prescribe certain mathematics courses for admission. Four name at least one unit of algebra as a requirement, and one specifies geometry in addition to algebra. One school requires one unit in algebra and one unit in geometry for mathematics and science majors and minors.

Eleven colleges recommend mathematics for admission to the freshman class of their teacher training programs. The mean of these recommended units is 1.82. One catalog advises two units in algebra and one and one-half units in geometry for mathematics and science majors and minors. Another catalog suggests that the student present one unit in algebra and one unit in geometry if he plans upper grade teaching, and one and one-half units of each for a teaching field in mathematics.

Several special cases of admission seem pertinent. One college stipulates that the student must take a three semester hour college course in elementary algebra and a three semester hour course in plane geometry unless he has presented one high school unit in each of these subjects for college entrance. Another institution

* Based upon a paper presented at the Eleventh Christmas Meeting of The National Council of Teachers of Mathematics at Gainesville, Florida, December 27–29, 1950. Some related studies were discussed in the original paper which are not included in this more abbreviated version.

has a similar requirement except that the student must have earned a grade of C or better on the final semester of each high school course. College credit will be granted on the college elementary algebra and plane geometry courses unless the corresponding high school course was completed with a grade of C or better in which case the repeated course will be classified as a "refresher" course carrying no college credit. Students who have one semester of high school credit in trigonometry, intermediate algebra or solid geometry may be permitted to repeat the corresponding course in college for one and one-half units of college credit.

An admission practice described in another catalog states that the student who does not have the one unit in algebra, geometry or trigonometry required for entrance must make up the deficiency by enrolling in non-credit courses before full admission is granted.

The last special case to be mentioned is that of a college which requires no mathematics for admission. However, students who will elect mathematics in college must present three high school units of mathematics of which algebra and plane geometry must have been taken in the senior high school.

Closely related to the problem of admissions is the practice of giving proficiency tests in the fields generally taught in the elementary school. Each of the following practices described is peculiar to one institution:

(1) Students when applying for admission to the teacher training program must display competence in the use of the English language, in arithmetic, handwriting, reading and spelling, indicated by having passed the "Fundamentals Test." The college offers non-credit courses for those who fail to qualify on the test.

(2) All students who seek the kindergarten-primary, general elementary, or junior high credential must stand an examination in arithmetic. Students who fail this test may prepare for a retest by self study, private coaching, or a sub-collegiate arithmetic course.

(3) The student must either display sufficient proficiency in arithmetic on the basis of a test or take a course in fundamentals of mathematics as a basis for further required mathematics.

(4) All students showing a deficiency on the pre-enrollment mathematics test must take a non-credit course in arithmetic.

(5) Entrance examinations are given in English, reading, American history and problems of democracy, general mathematics, general science, and speech.

(6) Preliminary non-credit courses in art and music are arranged for students who are not prepared to carry the credit courses in these fields without further instruction.

(7) The student may be required to take preliminary non-credit courses in art unless satisfying scores are made on qualifying tests.

(8) All students must pass an examination on the subject matter of arithmetic.

(9) Those who do not make a satisfactory showing on the entrance test in English are required to take a non-credit course in fundamentals of English.

(10) Upper elementary students must prove their proficiency in the fundamentals of mathematics for the upper grades. Those failing to pass a proficiency test must take a non-credit course in arithmetic for teachers. Any student who has successfully completed a college course in mathematics is excused from the test.

The major portion of this paper is concerned with the mathematical training prescribed by colleges in the preparation of elementary teachers. A majority of the eighty-five colleges studied offer the four-year curriculum for elementary teachers. It is interesting to notice in Table 1 that the mean of the mathematics content requirement in the various four-year curricula is 1.63 semester hours, while the means for art, English and geograpyh content are, respectively, 4.28, 11.46, and 4.30 semester hours. Mathematics stands at

the foot of the list. The requirement in art and also in geography is more than two and one-half times that in mathematics. The mean in English is approximately seven times the mean in mathematics.[1] However, thirty-five schools have a requirement of one or more courses in mathematics content in their four-year programs of study. This requirement ranges from zero to eight semester hours. We must realize in this connection that a number of the eighty-five colleges studied require courses which are partly content and partly methods in mathematics. In fact, twenty-eight colleges require courses of this nature. The range for these part content-method courses is from two to four semester hours. A total of sixty-three of the colleges require some mathematics content in their four-year curricula either in the form of purely content courses or in connection with courses which are partially method. It is doubtful, however, that the content mathematics treated as part of a methods course receives the serious attention it merits.

Turning now to the mathematics methods required we learn from Table 1 that the mean is .47 of a semester hour for all colleges having four-year curricula. Twenty-six colleges require courses in mathematics methods. Sixteen institutions require training in mathematics methods as a portion of a methods block which includes various other elementary school subjects. Typical courses along this line include methods in reading, language, and arithmetic, or in language, social studies, and arithmetic, or possibly in arithmetic and science. Counting the twenty-eight schools already mentioned as requiring mathematics courses which are part content—part method, we arrive at a total of seventy colleges which call for some contact with mathematics methods on their four-year curricula.

Ranking second among the curricula most frequently offered is the two-year program for training elementary teachers. According to Table 1, the mean of mathematics content required by all institutions having two-year curricula is .54 of a semester hour. The corresponding means for art, English, and geography are 2.84, 7.08, and 2.55 semester hours. Again mathematics holds the low position as far as requirements are concerned. Its mean is only about one-fifth as large as that of art and also of geography, while the mean of English is thirteen times as great as the mean in mathematics. Of the twenty-five schools having two-year programs only six require at least one course in content mathematics with a range from zero to three semester hours. Nine schools specify courses combining mathematics methods with content. Thus fifteen institutions call for some kind of training in content mathematics on the two-year programs.

Only six schools which have two-year curricula for elementary teachers require methods courses in mathematics. As is shown in Table 1, the mean requirement in mathematics methods for all two-year curricula is .39 of a semester hour. Four colleges require courses combining methods in several elementary subjects and nine treat mathematics content and

TABLE 1

*Means in Semester Hours of Certain Courses Required in Curricula for Elementary Teachers**

Curriculum	Mathematics Content	Methods in Mathematics	Art Content	English Content	Geography Content
4 year	1.63	.47	4.28	11.46	4.30
2 year	.54	.39	2.84	7.08	2.55

* Courses which are part content and part method are not included in this table.

[1] Speech and dramatics are not included in the English content requirements.

method in the same course. Therefore, nineteen of the twenty-five colleges with two-year curricula specify mathematics methods in their programs.

Of the eight schools which have three-year curricula for training elementary teachers, six require mathematics content. This requirement ranges from the content included in a two-hour combined content-methods course to four semester hours of pure content. No courses which deal strictly with mathematics methods are required for the three-year curricula, but five colleges call for two-, three-, or four-hour courses which are part content and part method.

Seven institutions list one-year training programs most of which are designed for the preparation of rural elementary school teachers. Only one of these curricula requires mathematics content. Two catalogs specify methods in mathematics—one and one-third and two and two-thirds semester hours, and another requires a methods course which may include mathematics along with other subjects. One college calls for a two and two-thirds semester hour course in mathematics content and methods combined.

Of the three schools which have two and one-half year curricula, one calls for three semester hours in content mathematics and another for an unspecified amount in that subject. None of these mentions mathematics methods.

Two schools have plans calling for two and one-third years or seven quarters of training for elementary teachers. While one of these has no requirements pertaining to mathematics, the other specifies two and two-thirds semester hours in content and one and one-third hours in mathematics methods.

One college has a six-week program, one a twelve-week, and one a one and one-third year plan for those who will teach on the elementary level. None of these has any type of mathematics requirements. The six-week program is to be discontinued after September 1, 1952.

Of special interest in this paper is the five-year program leading to the master's degree in elementary education. Of the eighty-five colleges studied, twenty which offer a master's degree in this field furnished sufficient data for evaluation. Not one of these schools specifies any courses in mathematics content or methods in its fifth year. This is shown in Table 2. Three of these twenty just referred to did not furnish information concerning their undergraduate training for elementary teachers. Of the remaining seventeen, twelve colleges or 71 per cent, require no mathematics content courses in all of their four-year elementary education programs. Of the same seventeen colleges, 65 per cent require no courses in methods in mathematics as revealed in Table 2. Frequently when it is suggested that some mathematics content be required in the curriculum for training elementary teachers the cry goes up, "There is insufficient time to crowd in all the courses we would like to include." It seems significant to observe in Table 2 that of the colleges supplying sufficient data on both their undergraduate and graduate programs, 71 per cent had no place in either their four-year or their five-year degree curricula for any

TABLE 2

Per Cent of Colleges not *Requiring Certain Courses in Curricula for Elementary Teachers**

Curriculum	Mathematics Content (%)	Methods in Mathematics (%)
Master's Degree	100	100
Four Year	71	65
Five Year	71	65

* Courses which are part content and part method are not included in this table.

mathematics content courses. This is probably the trend followed by a majority of institutions which have four- and five-year degree programs for elementary teachers.

Only one college has a requirement in mathematics methods for its master's degree. This is a two and one-half semester hour course called natural science and mathematics curriculum. Since this course does not deal solely with methods in mathematics, it is not included in Table 2.

Let us now examine the titles of the various mathematics content courses required in one or more of the curricula for those who will teach in the elementary school. Ten of the colleges specify general mathematics and three arithmetic for teachers. The remaining titles which will be listed below are each required by one college. These titles are: applied mathematics, fundamental mathematics, socialized mathematics, basic mathematics, practical mathematics, remedial and functional mathematics, background of mathematics, introductory course in mathematics, introduction to college mathematics, mathematics in the upper grades, mathematics for elementary teachers, mathematics for elementary grades, foundations in arithmetic, fundamentals of arithmetic, curriculum in arithmetic, college arithmetic, arithmetic, arithmetic in modern life, college algebra or college arithmetic, college algebra or basic mathematics, college algebra and/or trigonometry or business mathematics. Thus, college algebra is never required outright but may be taken as an alternate with one or more other courses. One of the courses in this list, entitled introduction to college mathematics, perhaps deserves special mention inasmuch as the content of the course varies according to the previous preparation of the students.

It is interesting to note that one college offers a fine arts four-year program which does not include specialization in music. This fine arts curriculum prepares for teaching fine arts in the elementary schools and in junior and senior high schools. The study revealed no specialized curriculum for prospective elementary school teachers of mathematics.

Two catalogs, in describing the requirements for a certificate valid in both the elementary and secondary schools, call for a minimum of twenty-four semester hours in each of two teaching fields with the exception of the social sciences and mathematics. These fields require thirty-two and twenty-two semester hours respectively. Here again, mathematics seems to be discriminated against with respect to the other subject fields. In one of these schools two semester hours may be deducted for each high school unit in mathematics, the maximum deduction to be six hours. This practice would seem to weaken further the college training in mathematics received by those who take advantage of the allowable deductions referred to above.

Summary

Only one-fourth of the catalogs studied specify mathematics as a requirement for college entrance for freshmen who enter with a teacher training objective. This seems to indicate a weak foundation for our potential teachers of mathematics in the elementary school as far as admission requirements are concerned.

In the four-year curricula, the requirement in content art and also in geography is more than two and one-half times that in mathematics. The mean in English is approximately seven times the mean in mathematics content.

In the two-year programs, the content mathematics requirement is only about one-fifth as large as that of art and also of geography, while the mean of English is thirteen times as great as the mean in mathematics content.

In both the two- and four-year curricula mathematics stands far below the other content subjects considered. A nation-wide study of certification requirements for teachers of mathematics was made by

the writer and published in 1949.[2] Some of the results contained in this book were later published in THE MATHEMATICS TEACHER.[3] The findings of the present investigation parallel those of the certification study in which mathematics stood at the foot of the list in terms of the requirements for certifying those who teach mathematics in the elementary school. At the time the results of the certification study were made public there were those who commented, "But the colleges provide far better programs for the training of teachers of mathematics in the elementary school than the state certification laws prescribe. Actually, mathematics is not slighted by the colleges nearly as much as your certification study would indicate." The means computed in the present study and those in the certification study are not directly comparable, but according to the present study, mathematics is still at the foot of the list of the required content subjects considered—English, geography, art, and mathematics. In fact, there is quite a big gap between the mathematics content requirement and that in the other subjects. This inadequate preparation in mathematics for teachers in the elementary school is an alarming situation not only in view of the over-all future of our civilization, but even more especially now with the increased demands placed upon mathematics due to the present national emergency. It would seem that this mathematical crisis is not going to be corrected until strong organizations like The National Council of Teachers of Mathematics and The Mathematical Association of America together with the state organizations of mathematics teachers put their shoulders to the wheel and act through the state certification channels to improve this pitiful situation in which mathematics finds itself. Probably the colleges will not improve their standards in mathematics for the training of elementary teachers until the state certification laws require them to do so.

On the basis of the present paper the graduate schools considered which offer degrees in elementary education do not require mathematics content in their fifth year of training, and the majority of these same schools do not require mathematics content courses in their four-year programs. It seems very regrettable that with five years being devoted to the preparation of elementary teachers in these schools, there is still no time for required courses in content mathematics. With more and more states looking toward a five-year program for training elementary teachers, it might behoove the mathematical organizations to devote some attention to the inclusion of mathematics content in these programs.

[2] William Isaac Layton, *An Analysis of Certification Requirements for Teachers of Mathematics* (Nashville, Tennessee: Bureau of Publications, George Peabody College for Teachers, 1949).

[3] W. I. Layton, "The Certification of Teachers of Mathematics," THE MATHEMATICS TEACHER, XLII, No. 8 (1949), 377-380.

The 1951 Christmas Seal Sale of the National Tuberculosis Association will be conducted from November 19 to December 25. Christmas Seals have become a holiday tradition in this country where they have been sold since 1907. Each year 94 percent of the money raised by the sale of Christmas Seals remains in the state where it was contributed to support local and state tuberculosis control. The remaining six per cent is forwarded to the National Tuberculosis Association. At least one-sixth of this percentage is used for medical and social research.

Tuberculosis kills more people in the United States than all other infectious diseases combined. It killed approximately 40,000 persons in the United States in 1949, at the rate of 100 a day, one person every 13 minutes. Estimates place the number of tuberculosis deaths all over the world at almost 5,000,000 a year. Tuberculosis is the chief cause of death among children throughout the world, according to the United Nations International Children's Emergency Fund. In the United States, it annually kills approximately 1,600 children under 15 years of age.

Since the campaign for tuberculosis was inaugurated almost 50 years ago, the death rate has been forced down 85 per cent. This has meant a saving of 5,000,000 lives for, if TB had continued to kill at the rate of 50 years ago—194 per 100,000 population—it would have killed 5,000,000 more *people* than have died of tuberculosis in the intervening years.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

36. Additive and Multiplicative Magic Squares.

Miscellanea 16.[1] showed how magic squares could be used in a new version of the old game of Tick-Tack-Toe. At the end of the explanation of this game was the statement, "The connection with magic squares is also obvious and might well motivate a report or project dealing with this latter topic." The purpose of this note is to aid in the realization of this project. It is my opinion that the device of the magic square has immense possibilities in mathematics teaching, not merely for the mathematics club where it has long been a favorite topic, but also for the mathematics class.

However, if magic squares are to be utilized to their greatest advantage, two simple but fundamental principles must be followed. I stumbled across these ideas about a year ago and have developed them in actual classroom teaching. The ideas are so simple that others must surely have come across them in their study of magic squares. However, I have not seen them set forth anywhere. I would be interested in knowing if they are embodied in any text or article.

If the first three numbers to be used in a 3×3 magic square are selected in accordance with the following two principles, the magic square will "check all the way around." Furthermore, the magic square may be multiplicative or additive.

PRINCIPLE I: In the case of the additive magic square, the three numbers first chosen must constitute a row, column, or a diagonal. The number which occupies the middle cell of the square must be the arithmetic mean of these first three, or, if the first three are placed on a row, column, or diagonal containing the center cell, the center number must then also be the arithmetic mean of the other two.

PRINCIPLE II. In the case of the multiplicative magic square, the three numbers first chosen must constitute a row, column, or a diagonal. The number which occupies the middle cell of the square must be the geometric mean of the first three, or the mean proportional to the other two if the first three are placed on a row, column, or diagonal containing the center cell.

If the first three numbers are placed in this latter diagonal position the fourth number may be chosen at random and entered at random. The remaining cells may then be filled in accordance with the operation to be employed.

If the first three numbers were placed in a row or column containing the center cell then the fourth number may be chosen at random but should be placed in a corner cell. This latter restriction is not mathematically essential, but ignoring it requires the determination of a fifth number which is undesirable for teaching purposes in junior or senior high school.

To prove these principles, enter a, b, c and d in four of the cells placing the first three in a row, column, or a diagonal. For the purpose of illustration, a diagonal has been used in Figure 1.

a		
	b	
d		c

FIG. 1. Magic Square at the Start.

If our magic square is to be additive, the

[1] THE MATHEMATICS TEACHER, vol. 44 (Jan. 1951), p. 43.

sum in any direction must be $(a+b+c)$, while if it is to be multiplicative, the product in any direction must be (abc). Using these properties, fill in three additional cells as shown in Figures 2 and 3.

a		$(a+c-d)$
$(b+c-d)$	b	$(b+d-c)$
d		c

FIG. 2. Additive Magic Square.

a		$\frac{ac}{d}$
$\frac{bc}{d}$	b	$\frac{bd}{c}$
d		c

FIG. 3. Multiplicative Magic Square.

If the middle row of the additive magic square is now added, the sum will be found to be $3b$. Since this must be equal to $(a+b+c)$, it follows that b must be the average or the arithmetic mean between a and c.

If the middle row of the multiplicative magic square is now multiplied, the resulting product will be found to be b^3. Since this must be equal to (abc), then b must be the geometric mean of a and c, or the mean proportional between them.

Note that the fourth number drops out in the process of obtaining the final relationship. Consequently, the choice of this number is arbitrary in this case. Further note should be taken of the similarity in the three additional cells in both types of magic squares provided they are entered by using the same directions.

The two sheets containing magic squares indicate the variety of purposes that such a device may serve.[2] Their history and their property of being self-checking all the way around furnish strong motivation in teaching. Additive magic squares enable a group to cover extensively the principles involved in the operations of addition and subtraction. The latter operation is involved in the process of obtaining any new number in the set. Similarly, multiplication and division may be given valued development through multiplicative magic squares.

Additive squares may include integers, fractions, decimals, percents, and denominate units.[3] Negative numbers have their place in the exercises. The need for subtracting in the obtaining of the desired sum has proved to be a worthwhile means of introducing the topic of signed numbers. Algebraic processes may be utilized by representing the unknown number as "x" and securing the equation needed to obtain the number.

Similarly multiplicative squares may be used for the presentation and development of the principles of algebra and arithmetic concerned with the inverse operations of multiplication and division. Work with square roots and irrationals may be treated through this type of square. The replacing of literal numbers by numerical values affords valuable practice in evaluation.

[2] Space limitations prevent the printing of the mimeographed sheets used by Dr. Rich in his classes. Interested readers may write him for copies.

[3] These are variations on the classical magic squares which contained only positive integers. Classically the subject has also been expanded to higher order squares, also magic cubes and circles. See for example W. W. Rouse Ball, *Mathematical Recreations and Essays*. (Macmillan and Co., 1940) pp. 193–221.

3	8	7
10	6	2
5	4	9

FIG. 4. Original Magic Square.

$3x$	$8x$	$7x$
$10x$	$6x$	$2x$
$5x$	$4x$	$9x$

$3x^2$	$8x^2$	$7x^2$
$10x^2$	$6x^2$	$2x^2$
$5x^2$	$4x^2$	$9x^2$

FIG. 5. These new squares may then be evaluated for varied values of x.

.3	.8	.7	3%	8%	7%	3/3	8/3	7/3
1.0	.6	.2	10%	6%	2%	10/3	6/3	2/3
.5	.4	.9	5%	4%	9%	5/3	4/3	9/3

FIG. 6. Arithmetic type magic squares obtained from the first one. The last of this group should be changed by the student into one consisting of mixed numbers.

3 ft.	8 ft.	7 ft.	which may be changed into	1 yd.	2 yd. 2 ft.	2 yd. 1 ft.
10 ft.	6 ft.	2 ft.		3 yd. 1 ft.	2 yd.	2 ft.
5 ft.	4 ft.	9 ft.		1 yd. 2 ft.	1 yd. 1 ft.	3 yd.

FIG. 7

Creative formulation of magic squares may be carried on by students as long as the two fundamental principles are observed. Self-reliance is strengthened through the process of making up new magic squares, completing the ones made up, and then checking them "all the way around." Note how many new magic squares may be invented from one that has already been finished:

Finally, the two types of magic squares can be interrelated by using the values of the additive magic square as the exponents in a multiplicative magic square as shown in Figures 8 and 9.

3	8	7
10	6	2
5	4	9

FIG. 8. Additive Magic Square.

x^3	x^8	x^7
x^{10}	x^6	x^2
x^5	x^4	x^9

FIG. 9. Multiplicative Magic Square.

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37. Still More About Nedians

The following letter to the editor is a good preface to some additional notes on nedians as well as being of interest in itself:

1401 East 35th Street
BROOKLYN 34, N. Y.
May 29, 1951

DEAR SIR:

I am a student at James Madison High School in Brooklyn. While glancing through the January 1951 issue of THE MATHEMATICS TEACHER, my attention was attracted to the article entitled *Nedians of a Plane Triangle.* Upon inquiring, I was told that several other articles had appeared in your more recent issues.[4]

This subject was of particular interest to me because I had prepared a thesis on *inwoven polygons* for the Westinghouse Science Contest of December, 1950.

In my thesis, using plane geometry, I not only developed the solutions which your magazine printed, but went further to establish a general formula for all inwoven regular polygons. I ended by proving by analytical geometry that the parallelogram formula does not apply to irregular quadrilaterals.

[4] Notes on nedians and associated topics are to be found in *Miscellanea 18, 26, 32, 34,* in THE MATHEMATICS TEACHER, Vol. 44 (January 1951), pp. 46–48; (May 1951), pp. 310–312; (Nov. 1951), pp. 496–500.

Mr. George Morgenstern, the faculty adviser of the James Madison High School "Math Team," has urged me to send you a copy of my thesis which I am enclosing.

Respectfully yours,
MARILYN R. TAIG

We regret that we cannot print all of Miss Taig's thesis, but we do reproduce her last theorem because it provides an interesting addition to those which we have printed so far and also displays her method of procedure.

THEOREM. If in any parallelogram, $RSTU$, n segments of a units length and n segments of b units length are laid off alternately from each vertex in a cyclic order, and if each point of division is connected to the opposite vertex, then the area of the inwoven parallelogram, C, is

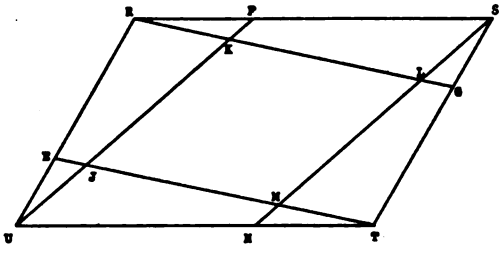

FIG. 10

given by $C=(n-1)^2/(n^2+1)\cdot K$, where K is the area of the parallelogram.

In Figure 10:

$$\triangle STH=\triangle TUE=\triangle URF$$

(1) $$=\triangle RSG=\frac{K}{2n}$$

(2) $\triangle SGL=\triangle UEJ$; $\triangle THM=\triangle RFK$

(3) $MLGT=JMHU=KJER=KFSL.$

Let B represent the area of these latter quadrilaterals, then

(4) $$\frac{\triangle UEJ}{\triangle UEJ+B}=\frac{a^2}{a^2n^2}=\frac{1}{n^2};$$
$$\frac{\triangle RFK}{\triangle RFK+B}=\frac{b^2}{b^2n^2}=\frac{1}{n^2}.$$

(5) Hence $\triangle UEJ=\triangle RFK$.

Let A represent their area.

Then $\frac{A}{A+B}=\frac{1}{n^2}$, and

(6) $$B=A(n^2-1).$$

Substituting from (6) in $C+4B+4A=K$ we have

(7) $$K=C+4An^2.$$

But $2A+B=K/2n$ from (1) and (5) which, by substituting from (6), becomes

(8) $$A(n^2+1)=\frac{K}{2n}.$$

Eliminating A from (7) and (8) finally gives

$$C=\frac{(n-1)^2}{n^2+1}\cdot K.$$

Miss Taig's attack on the trapezoid via a coordinate system and the use of determinants to express both collinearity and area lead to the conclusion which she noted in her letter in regard to the general quadrilateral. Perhaps someone else has a neat formula for the *inwoven*, *cevian*, or *redian* figure of this polygon?

88. *Monkeys and Coconuts*

THE PROBLEM: A group of persons have a quantity of coconuts to be divided equally among themselves. Each person in turn, not trusting his friends, surreptitiously, sorts the coconuts into piles, buries his own share and jumbles the remainder together. However, in order to have all piles equal each person had to give the same number of coconuts to some waiting monkeys. How many coconuts were there at the start?

Most forms of the "monkeys and coconuts" problem not only can be appreciated but can be solved by the alert student of high school mathematics. In support of that opinion some solutions are offered.

NOTATION: $A, B, C, D, E, \ldots$ in succession are the numbers of nuts in a pile, A at the start, B after the first operation, and so on.

EXAMPLE 1. Three men, one monkey, four "operations." In this case *operation* means to give one coconut to the monkey and then to carry away one-third of the remaining pile.[5]

[5] This problem is to be found in Maurice Kraitchik, *Mathematical Recreations*. (New York: W. W. Norton and Co., Inc., 1944), p. 32.

The problem is to find positive integers, A, B, C, D, E, to satisfy

$$(1)\qquad \begin{aligned} 2(A-1)&=3B,\\ 2(B-1)&=3C,\\ 2(C-1)&=3D,\\ 2(D-1)&=3E. \end{aligned}$$

From equations (1) the student can eliminate B, C, and D and arrive at

$$(2)\qquad 16A=81E+130.$$

If he cannot solve this by trial-and-success, the problem must be abandoned at this point. It will be of little use to tell him about congruences, or continued fractions, or "indeterminate" equations.

Notice now that equations (1) can be written in the form:

$$(3)\qquad \begin{aligned} 2(A+2)&=3(B+2),\\ 2(B+2)&=3(C+2),\\ 2(C+2)&=3(D+2),\\ 2(D+2)&=3(E+2). \end{aligned}$$

This amounts to importing two blue coconuts. They will make the arithmetic easier for the men and make life harder for the monkey. They are given a distinctive mark because they are to be exported at the end. They may be regarded as catalysts; they make the game easier without really playing in it "for keeps."

From equations (3) by multiplication,

$$(4)\qquad 16(A+2)=81(E+2).$$

Naturally this is the same as (2) but it is much more convenient.

Since E is a positive integer, so also is $(E+2)$. It follows that $16(A+2)$ is divisible by 81. Now 16 and 81 have no useful common factor and therefore $(A+2)$ is divisible by 81. By similar reasoning, $(E+2)$ is divisible by 16.

Note that (4) can now be written in the form

$$\frac{A+2}{81}=\frac{E+2}{16}=K$$

where K is any positive integer.

The chestnut is cracked. By going back to equations (3) it is easy to show

$$\frac{A+2}{81}=\frac{B+2}{54}=\frac{C+2}{36}=\frac{D+2}{24}=\frac{E+2}{16}=K$$

where $K=1, 2, 3, 4, \ldots$.

We have thus the "one-parameter" set of solutions:

$$\begin{aligned} A&=81K-2,\\ B&=54K-2,\\ C&=36K-2,\\ D&=24K-2,\\ E&=16K-2, \end{aligned}$$

where K is any positive integer.

Quod erat inveniendum.

INTERMISSION: It is recommended that the student take time to tabulate a few successive solutions.

K	1	2	3	4	5
A	79	160	241	322	403
B	52	106	160	214	268
C	34	70	106	142	178
D	22	46	70	94	118
E	14	30	46	62	78

He should find it easy to discover what law governs the numbers in any row and what law governs the values of $(A-B)$, $(B-C)$, $(C-D)$, $(D-E)$, taken from any column.

EXAMPLE 2. Three men and 403 monkeys arrive at twilight on a tropical island where there is a pile of coconuts too big to be counted before dark. It is assumed that the men know enough mathematics to be able to "deal," and that the monkeys are sufficiently polite to refrain from snatch and grab. One man, while the others sleep, gives a coconut to each monkey, deals the rest into three equal piles, takes away one pile for himself, puts the other two piles neatly together and goes back to sleep. In succession the second man and the third do likewise. How many nuts were in the original pile?

For finding A, B, C, D, we have

$$2(A-403)=3B,$$
$$(6)\qquad 2(B-403)=3C,$$
$$2(C-403)=3D.$$

Import 806 blue coconuts observing that

806 = (number of sleeping men) · (number of monkeys).

Equations (6) in handier form lead to (7) and (8)

$$2(A+806)=3(B+806),$$
$$(7)\qquad 2(B+806)=3(C+806),$$
$$2(C+806)=3(D+806).$$

$$(8)\qquad 8(A+806)=27(D+806).$$

By the sort of reasoning used before, $(D+806)$ must be divisible by 8. We have as possible values of D: 2, 10, 18, 26, . . . That is $D=2+8t$, where $t=0$, 1, 2, 3, 4, . . . Put this value of D into equation (8).

$$8(A+806)=27(2+8t+806)$$
$$=27(808+8t)$$
$$A+806=27(101+t)$$
$$A=1921+27t.$$

The pile contains at least 1921 coconuts but may contain 1948, 1975, 2002, . . .

What we wanted.

NOTE: There is no magic about choosing 806 instead of any other auxiliary number. If we set

$$2(A-403)-3B=2(A+x)-3(B+x),$$

we shall find $x=806$.

EXERCISE 3. n humans, 1 monkey, n "operations." This problem is used to introduce an interesting article on the theory of numbers in the *Scientific American*.[6] The alert student can now solve this problem. The student who never wants to see another coconut problem may still want to verify the answer:

$$A+n-1=K(n^n).$$

The number K is to be a positive integer and it can be chosen large enough to make the last population of the pile a *positive* integer. After that has been attended to, any integer larger than that value of K will serve equally well.

For the teacher and the insatiable student there remains

EXERCISE 4. n humans, s simians, $(n+1)$ operations.

FOOTNOTE: The trick of "importing" is not novel. For instance you may recall the whimsy about Omar ben Solomon, the sheik who was called in to distribute an estate of 17 camels giving one-half to the eldest son of the testator, one-third to the second son, and one-ninth to the remaining son. He did it without bloodshed and enhanced his reputation for wisdom by putting his own camel in the corral with the others. After the sons had been given 9, 6, and 2 camels, he embarked on his own and sailed away.[7]

NORMAN ANNING
University of Michigan
Ann Arbor, Michigan

[6] Paul S. Herwitz, "The Theory of Numbers." *Scientific American*, Vol. 185, No. 1 (July, 1951), pp. 52–55. Since this note was written, *The Scientific American* for September, 1951 has appeared with letters about the article and a *general* solution by the original author on pages 2, 4, 6. P.S.J.

[7] *Department Editor's note:* Don't miss the difference however. Omar ben Solomon's "wisdom" not only avoided "halving" a live camel but also politely ignored the bad arithmetic of the deceased. Professor Anning's "imported" coconuts facilitate good mathematics.

HAVE YOU SEEN?

"Engineers" by Karl T. Compton in *Scientific American*, September, 1951. "There are 400,000 engineers, but they are in acutely short supply. An analysis of this critical situation, with some suggestions as to how it might be alleviated in the future."

"Scientists" by M. H. Trytten in *Scientific American*, September, 1951. "We have 175,000 and need more. The problem is complicated by the fact that applied science takes workers away from pure science, the wellspring of our technological progress."

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. Or if that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

TINKERTOY MODELS[1]

Editor's Note: An important aspect of the "building" procedure outlined in this article is its developmental nature. The technique is unique; the contributor illustrates with a solid geometry proposition how the building of a model in itself becomes part of the proof. The reader will note as he proceeds with this article that the "Given" is built first. E.J.B.

As an illustration of the use of TINKERTOY in the solid geometry classroom consider the proof of the following proposition: "If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to their plane."

Begin by compressing two rods, BC and BF, into the rim holes of a spool. Let these two rods represent the given intersecting lines. (See Fig. 1.) Next compress a third rod AB into the through hole of the spool; AB will be perpendicular to lines BC and BF. Thus the "Given" part of the proof is now completely illustrated.

By compressing a fourth rod BE into a rim hole it is possible to represent "any line through the given point lying in the plane of the given intersecting lines." Extend line AB through B to A′ by compressing a fifth rod into the through hole of the spool on the side which is opposite A. Rod A′B should be selected so that AB = A′B. At this point the student should be asked to note that end A of rod AB represents a point on a line of unlimited length and that AB represents only a segment of that line.

The model may be completed by using colored pieces of ribbon with knots tied on their ends. The pieces of ribbon slip readily into the slots at the ends of the rods and are held in place by the knots. The ribbons may be thought of as representing the construction lines in the figure. They are helpful in locating the various triangles which must be considered in stating the proof.

One advantage of the model building idea suggested in this article is that students can see how the figure is related to the statement of the proposition. Only the materials are brought to class—not the completed model. The given facts are shown first. Then the construction lines are added. After this the proof usually follows with comparative readiness.

The building procedure described above will be found helpful in nearly all work with lines and planes and to a limited extent with prisms.

HELEN MACDONALD SIMMONS
Governor Dummer Academy
South Byfield, Massachusetts

RIGHT TRIANGLE MODELS

Illustrated in this article is a set of models which can be used to help develop the statements and proofs of those right triangle theorems which have to do with the altitude upon the hypotenuse. The set

[1] Directions for construction in this article are given in terms of the language employed in the "Direction Sheet for Wonder Tinkertoy," The Toy Tinkers Inc., Evanston, Illinois, 1950. TINKERTOY is available from the manufacturers at this address.

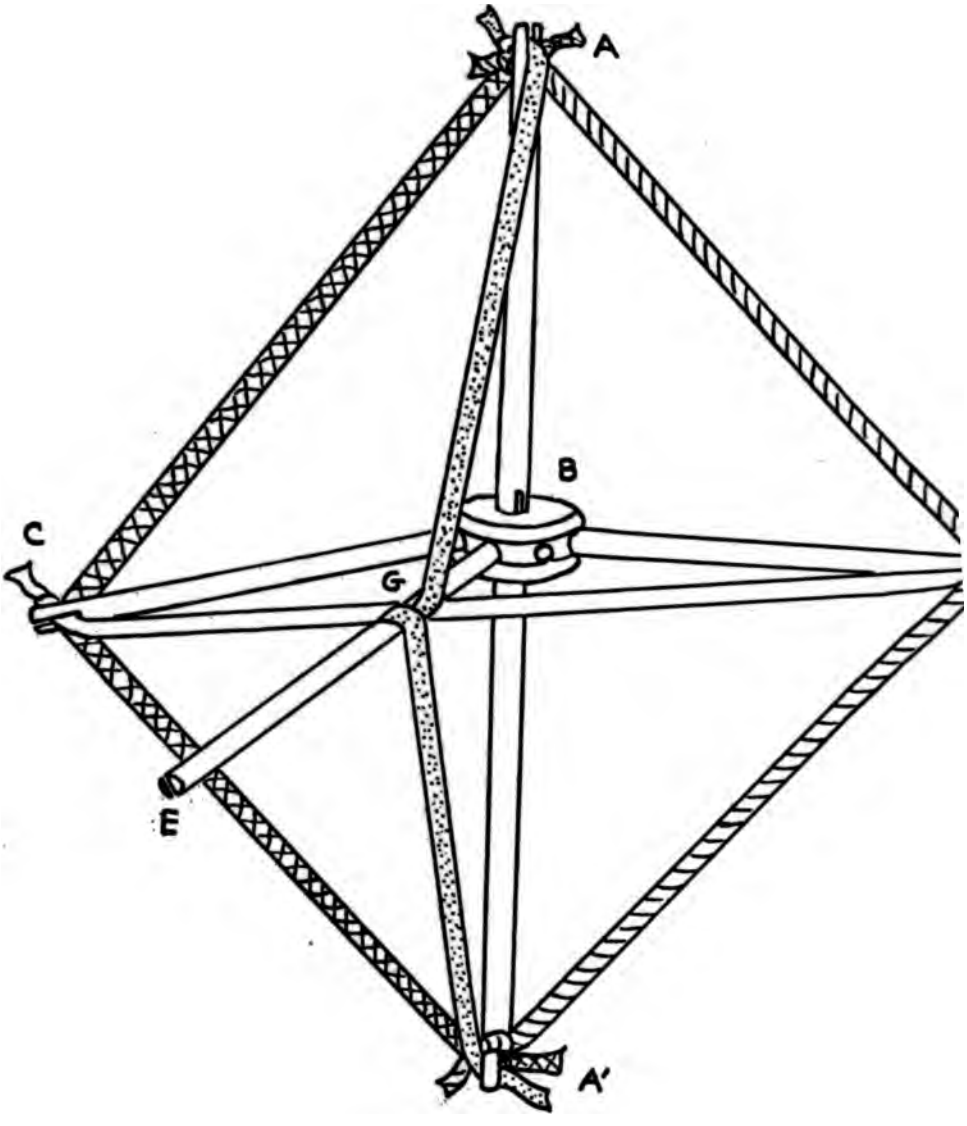

FIG. 1

consists of seven right triangles which may be made from plywood and painted different colors as illustrated by the shading in the diagrams, or cut from heavy c cardboard.

For the sake of convenient refere

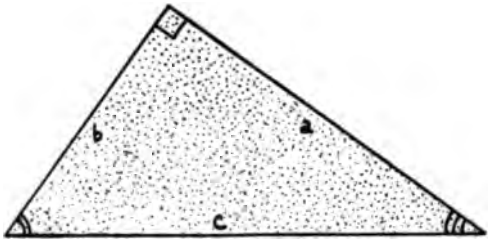

FIG. 2

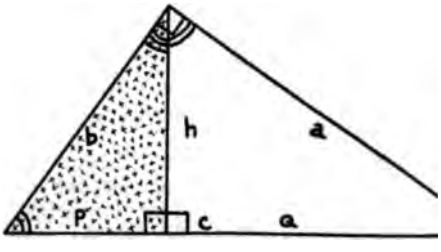

FIG. 3

the drawings let us refer to the largest of the right triangles as the "original" one. Suggested dimensions for its hypotenuse and altitude are 18″ and 8″ respectively. Three of these and two each of the smaller right triangles should be constructed.

Figure 2 represents the original right triangle; Figure 3, the original triangle with the altitude upon the hypotenuse as the common side between the other two tri-

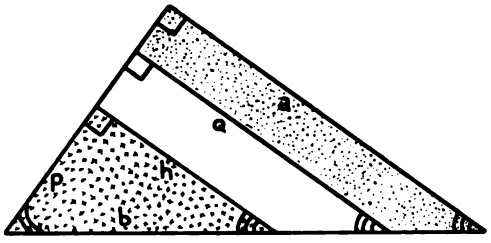

FIG. 4

angles, and Figure 4 shows the three right triangles separated. Please note that there are seven triangles in the three illustrations.

By matching the three triangles illustrated in Figure 4 with their duplicates it is possible to show that the triangles of Figure 2 and 3 are all similar.

By employing the set of models described above in appropriate laboratory experiments the following geometric facts may be developed and demonstrated:

(1) The altitude upon the hypotenuse of a right triangle divides the triangle into two triangles which are similar to each other and to the original triangle.
(2) If the altitude to the hypotenuse of a right triangle is drawn:
    (a) The altitude is the mean proportional between the segments of the hypotenuse;
    (b) Either leg is the mean proportional between the hypotenuse and the adjacent segment.
(3) The product of the two legs is equal to the product of the hypotenuse and altitude (twice the area of the original triangle).
(4) The mean proportional construction.
(5) The Pythagorean Theorem.

ISADORE CHERTOFF
Daniel P. Sweeney High School
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BILLIARD MARKERS FOR ABACUSES

Mr. Frank Hawthorne of Hofstra College has informed us that wooden billiard markers may be procured in quantity from The Brunswick-Balke-Collender Company, 109 Fifth Avenue, New York 3, N. Y. These beads are very good ones for making abacuses and fact finders. The current retail price for a set of 100 markers complete with wire hooks and stretchers is $3.30 regardless of quantity. The 100 markers may also be purchased separately at a cost of $2.30.

A SIMPLE STADIA TUBE

Measuring distances with a stadia tube is an interesting application of similar triangles. The instrument as usually described in books on field work consists of a tube with a peep slight at one end and a pair of parallel cross wires at the other. Depending on the materials used in construction it is possible to produce some very precise instruments.

The device suggested here can be pro-

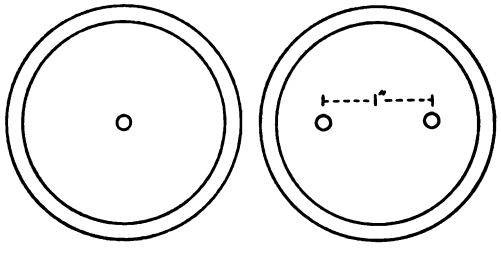

FIG. 5

duced with two two-piece Kerr Mason fruit jar caps, a piece of tagboard 12″ square, and a small roll of cellophane tape.

Disassemble the two jar caps—that is, remove the lids from the screw bands. Drill a $\frac{1}{8}$″ hole in the center of one of the lids, and drill two $\frac{1}{8}$″ holes 1″ apart (from

center to center) in the other lid. (See Fig. 5.) The metal in the lids is so soft that the holes can also be made with the metal point of compasses or a paper punch.

To assemble the tube replace the lids in the screw bands, roll the 12″ square of tagboard into a tube and fit the assembled jar caps around the ends of the tagboard tube. (See Fig. 6.) A little cellophane tape will secure the jar caps to the tagboard.

For most practical purposes this device is rather crude, but it has the advantage of simplicity and the materials needed to produce it cost very little. Every student in class can produce one in a few minutes. This fact in itself would seem to compensate for its lack of fineness. It is not exactly a secret among mathematics teachers that lessons involving field work are difficult to conduct because of a lack of equipment. Usually there are too many spectators and not enough participants.

To measure any distance such as D in Fig. 6 procure a broom stick, fish pole, or other long rod, and two red ribbons. Tie the ribbons around the rod as shown in the figure. If one student holds the rod at eye level in a position which is horizontal with the ground and parallel to the end of the tube through which another student is sighting, the distance D may be readily computed. The student doing the sighting must line up the ribbons through the two holes in the far end of the tube. It is his responsibility to direct the student holding the rod to slide the ribbons together (or apart) until the line-up is effected.

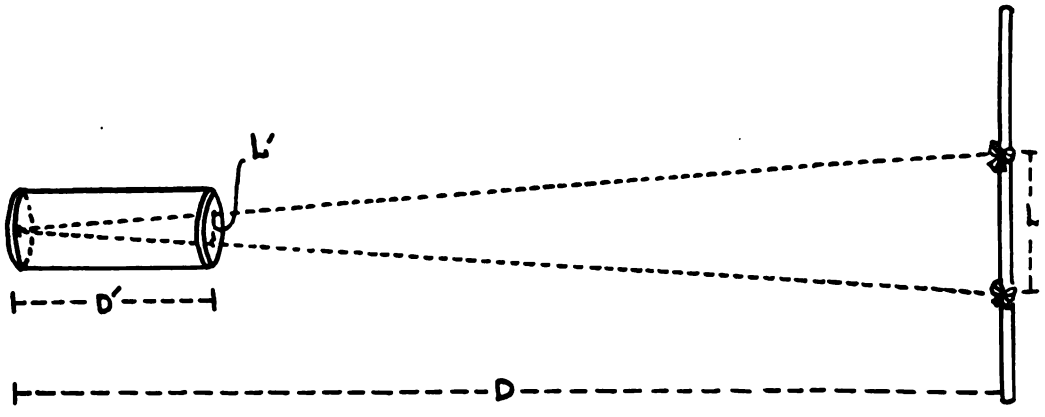

FIG. 6

After measuring the length L, the distance D may be obtained directly from the following proportion by making the appropriate substitutions for L, L', and D':

$$\frac{D}{D'}=\frac{L}{L'}.$$

Note that $L'=1''$, and $D'=12''$. The proportion is a direct application of the fact that the altitudes of two similar triangles are to each other as their bases.

THE MATHEMATICS LABORATORY
Monroe High School
St. Paul, Minnesota

HAVE YOU SEEN?

"Geometry in Flags" by Alice Lamb, and "Geometric Designs in Life" by Octavia Beard in *Student Life*, November, 1951.

"Classroom Atmosphere Stimulates Arithmetic Study" by Edna M. Nolan in *Educational Progress in the Chicago Public Schools*, October, 1951.

"Swimming is a Science" by Georg Mann in *Science Digest*, July, 1951. "The square of the maximum speed of the total stroke is the sum of the squares of the separate swimming speeds of the arms and legs."

"Beware of Bumper-Chasers" in *Science Digest*, July, 1951. "A rule of thumb the experts use for stopping on dry pavement is to square the speed you're driving, and divide it by 10."

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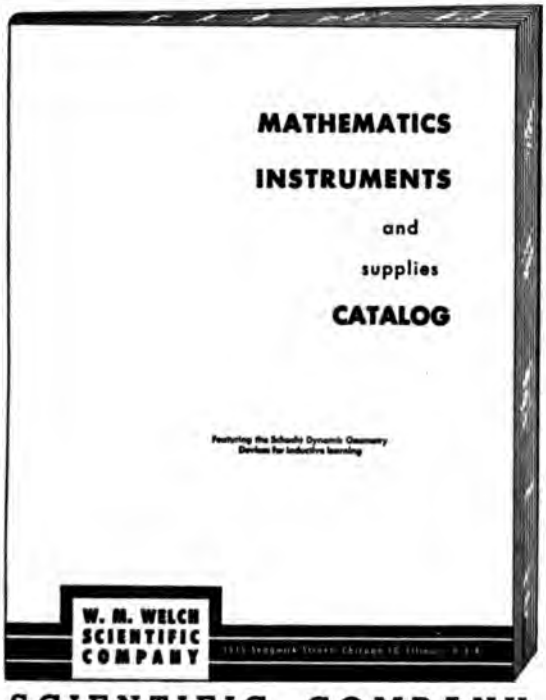

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RADICALS *and* EXPONENTS
ENTER INTO CIRCUIT PROBLEMS.....

Coupling of Two Circuits with a Common Capacity

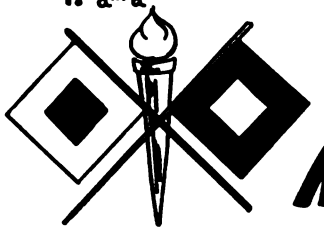
THE MATH:

The Algebraic Form

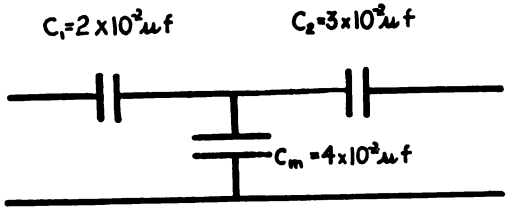
$$X = \frac{\sqrt{(a\,b)}}{\sqrt{(c+a)}\,\sqrt{(c+b)}}$$

The operations appearing in this radio application:

1. $\sqrt{(ac)(bc)} = c\sqrt{(a\,b)}$
2. $\sqrt{(ac^2+bc^2)} = c\sqrt{(a+b)}$
3. $\sqrt{(a^{-2m})} = a^{-m}$
4. $\frac{a^{3m}}{a^m} = 1$



PREPARED IN COOPERATION WITH THE BOYS' AND THE WOMEN'S MATHEMATICS CLUBS OF CHICAGO



C_1, C_2 and C_m are capacities of the condensers, and μf stands for microfarad (one millionth of a farad, the unit of capacity).

THE RADIO:

Coefficient of Coupling(k)

$$k = \frac{\sqrt{(C_1\,C_2)}}{\sqrt{(C_m+C_1)}\,\sqrt{(C_m+C_2)}}$$

$$= \frac{\sqrt{(2\times10^{-3})(3\times10^{-3})}}{\sqrt{(4\times10^{-3})+(2\times10^{-3})}\,\sqrt{(4\times10^{-3})+(3\times10^{-3})}}$$

$$= \frac{10^{-3}\sqrt{(6)}}{10^{-3}\sqrt{(6)}\;10^{-3}\sqrt{(7)}} = \frac{1}{\sqrt{(7)}}$$

$$= .38 -$$

MATH FOR THE SIGNAL CORPS!

PREPARED BY SIGNAL SCHOOLS
AIR SERVICE COMMAND

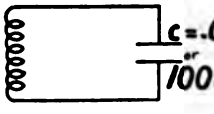
POSTER № 15

EXPONENTS and RADICALS enter into STUDY of RESONANCE

THE ELECTRICAL PROBLEM:

To what frequency is this circuit resonant?

find f



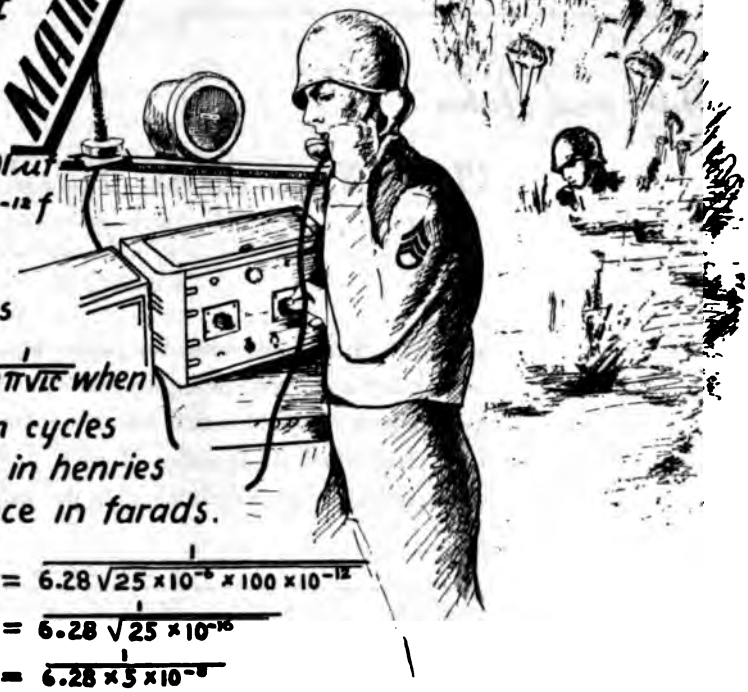
$L = 25\ \mu h$
or $L = 25 \times 10^{-6}$ henries

THE FORMULA $f = \frac{1}{2\pi\sqrt{LC}}$ when

f = frequency in cycles
L = Inductance in henries
C = Capacitance in farads.
$\pi = 3.14$

MATH IN RADIO

When you tune a radio receiver to the signal from a broadcast station, you adjust the input circuits of the receiver to resonate at the frequency of that station.



$$f = \frac{1}{6.28\sqrt{25 \times 10^{-6} \times 100 \times 10^{-12}}}$$

$$= \frac{1}{6.28\sqrt{25 \times 10^{-18}}}$$

$$= \frac{1}{6.28 \times 5 \times 10^{-9}}$$

$$= \frac{10^9}{31.4} = 3.18 \times 10^6 \text{ cycles}$$

or 3.18 megacycles.

PREPARED IN COLLABORATION WITH THE MEN'S' AND WOMEN'S' MATHEMATICS CLUBS OF CHICAGO

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POSTER # 14

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APPLICATIONS

Edited by SHELDON S. MYERS
Department of Education, Ohio State University, Columbus, Ohio

WE HAVE received a number of request for military applications of high school mathematics. There are unlimited possibilities in this direction, and this department can only present a few examples suggestive of the larger field. We are using the examples in this issue to provide references to some sources of applications of a military nature.

P.G. 5 Gr. 10–12 *Plane Geometry and Artillery Firing*

The following application is number 5 in the pamphlet, "Some Military Applications of Elementary Mathematics," by the Department of Mathematics, the United States Military Academy, June 1942, prepared for The Institute of Military Studies, The University of Chicago.

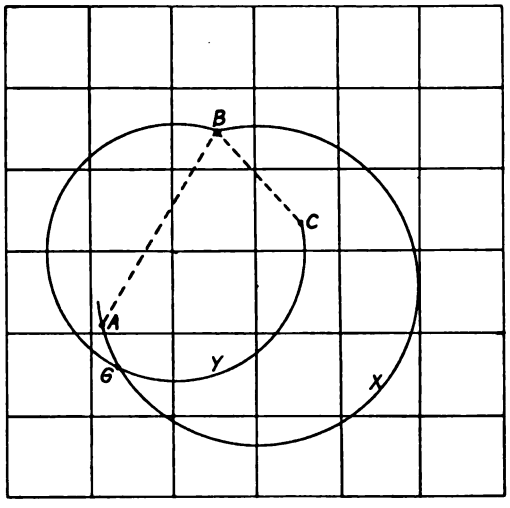
This problem is often encountered in military surveying. In the field, the problem is usually solved by another method, depending on certain items of surveying equipment. However, the solution given below, based on compasses and straightedge, illustrates the basic principle of solving.

"*Mathematical Features*: Construction of a segment which shall contain a given angle; intersection of loci.

"*Problem*: During an artillery reconnaissance, a gun position was selected, and the reconnaissance officer was instructed to plot its location accurately on the map. From the gun position, he was able to see three prominent landmarks, A, B, and C, which had previously been accurately located and plotted on the map. (See accompanying grid sheet.) Setting up an instrument at the gun position, he read angles as follows:

A-Gun-B = 800 mils or 45°
B-Gun-C = 533 mils or 30°

"*Requirement*: from the foregoing data, determine the map location of the gun position.

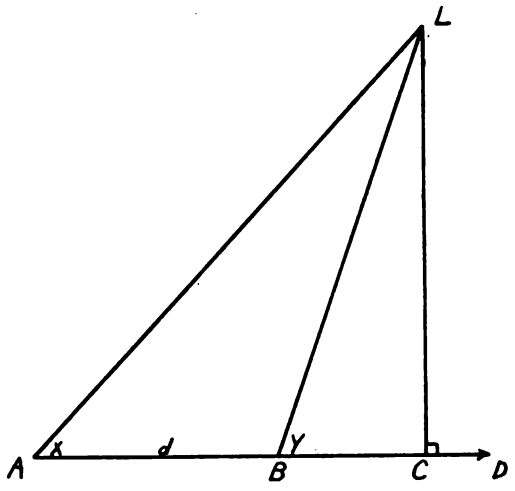


"*Solution*: Upon AB, construct a segment (AXB) which shall contain inscribed angles of 800 mils or 45°. Upon BC, construct a segment (BYC) which shall contain inscribed angles of 533 mils or 30°. The intersection of arc AXB with arc BYC is the gun position, G."

T. 3 Gr. 10–12 *How Far Is the Lighthouse?*

Problems of navigation usually play an important part in war time mathematics. Here is an application involving simple trigonometry and algebraic manipulation from "Wartime Applications of Mathematics," by John J. Kinsella, University School Series No. 1, Ohio State University, 1945, 31 pp. From page 23:

"A ship sailing in the direction AD observes that angle $LAD = x°$. After a run of d miles to B it notes that angle $LBD = y°$. What will be its distance from lighthouse L when it is "abeam" at point



C, so that angle $LCD = 90°$? Show that:

$$CL = \frac{d}{\cot x - \cot y}."$$

Ar. 12 Gr. 6–10 *Changing Metric Information about a Famous German Warplane to English Units*

The War Service Program of the Civil Aeronautics Administration was one aspect of a stepped-up program of youth education for the air age precipitated by the last war. Complementing this program was the publication of a number of textbooks in basic mathematics for aeronautics. One of these was *Basic Mathematics for Pilots and Flight Crews* by Newsom and Larsen, Prentice-Hall, 1943, 153 pp. The following is from page 73.[1]

"The following information is listed in regard to the famous plane, the Ju 49, developed in Germany in 1928.

Span: 28.26 m.
Length: 17.20 m.
Height: 4.75 m.
Weight: Loaded, 4,250 kg.
Maximum speed: 146 km./hr.
Greatest height reached: 12,739 m.

"Rewrite this information using the English system of units. Students would probably be eager to look up whatever metric data is available on the new Russian jet planes and convert it to English units.

[1] Reprinted by permission of Prentice-Hall, Inc. and the University of New Mexico Press.

Al. 10 Gr. 9–10 *Ratios of Planes at a Training Base*

Another problem is taken from page 41[1] of the above book.

"At a bombardier training base, there were ten AT-11's for each B-24 stationed at the base; but there were two of the B-24's for each B-17. How many planes of each variety were located at the base if there were 138 planes in all?"

C. Al. 5 Gr. 10–12 *Finding the Temperature of an Explosion*

The book *Engineering Problems Illustrating Mathematics* by John W. Cell, McGraw-Hill Book Company, 1943, 172 pp., contains an extensive collection of valid engineering problems arranged in five parts covering college algebra, trigonometry, analytic geometry, differential calculus, and integral calculus. Each of these parts are subdivided into subtopics. There are great numbers of problems which can be used on the high school level such as the following from page 4:[2]

"The temperature t of explosion (in degrees centigrade) of nitroglycerin is given by a solution of the equation:

$$(a+bt)t = 1000Q_v + 15(a+15b)$$

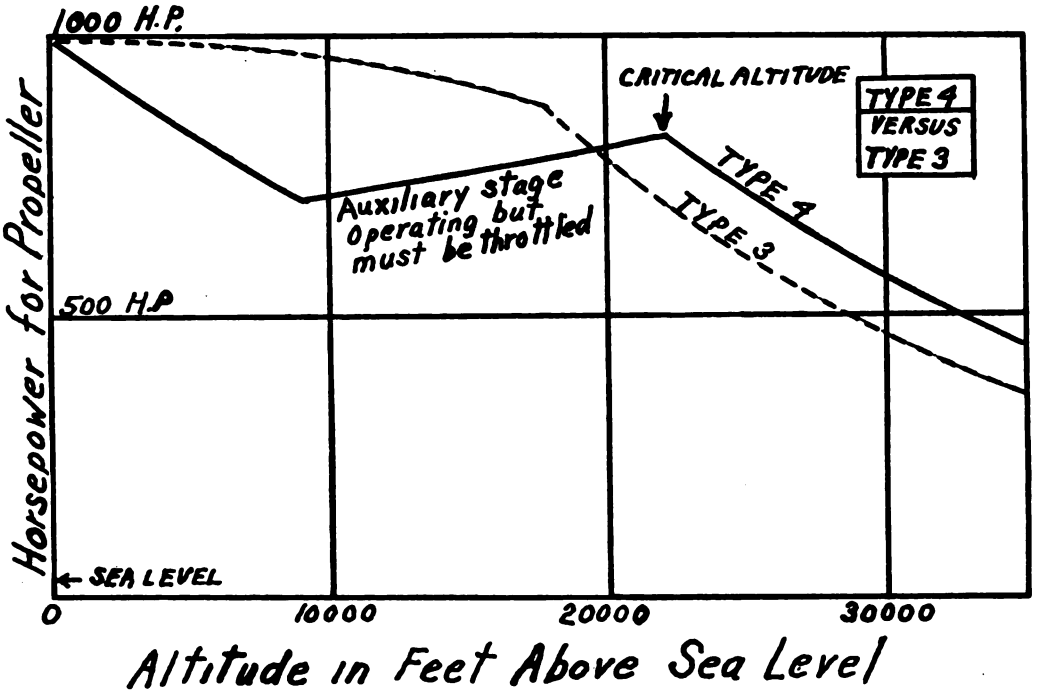
where, for nitroglycerin, $a = 43.705$; $b = 0.01775$; $Q_v = 346.5$. Determine the temperature t. Determine the temperature of explosion for ammonium picrate where the data for the formula are as follows: $a = 54.04$; $b = 0.01212$; $Q_v = 175.3$."

These problems come under "Quadratic Equations" in the "College Algebra" part.

Al. 11 Gr. 9–10 *Performance Graphs of Superchargers*

The ability to interpret graphs is one of the most important outcomes of secondary mathematics. Why confine the study of graphs in ninth and tenth grade algebra to the abstract variables X and Y? The study and visualization of functions through graphs of interesting concrete var-

[2] Reprinted by permission of the Editorial Committee of the Mathematics Division of the Society for the Promotion of Engineering Education, John W. Cell, Chairman, and the McGraw-Hill Book Company.



iables is mathematically fruitful and psychologically motivating. In this connection a source which has proved to be very worthwhile is the 80 page pamphlet entitled "Airplane Power, with Special Reference to Engines and Altitudes" published by General Motors in 1943. Boys in particular have been challenged by this clearly written manual on seven types of airplane engine superchargers and their altitude performance graphs. A teacher without any background could easily read the manual in one or two hours and understand its essential features. The manuals were free and could be ordered in quantity by writing Room 11-230, General Motors Corporation, Detroit, Michigan.

Superchargers are special blowers attached to the fuel intake systems of aircraft engines to offset the effects of low oxygen due to high altitudes. Seven different, graduated types are clearly and simply diagramed and explained. The performance of each type is compared graphically with the preceding type of supercharger. The graph above,[3] taken from the manual, compares Type 4 with Type 3. Type 4 is a two-stage supercharger with mechanical clutch, while Type 3 is a single stage supercharger with variable speed clutch. Type 4 consists of two blowers, one running continuously, while the other one is put into operation at 9000 feet. Notice what happens to the graph for Type 4 at 9000 feet. Students can be asked to compare the two types, giving the advantages and disadvantages of each as shown by the graph.

Some "Tail End" Data

Research has shown that a cocker spaniel wags his tail with a frequency of about 300 vibrations, or wags, per minute. More profound investigation of the problem revealed that the end of a four-inch tail traveled 3 inches with each wag, giving a tip speed of 900 inches per minute or 85 hundredths of a mile an hour.

[3] By permission of General Motors Corporation.

"How Children Use Arithmetic" by Effie G. Bathurst is a new Office of Education Publication which shows how children develop arithmetic abilities. It is listed as Bulletin 1951, No. 7, contains 13 pages and may be purchased from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. for 15 cents. Catalog No. FS 5.3:951/7.

AIDS TO TEACHING

Edited by

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and

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BOOKLETS

B. 83—Use, Care, and Adjustment of Polar Planimeters

Keuffel & Esser Co., 127 Fulton Street, New York, N. Y.

Pamphlet; 9″×6″; 14 pages; $.50

Description: After a brief description of the planimeter and its parts, the operation of ascertaining an area is presented in detail. This operation consists in the proper placement of the device, the movement of the tracer on the perimeter of the area to be measured and the reading of the measuring wheel. Variations of these instructions are given for different types of planimeters. Suggestions are included on the care of the instrument so that maximum accuracy and use will be attained from each instrument.

Appraisal: This pamphlet will give the teacher a simple explanation of how the planimeter works as well as how to use it to measure area. Probably the reason planimeters have so seldom been used by mathematics teachers as an example of a measuring device or as a check of area computation has been a lack of knowledge of how they work. This pamphlet will be useful in showing teachers another device to be used for measurement activities.

B. 84—Ranger 'Rithmetic

U. S. Department of Agriculture, Forest Service, Washington, D. C.

Booklet; 8″×10″; 12 pages; Free

Description: The main part of this booklet is concerned with 21 problems in arithmetic. It is stated on the cover that these are intended for seventh grade classes, but the numbers and vocabulary may not be suitable for some groups at that level. Each problem is accompanied by a black and white drawing, and has the answer printed under it in smaller type. Then there follows a message about conservation which pertains to the problem. Here is one of the shorter ones: "Problem No. 6. There are 4,200,000 small forest ownerships in the United States and 97% of them are east of the Great Plains. How many are east and how many are west of the Great Plains? Answer: 4,074,000 east; 126,000 west. Small forests are the greatest part of our forest problem because there are so many of them and because the owners are often indifferent toward good care of them." There is a list of materials to help teach forest conservation.

Appraisal: The problems are often as artificial as many of the common problems in arithmetic books, but the cumulative effect of so many on one subject, and the reality of the numbers makes this booklet a good supplement in teaching. The little pictures add a great deal to the interest. There is no reason why conservation should not be taught every time that it is possible to do so. It is difficult to assay the actual value of forest saved by the publication of this booklet, but then much of the expenditure of the government is removed from its ultimate intended effect. There is no doubt that the objective is worth while (but what equally worthy projects were

excluded by this one?). When materials such as this exist, teachers should certainly make use of them.

B. 85—Standard Time Throughout

Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

Booklet; 6″×9″; 30 pages; $.15

Description: This booklet, by R. E. Gould, is a publication of the National Bureau of Standards. It begins with a short historical sketch of standard time, and then contains two long sections on "Standard Time in the United States," and "Time in Foreign Countries." Some of the topics treated are time zones, time signals, international date line, time designations, and daylight saving time. There is a long table comparing the standard times in countries throughout the world. A bibliography is included.

Appraisal: It is not important to consider whether *all* the material in the booklet would be used in a mathematics classroom. So many of us are textbook minded that it seems wrong to use advanced material which is read only in selections. And yet, this process of selecting and discarding parts of a work is seldom taught in mathematics classes. The exercise of judgment is neglected. For this and other reasons there is a place for this interesting booklet. The use of plus and minus signs to signify zone numbers is practical and enlightening. The understanding of denominate numbers, hours—minutes—seconds, is important. This would be a very valuable addition to a classroom library of supplementary materials.

B. 86—How to Choose a Slide Rule

Keuffel and Esser Company, 127 Fulton Street, New York, N. Y.

Booklet; 5½″×8¼″; 22 pages; Free

Description: The booklet begins with a rather general discussion of advantages of performing operations on a slide rule. Then the advantages and disadvantages of specific types of rules are presented. The models included are the Mannheim Polyphase, Polyphase Duplex Trig (and Decitrig), Log Log Duplex Trig (and Decitrig), and the Log Log Duplex Vector.

The story-like account is very humorously written and interestingly presented. The booklet is printed in black and red upon white pages. It is excellently illustrated by fifteen cartoons and photographs of each rule which is discussed. The last five pages are devoted to advertising which is hardly objectionable because it is so informative about prices and availability of slide rules in general.

Appraisal: The booklet is excellent for teaching high school students to select a specific slide rule to fulfill predetermined desires. Although the discussion is intended primarily for college students, it is perfectly suitable for college preparatory students in the later years of high school; slide rules should become familiar instruments by the time a student enters college, and proper instruction on the high school level serves to acquaint the student with the model he needs. The necessary facts are presented in the booklet in a form simple enough for the student to understand.

The booklet is not suitable for use in the ninth and tenth grades because students have not yet encountered many of the problems which make the slide rule a beneficial instrument. Several copies of the booklet are sufficient to satisfy the needs of the entire class; they may be circulated among the pupils. (Reviewed by Bernard Singer, Hyannis, Mass.)

B. 87—Map Projection Studies, No. P208

Denoyer Geppert Company, 5235 Ravenswood Avenue, Chicago 40, Ill.

Booklet; 8½″×11″; 20 pages; $0.50

Description: Many types of map projections are compared by means of illustrations, questions to the reader and a short textual description. The following projections are discussed: orthographic,

stereographic, globular, gnomonic, polar equidistant, polar equal area, stereographic cylindrical, gnomonic cylindrical, mercator, homolographic, sinusoidal, semi-elliptical, homolosine, simple conic, conic two standard parallels, conformal conic, polyconic, Albers, and Bonne.

Appraisal: Naturally the meaning, definition and derivation of these projections is not intended at the secondary school level. However, it is very convincing to thumb through these pages and see how the facts of plane and solid geometry are put to use in geography. The illustrations are remarkably clear and helpful.

B. 88—Study Guide and Work Book (Life Insurance)

B. 89—Set of Questions (Life Insurance)

Institute of Life Insurance, 488 Madison Avenue, New York 22, N. Y.

Booklets; $8\frac{1}{2}'' \times 11''$; Free

Description of B. 88: This is a workbook (40 pages) to be put into the hands of the pupils in a class which has studied the "Handbook of Life Insurance" (B. 12—November, 1948). It is framed in terms of the meanings and descriptions of life insurance; very little inspiration for computation is included.

Appraisal of B. 88: This is an extremely complete booklet on the facts on life insurance—far too complete. Few schools would find it worth their while to spend the amount of time on the subject which this booklet would demand. It will be tempting to do so, to the detriment of other subjects, but if proper selection is made, this booklet should be a tremendous help.

Description of B. 89: On 8 pages there are 20 true-false, 20 multiple choice questions and 20 completion questions with answers and references for instructors to use.

Appraisal of B. 89: The questions are not too well written, but suffer from the common ambiguities of objective-type questions when poorly written: they often make little sense without their answers. The amount of detail expected is much too much for the time spent on insurance in today's schools. However, by selecting and adapting, the material will become useful to teachers.

B. 90—Elementary Instructions for Operating the Slide Rule

B. 91—Graph Sheets

B. 92—Templates and Other Drafting Tools

B. 93—Drafting and Reproduction Equipment and Materials

Keuffel and Esser Co., 127 Fulton Street, New York, N. Y.

Booklets; $5\frac{3}{4}'' \times 8\frac{3}{4}''$; Free

Description of B. 90: This 19-page booklet is useful only to those who have slide rules in their hands, and most useful to those with K & E rules. It tells how to read scales, how to multiply, divide, square, find square root, use proportion, the CI scales, cube and cube root scales, and trigonometric problems. It is all based upon the Mannheim-type rule.

Description of B. 91: This 81-page booklet is a mine of information concerning the types of graph papers and cloths now available. It is beautifully illustrated in color. All the usual types and many special ones are shown.

Description of B. 92: This 12-page booklet shows many interesting drafting aids which have some mathematical content.

Description of B. 93: This 254-page booklet is Part 1 of the 41st Edition of the K & E catalog. It lists and illustrates drawing and tracing papers and cloths, graph sheets, drawing instruments, drafting tools, printing and developing equipment, drawing boards and furniture, slide rules and other calculating instruments.

Appraisal of B. 90 through B. 93: In all of these booklets the clarity of the illustrations and the completeness of the coverage is outstanding. B. 90 should not be sent for unless a school has or intends to buy slide rules. In order to follow the instructions a slide rule should be in front of you. B. 91 is a real education in the ingenious and useful types of graph ma-

terial. B. 92 has some small amount of application, but may suggest some instruments which will improve the drawing of geometric plates. B. 93, the complete catalog, will deserve a great deal of study on the part of both teachers and pupils to see the equipment which is used to apply mathematics to the field of drafting and engineering.

CHARTS

C. 27—Road Maps of Industry

National Industrial Conference Board, 247 Park Avenue, New York 17, N. Y.

Graph charts; 8½″×11″; Free

Description of C. 27: These charts are really splendid examples of mathematical graphs on such subjects as U. S. Saving Bonds, Farm prices and parity, wholesale prices, military manpower, personal income, federal expenditures, world steel capacity, and gross market value of products of the United States. They are printed in clear and bright colors and are punched with three holes to put into notebooks. These charts are available free only to high school teachers, staff members of teachers colleges, and administrators at the above levels. They must be sent to school addresses. Sometimes additional copies of certain charts are available for pupil use.

Appraisal of C. 27: It is hard to overemphasize the attractiveness of these graphs; they are very suitable for mathematics classes since a variety of types are constantly used. The fact that they present the numerical facts of economics with up-to-date precision makes it mandatory that teachers use them to supplement the graphs which appear in textbooks. Even if they are merely displayed on bulletin boards they will improve the teaching of mathematics, but it would be a shame not to help the class to interpret them and find the many conclusions which can result.

EQUIPMENT

E. 46–E. 66—Grove's Moto-Math Set

BASIC

E. 46–(250-M)—Moto-Math Manual
E. 47–(250-1)—Blackboard Graph Chart
E. 48–(250-2)—Stainless Steel Split Pins
E. 49–(250-3)—Heavy Multi-colored Elastics
E. 50–(250-4)—Beam Blackboard Compass
E. 51–(250-5)—Two Semicircles—16 sectors
E. 52–(250-6)—Three Triangles
E. 53–(250-7)—Two Congruent Trapezoids
E. 54–(250-8)—Navigation Protractor
E. 55–(250-9)—Standard Protractor
E. 56–(250-10)—Combination Curve Drawing Instrument
E. 57–(250-11)—Straight Edge and Two Spring Clips With Cord
E. 58–(250-12)—Linkage Set
E. 59–(250-13)—Wooden Case for Accessories
E. 60–(250-14)—Miniature Fibre Airplanes

EXTRA

E. 61–(250-15X)—Hard Fibre Fractions Discs
E. 62–(250-16X)—30 Counting Discs
E. 63–(250-17X)—Abacus
E. 64–(250-18X)—Clock Face
E. 65–(250-19X)—Pantograph
E. 66–(250-20X)—Easel

Developed by Prof. J. Seth Grove, Head of Mathematics Department, State Teachers College, Shippensburg, Penna.

Purchased through Yoder Instruments, East Palestine, Ohio.

Description of E. 46—(250-M): This 30 page manual is designed to assist the teacher in the use of the Moto-Math Set. It presents many diagrams with descriptive instructions on how to use the material.

The manual lists demonstrations as follows: plane geometry—36, solid geometry—4, linkages—12, physics and mechanics—7, algebra—12, trigonometry—8, navigation—6, arithmetic—8, analytical geometry and calculus—18. Additional demonstrations can be devised by teachers and students. The price of the manual is 30¢ each—4 copies for $1.00.

Description of E. 47–(250-1): This blackboard graph chart is of 20 gauge sheet steel measuring 32″×34″ in a hardwood frame—ruled 1″ squares with holes punched at intersections of all lines and at other places for various conic section curves. There are 977 holes in all. This part of the equipment is the basic unit upon which the set is built. It is used in practically all demonstrations. The price of the graph chart is $35.00.

Description of E 48–(250-2): These 25 Split Pins are used in the holes on the Graph Chart to support various devices on the board and to serve as vertices of diagrams when elastic bands are used. The price for the set of 25 is $1.00.

Description of E. 49–(250-3): This set of 50 multi-colored elastics are of good rubber and serve the purpose of forming geometric figures on the graph board when stretched between the pins. The price for the 50 elastics is $.50.

Description of E. 50–(250-4): These compass arms are of flexible steel 12″ and 30″ long. An adjustable pin is provided, which makes it possible to mount the compasses on the blackboard graph. The price for the set is $3.00.

Description of E. 51–(250-5): Two semicircles, each of ¼″ composition material with glazed surface. Together they form a circle of 12″ diameter. Each semicircle is divided into 8 equal sectors, which are attached by leather at the circumference. These semicircles are used, when supported on the blackboard graph chart by pins and the ruler, to show area of a circle and its relation to the area of a parallelogram. The price for the set is $5.00.

Description of E. 52–(250-6): These triangles are constructed of ¼″ composition material, with white glazed surface, with inch rulings vertically and horizontally. The three triangles may be used separately and also can be assembled to form a rectangle or a parallelogram. They are 45-45-90, 30-60-90, and 75-60-45 triangles. They may be used with the blackboard graph chart as background when supported by the ruler. The price for the set is $4.00.

Description of E. 53–(250-7): These two congruent trapezoids are constructed of ¼″ composition material, with white glazed surface. They measure 8½″ on the base and 7″ high. They can be used separately or to form a parallelogram when put together to show area relationships and congruency. They may be used on the blackboard graph chart when supported by the ruler. The price for the set is $3.50.

Description E. 54–(250-8): This Navigation Protractor is of laminated paper, 7″ in diameter and has markings around its circumference in units of 5°. It may be mounted on the blackboard graph chart to illustrate angle size especially in solving navigation problems. No. 250-14, the miniature airplanes, may be used with this protractor to make the diagrams more realistic. The price for the protractor is $.75.

Description of E. 55–(259-9): This is a standard circular protractor on a masonite base with a 7″ diameter. It has markings in units of one degree from 0 through 360 each way. It can be mounted on the blackboard graph chart to illustrate angle size. It can be used to draw cycloids and trochoids on the blackboard graph chart by using the proper holes for chalk and by revolving the protractor on the ruler. The price of this protractor is $2.50.

Description of E. 56–(250-10): This set consists of pins, cord of different lengths, a chalk holder, and a special wooden ruler in the form of a T square to be mounted on the ruler on the blackboard graph chart. The set is used for drawing the parabola, ellipse and hyperbola. The price for the set is $3.50.

Description of E. 57–(250-11): This is the straight edge ruled in inches by units of ⅛ on one side and metric units on the other. It is approximately 30 inches long and is used for measuring, for mounting on the blackboard graph chart by means of 2 spring clips which come with it, or for drawing lines. When mounted on the

blackboard graph chart, it is useful for supporting many of the objects used in a variety of demonstrations. The price for the set is $1.75.

Description of E. 58–(250-12): This linkage set consists of 12 strips of hard white fibre ⅜″ wide and in varying lengths, 1 piece 14½ inches, 4 pieces 12 inches, 2 pieces 8 inches and 2 pieces 6 inches. It is accompanied by 6 split pins, 6 brass nails, and 6 rubber spacers and a chalk holder. The strips may be used to demonstrate linkages on the blackboard graph chart in problems such as: If the base of a triangle is 12″ and the altitude is 8″, what are the limits in the length of the other two sides? Many locus principles can also be demonstrated. The price for the set is $4.50.

Description of E. 59–(250–13): This is the wooden case measuring 9″×33″×2½″. It is divided into 3 compartments and is used to store and transport the many items that make up the total equipment for the set. The price is $2.50.

Description of E. 60–(250-14): There are three white miniature airplanes measuring 4″×4″ made of hard fibre with holes for mounting on the blackboard graph chart. They are used to make problems in Navigation more realistic. The price for the set is $1.50.

Note on prices: If the individual prices for items 250-M through 250-14 are added, the total price would be $70.00. However, the complete set may be purchased for $60.00.

Extra Equipment—Sold Separately

Description of E. 61–(250-15X): These hard-fibre fraction discs consist of two circles, a red one and a white one, mounted together so that they may be rotated with respect to each other. When mounted at the center of the board all useful fractions may be quickly illustrated by rotation. The price is $2.50.

Description of E. 62–(250-16X): This set of counting discs consists of 30 pieces of white hard fibre 1½″ in diameter with a hole punched in the center for mounting purposes. Three spring clips are provided on which to assemble the discs. They may then be mounted on the chart to serve as objective material for teaching the meaning of number and the fundamental facts and processes of arithmetic. For example, showing 3 groups of 2 in the number 6. The price is $2.00.

Description of E. 63–(250-17X): This Abacus consists of three stainless steel strands each with 10 colored wooden beads. They may be mounted on the blackboard graph chart to teach the meaning and use of number. The beads can be slid up and down the wires. Each bead is constructed so as to remain in any position on the wires even when the wires are in a vertical position as illustrated. Each individual string of ten beads is easily removable from the abacus so that it can be used for more elementary work with numbers before the pupil is ready for the use of the abacus as such. The price is $4.00.

Description of E. 64–(250-18X): This clock face is plastic laminated cardboard 12″ in diameter with two steel hands. It may be mounted on the Graph Board and has many uses in teaching number and time. The price is $2.00.

Description of E. 65–(250-19X): The pantograph consists of four wooden (maple) bars (2–18″ and 2–20″) fitted to mount on the Graph Board. It is a mechanical instrument used to enlarge or reduce drawings, pictures and diagrams. With it comes pins, eyelets and a metal base for mounting purposes. The price is $3.00.

Description of E. 66–(250-20X): This is the hardwood easel which has three legs and a 34″ chalk tray. It is used to support the Graph Board. The price is $15.00.

Appraisal of E. 46–(250-M): The manual contains 30 pages and 112 key problems with illustrative and explanatory material on the use of the Moto-Math Set. By following its directions and observing the diagrams, the equipment can be effectively used to carry out the many demon-

strations listed in plane geometry, solid geometry, linkages, physics and mechanics, algebra and graphs, trigonometry, navigation, analytic geometry, calculus, and arithmetic. The diagrams, showing the use of the equipment on a graph background, are placed beside the explanatory material for convenience. Teachers and students will find the manual very helpful in carrying out the demonstrations. Additional problems with illustrations on the subjects listed and other subjects will be issued from time to time.

Appraisal of E. 47–(250-1): The blackboard graph chart is sturdily built, attractive, and will stand plenty of use. It may be hung by the two strong hooks provided or supported by the easel. It is a convenient and easy-to-use piece of equipment and can add much when used for class demonstrations or by individual pupils. It is a good blackboard graph that can be used for many mathematical purposes.

Appraisal of E. 48–(250-2) and E. 49–(250-3): The split pins and elastics are necessary pieces of equipment to carry out many of the demonstrations. The pins are well made of stainless steel and will stand up well. The elastics are of good rubber and add color to the demonstrations. It might be better to have elastics of two lengths—some being longer than those now provided.

Appraisal of E. 50–(250-4): These special compasses are invaluable in drawing circles on the Graph Board. A circle may be drawn at any point by inserting the split pin of the "center post" in any desired hole. A slight pressure on the cap of the "center post" releases the "compass arm" which may then be adjusted to any desired length, 1″ to 12″ with the short arm and 1″ to 30″ with the long arm.

Appraisal of E. 51–(250-5), E. 52–(250-6), E. 53–(250-7): The semicircles, triangles and congruent trapezoids are well made of composition material and are valuable aids when used either with or without the Graph Board for many demonstrations of area and linear relationships.

Appraisal of E. 54–(250-8) and E. 55–(250-9): The navigation and standard circular protractors are useful and valuable aids when used with the Graph Board for demonstrations involving angle size. When used with the colored elastics and the pins the protractors mounted on the graph board make an attractive and functional device. The protractors would probably be even more effective if they were slightly larger so that they could be easily seen from the back of a classroom.

Appraisal of E. 56–(250-10) and E. 57–(250-11): The set of pins, cord and chalk holder and the straight edge are indispensable pieces of equipment for drawing lines and measuring. They are well made and should last a long time. The straight edge has the additional value of its use for supporting other objects when attached to the graph board.

Appraisal of E. 58–(250-12): The linkage set is a valuable addition to the equipment since many area and linear relationships can be clearly demonstrated when the strips are used on the graph board. Many loci using linkage strips can be quickly and clearly demonstrated. The importance of this subject of linkages is often overlooked in the teaching of mathematics. This method of demonstrating them makes it a pleasant and worthwhile project. With the equipment provided, the strips are easily attached to the graph board.

Appraisal of E. 59–(250-13): The wooden case is a must since the many pieces of equipment should be carefully stored when not in use. The case is well built and worth the money.

Appraisal of E. 60–(250-14): These miniature planes are not indispensable, but do add a touch of realism when used on the graph board in navigation problems involving drift angle, heading and course. They are well-made of hard fibre and should last a long time.

Appraisal of Extra Equipment—E. 61 through E. 66–(250-15X through 250-20X): This equipment has many uses in demonstrations of various kinds. The fraction

discs, counting discs, abacus and clock face are valuable aids, especially at the elementary school level, for objective material in the teaching of number, the meaning of the fundamental facts, and the meaning of the fundamental processes of arithmetic. They can be used separately, but are more effective when used with the graph board as background. The pantograph is a helpful piece of equipment that is well-made and comes complete with the necessary parts. It is useful for enlarging or reducing drawings, pictures and diagrams.

There are many other uses for the set not listed here in a variety of subjects. It would, for example, be helpful to the arithmetic teacher in drawing different kinds of graphs, showing perimeters and areas, illustrating the meaning of per cent, decimals and fractions, as well as in making diagrams in teaching understanding in the solution of problems. (Reviewed by John K. Moulton and Ralph F. Ward of Brookline High School, Brookline, Mass.)

FILMS

F. 21A—What Is Money?*

Producer: Coronet Films, Coronet Building, Chicago 1, Ill.

Educational Collaborator: Paul L. Salsgiver

16 mm. film; Black and white; Sound; one reel; nine minutes; $45.00; colored, $90.00.

Description: Recommended grades from producer: Intermediate, Junior High, Senior High, and Adult.

In this film we follow the journey of a brand new five dollar bill through its many transactions. The bill starts in circulation with Tom receiving the five dollar bill from his father. He is to buy a gallon of paint with it at the hardware store. From the hardware store the five dollar bill goes to an electrician in payment for special services he has rendered. On the way home the electrician stops in at a gasoline station to get some gas and pays for it with the five dollar bill. Mrs. Moore comes in the gasoline station to cash a check and receives the bill as part of the transaction. After cashing her check, Mrs. Moore goes downtown to buy a pair of bookends which cost five dollars. The store is then shown paying off its employees and the five dollar bill goes to one of the employees who in turn takes it to a radio store. This employee had bought a radio on the installment plan and wants to make her weekly payment. The radio man upon receiving this bill gives it to a boy who has been working for him. We see that this boy is Tom, the same boy who started the five dollar bill on its journey. Tom deposits the money in the bank.

Interwoven among the journeys of the five dollar bill are pictures of how the barter system was used in ancient times and still is today. It shows two boys, one is trading a bicycle light for a catcher's glove. We also see how different articles have been used as money such as: spears, shells, and wampum. We are shown how people used to make their own money and how the government now makes it.

The film closes with a summary of the uses of the five dollar bill. First, it was a medium of exchange for goods; second, it was a medium of exchange for services; third, it was a standard of value; fourth, it was a standard of future payments; and fifth, it was a storehouse. It is shown that the bill could conceivably have gone to a world traveler and traveled all over the world.

Appraisal: Seven teachers evaluated this film and five of the teachers in considering the entire film said it was good for introducing new material, providing a common experience, and for motivation. All of the teachers thought the film was good for review. The grade level was placed between the Elementary and College level with the 7th and 8th grade being checked by 53%. One half of the evaluators favored arithmetic as the course best suited while the other half chose consumer mathematics. One teacher thought the

(*Continued on page 584*)

* This is the first aid to teaching which we have reviewed for the second time. Letters will follow the numbers to indicate repeat reviews.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* JOHN A. BROWN
The University of Wisconsin, Madison, Wisconsin

IN ADDITION to statistical information on school offerings, in this department during this year, there will be published accounts of interesting and unusual school activities which have been contributed directly to *The Mathematics Teacher*, or which have been brought to the attention of the editors in other ways.

FOURTH TERM ALGEBRA

The topic at one of the discussion groups of the summer session of the National Council of Teachers of Mathematics at Northfield, Minnesota, was "What Belongs in Fourth Term Algebra?" The variety of opinions and practices prompted several of the group to suggest that a report in *The Mathematics Teacher* might be a step in determining whether other teachers would be interested in further discussion at a subsequent meeting.

At least two factors are responsible for the wide variety of topics taught in fourth term high school algebra. One is the time the fourth term is taught. Some schools teach algebra the ninth and tenth years; others the ninth and eleventh years; still others the ninth, one semester of the eleventh, and one semester of the twelfth year. Another factor is the purpose of the course: a polishing up of previous work in algebra, a preparation for algebra in college, an introduction to the topics taught in algebra in college, or some combination of these purposes.

In our discussion we began with the list of topics offered in a widely used textbook, attempted to determine their value to a high school student, and tried to decide whether they belonged in fourth term algebra. By a show of hands we made a record of the practices in the schools represented. It must be remembered that the time of giving the course is often a deciding factor in choosing topics. For instance, if fourth term algebra follows trigonometry, obviously logarithms must be taught at another time.

The following topics, some of them continuations of third term work, were the only ones on which there was majority agreement: factoring, fractions, exponents, radicals, imaginaries, equations linear and quadratic in one and two variables, equations involving radicals, equations of degree higher than the second that can be solved with the factor theorem.

The topics taught by some of the teachers in the group, but not by a majority of them were: functions and their graphs, determinants in two and three variables, ratio, proportion, variation, arithmetic and geometric progressions, logarithms, permutations, combinations, and probability.

No one in the group did anything with analytical geometry or calculus and the college teachers present agreed that these topics were best left for college students.

Do you think that further investigation of the subject would be profitable?

ONA KRAFT
Collinwood High School
Cleveland, Ohio

NEWARK SECONDARY SCHOOLS SPONSOR MATHEMATICS FAIR

Come to the Fair! Come one! Come all! On April 25, 1951 Arts High School in Newark, New Jersey, was the scene of this unusual hubbub sponsored by the Newark Council of Teachers of Mathematics. Had you entered the school on that afternoon large posters would have invited you to

the "fair grounds". More pointedly, the decimals of π strung high on a wire would have directed you to the gym.

However you wouldn't have recognized the gym. The centerpiece, a huge solid of many faces, used a few of them to observe you as you pinned on the name tags handed to you by the "barkers". Posters of geometric designs in thread and wool tacked on large screens, and mathematical newspaper clippings adorning the walls, served as open invitations to the sidelines and byways of mathematics. These led next to the large tables displaying geometric models and applications and attended by students of Newark High Schools; students of Montclair State Teachers College; Mr. George Kays, an instructor at the College; Mr. Charles Goeller, an instructor at the Newark School of Fine and Industrial Arts; and various other teachers of Newark Secondary Schools.

At many of the tables you would have been forced to edge your way closer in order to get a good look at the soap film experiments, the statistical array of shot in columns, the surveying instruments, the various models of solids—some in paper form, others of ivory texture. Once aroused, your curiosity would not have been satisfied until you circled the fair grounds again and again, brushing shoulders here with young mathematics students, there with teachers, and perhaps even with a celebrity or two, like Mr. Michael McGreal, Assistant Superintendent of Schools of Newark, or Dr. Virgil S. Mallory, Head of Mathematics Department at Montclair State Teachers College, who found time in their busy schedules to meet us at the Fair. But once again the "barkers" were at work urging everyone on to the "big tent". On the way you must stop for your free refreshments, in the form of lollipops, and a "program", in the form of a mimeographed numbered pamphlet containing mathematics puzzles, problems, and diagrams.

In the auditorium Harold A. Gouss of Newark Central High gave a short welcoming address in his official capacity as president of the sponsoring organization. Next, the Ringmaster, Mr. Ernest R. Ranucci, Chairman of the Mathematics Department of Weequahic High and of the program committee for the Fair, introduced the main "acts".

First, Sanford Kahn, student of Weequahic High gave his magic demonstration of soap film experiments. Next Irving Eng, student of Arts High showed the use of the abacus in computation. Mr. Goeller then spoke on his process for constructing geometric models with artistic overtones. The final act guided us on a historical journey through the development of units of measurement and was led by a student of Montclair State Teachers College.

Mr. Max Sobel, teacher at Robert Treat Junior High, took over the main circle to introduce the final event of the Fair, a contest of mathematical skill. He determined five contestants by a drawing of numbers based on those of the program pamphlets. Peter Tango of Cleveland Junior High was adjudged the winner and awarded the grand prize, a 10″ slide rule accompanied by 10 humorous rules for happiness via its use.

In the Executive Council of the Newark Council of Teachers of Mathematics the usual post-mortems helped adjust the financial balance of this event from its physical debit side to its intangible benefits to students and teachers on the credit side. We teachers of mathematics in Newark had, in the past year, seen fit to band together to discuss our problems and attempt concerted efforts at their solution; to stimulate interest in mathematical education on all levels, administrative, teacher, student, and parent; to do so vividly and dramatically, not disdaining the well-proven methods of the salesman, the "barker", to achieve these worthwhile ends. Therefore we felt justified in our estimate of the credits accruing to the organization from this first public project. We were not unmindful of the wonderful

cooperation received from the administration at Arts High, from the Mathematics Department of Montclair State Teachers College and from all of the teachers and students who lent a hand in this achievement. In fact it was this very cooperation which has greatly encouraged us in contemplating plans for future activities based on the main objectives previously noted.

Perhaps others in your organization who feel a need for cooperative action on a local level will also be encouraged to join us in achieving them on a wider scale. Surely this will make for a more vital mathematical program by teachers in times when everything is in great need of vitality and nothing more so than the field of mathematical education in the secondary schools.

HAROLD A. GOUSS, President
The Newark Council of Teachers of Mathematics
Central High School
Newark, New Jersey

Aids to Teaching

(Continued from page 581)

speed of development of ideas and duration of scenes was too fast. No inaccuracies were found in the mathematical content. 57% said the film would hold the interest of the student and that the teaching methods in the film were conducive to learning. 86% thought that the content could not be just as effectively and efficiently presented in some other way. Six of the seven teachers said they would use the film in their classroom. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway)

F. 65—Angles

Knowledge Builders, 625 Madison Avenue, New York 22, N. Y.

16 mm. film; Black and white; Sound; one reel; eleven minutes; $40.00.

Description: This depicts various types of angles and their relationship to each other.

It is shown that two rays intersecting form four angles. Complementary and supplementary angles are defined and illustrated by animation. The bisector of an angle is defined and a circle with an angle of sixty degrees marked off. This is divided into two thirty degree angles to illustrate the bisecting of the angle. A drawing is made of a transversal through two parallel lines, and it is then pointed out that there are four acute and four obtuse angles as a result of this. Interior and exterior angles are defined and pointed out.

Various scenes such as buildings, bridges, etc., are shown throughout the film to illustrate the practical use of angles.

Appraisal: For the evaluation of this film there were thirty-seven teachers of mathematics present. Taking the entire picture into consideration 83% thought it was fair or poor for introducing new material, 83% said it was good or fair to augment explanations, 44% checked poor for the development of skills, and 35% said it was excellent for review. 57% favored grade 10 for this film while the other evaluators checked grades 7 through the college level. 75% checked Geometry as the course best suited for this film while 15% checked Shop Mathematics. Nine teachers thought that the speed of development of ideas and duration of scenes was too fast. 38% found inaccuracies in the film. Two-thirds of the evaluators thought the film would partially hold the interest of the students, but 80% found the teaching methods conducive to learning. 50% found that the photography and sound was good. One-half of the teachers thought that the amount of material was not reasonable for comprehension in one

(Continued on page 588)

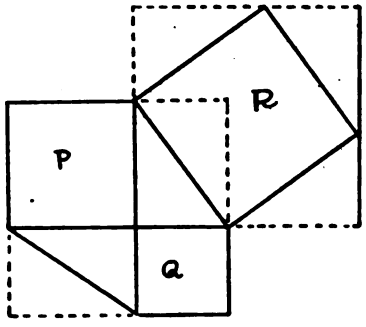
REFERENCES FOR TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

The Theorem of Pythagoras

JUSTLY called the most celebrated theorem of geometry, the Pythagorean proposition is probably the one bit of mathematics which millions of laymen will remember long after they have forgotten whatever other mathematics they may once have known. The theorem is notable first because of the rich historical associations with which it is attended; secondly, because of the amazing variety of proofs which have been given; and thirdly, because further exploration quickly leads to interesting and unexpected byways, such as the Golden Section, dynamic symmetry, logarithmic spirals, angle trisection, duplication of the cube, squaring the circle, determining the value of π, the concept of irrational number, regular and star polygons and polyhedra, theory of numbers, constructibility of angles and polygons, continued fractions, phyllotaxy, musical scales, Diophantine equations, Heronian triangles, and Pythagorean number lore.

Thus the theorem of Pythagoras lends itself admirably as a topic for discussion in mathematics clubs or for further study by mathematically minded students. Many proofs are readily accessible, particularly in the well known monograph by Loomis and in recent volumes of *Scripta Mathematica*. Some of these analyses are challenging to able minds, as for example, the alleged proof of *Multatuli* (1867), than which, the author asserted, no other proof could possibly be simpler. In view of these observations, teachers may find the following references of more than passing interest.



MULTATULI'S "PROOF": $P+Q=R$

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Aids to Teaching

(*Continued from page 584*)

showing, and 97% said that the content could be just as effectively and efficiently presented in some other way. No distinction was made between "line" and "line-segment" and the definition of "adjacent angle" was not clear. The general comment seemed to be that the film definitely had many weak points. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway.)

F. 66—Polygons

Knowledge Builders, 625 Madison Avenue, New York 22, N. Y.

16 mm. film; Black and white; Sound; one reel; twelve minutes; $40.00.

Description: The film opens by showing real life polygons such as an air view; cut stones; honeycombs, which have the hexagon as the favorite design; and snowflakes, which are generally six-sided polygons.

The definition of a polygon is given and the ideas of convex and concave are clarified by animation. Pictures of irregular quadrilaterals, isosceles trapezoids, the rectangle and parallelogram are shown. A figure is then depicted with three sides, it is then changed to four sides, five sides, six sides and on up to twelve sides. By animation it is shown that as the sides are increased the polygon becomes more and more like a circle and that an infinite number of sides forms a circle. The terms "equiangular," "equilateral," and "regular" polygons are defined.

It is shown that in a regular polygon (a six-sided figure is used) the perpendicular bisectors of two sides meet in a point which is the center. The radius of a circumscribed and inscribed circle is explained. A description of the inscribed circle and the circumscribed circle of a regular polygon is given.

The sum of the interior angles theorem and the sum of exterior angles theorem are covered in detail.

Appraisal: There were twenty-five teachers present at the evaluation. Taking the entire film into consideration 90% thought the film was good for providing a common experience, 72% checked it was good to augment explanations, and 60% said it was good to introduce new material, to develop skills, to motivate and to review. 88% of the evaluators checked grades nine through eleven as best suited for the film, while 54% favored grade ten. The film is especially suited to Geometry with four teachers checking Shop Mathematics. Four teachers thought the speed was too fast for the development of ideas and duration of scenes, while one thought the speed was too slow. Four evaluators found inaccuracies in the film. 91% of the teachers said they would use this film in their classroom. The general comment was that the film failed to distinguish between essence and consequence in places. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway)

Correction: I.34—*Slide Rule and Mathematical Tables* should not be ordered from Standard Transformer Corp., Chicago. It may be secured from distributors of electronic parts such as Allied Radio Corp., 833 W. Jackson Blvd. Chicago 7, Illinois for 98 cents.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
135-12 *77th Avenue, Flushing* 67, *N. Y.*

MANY teachers of mathematics believe that a mathematical recreation is a species of its own. In other words, not all mathematical situations may be turned into mathematical recreations. This department proposes to explode this misconception once and for all. Any mathematical situation, if appropriately treated, may become a mathematical recreation. While "exploding" the above misconception, this department will reveal some of the "secrets of the trade" so that any mathematics teacher may become a creator of mathematical recreations.

The program for the creation of mathematical recreations is comparatively simple. . . . Its simplicity is predicated on two relatively important factors. One, a sense for the incongruous. The second, a slight affliction with imaginativeness. These two, if practiced properly, and, if employed timely, should enable any teacher to turn a mathematical situation into an interesting pastime. We shall illustrate the above "propositions" with a few examples.

Consider the following geometric original. *Every vertex of a parallelogram is joined by straight lines with the midpoints of the opposite sides. What portion of the area of the parallelogram is bounded by these straight lines?*

We shall defer the proof of this original for the time being. Suffice it to say that the bounded area will be equal to one-sixth of the area of the parallelogram.

Now, let us consider what facts are here before us. These are tabulated below.

(*a*) A parallelogram and its area.
(*b*) The midpoints of the sides of the parallelogram.
(*c*) Lines joining the vertices of the parallelogram with the midpoints of the opposite sides.

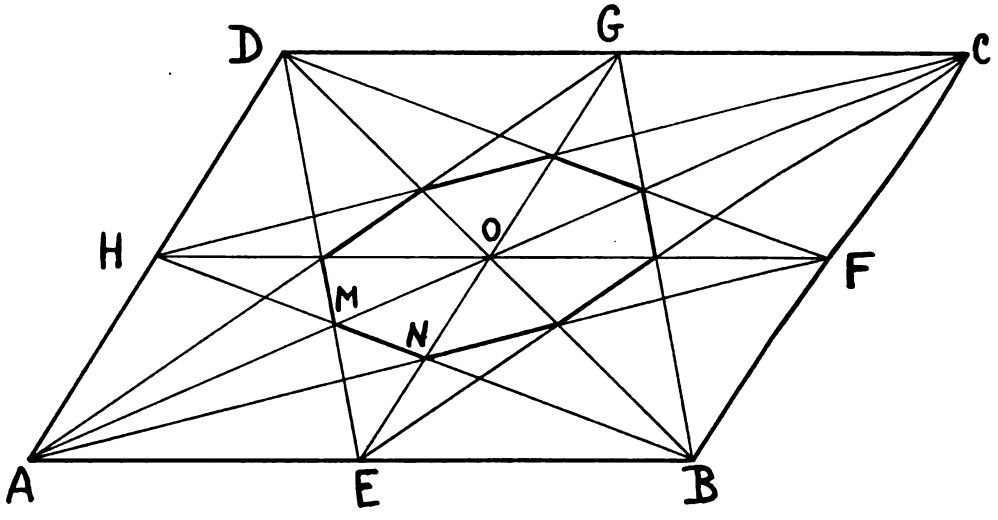
Let us also take into consideration the final result of the original, that is, that the area bounded by the straight lines so drawn is equal to one-sixth of the area of the parallelogram.

Let us restate the above original in a form with is more amenable to the requirements of a puzzling situation. We have a parallelogram. We have the midpoints of the sides. We want to obtain a figure whose area is one-sixth of that of the parallelogram. We will not indicate that the vertices of the parallelogram should be joined with the midpoints of the opposite sides by means of straight lines. But we will impose a definite restriction to the effect that no measurements of any sort are permitted. Thus far we have made only a slight breakaway from the rigid bounds of a geometric original. Now, let us *dress-up* our problem into a more human interest cloak. In other words, let us make up a story and wrap it around the geometric situation. Here is a suggested situation.

Farmer Jones has four sons. He also has a farm whose shape is that of a parallelogram. Farmer Jones decides to divide his farm among his sons so that every one of them will receive an equal acreage. Having reached the ripe age when rest and relaxation are very welcome, Jones plans to subdivide his land before he dies. But knowing how much his supervision of his sons' work is essential in the upkeep of the farm, he feels that he should keep for himself a small portion of the land. He decides to retain one-sixth of the land and that this part of his farm should be centrally located. Now Jones had no use for sur-

veyors. He never had permitted anyone to measure his land. The only markers he ever put on his farm were placed at the corners and at the midpoints of the boundary. How was the division of the land accomplished?

The above restatement of the geometric original may be only one of several other



and probably more effective and intriguing forms of the problem.

As far as the correctness of our assertion that the area of the portion of the parallelogram which is bounded by the straight lines drawn from the vertices of the parallelogram to the midpoints of the opposite sides is equal to one-sixth of the area of the parallelogram this may be established from the following considerations. In triangle ABD the straight lines AO, DE, and BH are medians. Then OM is equal to one third of OA. Now, $ABFH$ is a parallelogram. From this it follows that ON is equal to one half of OE. Compare the area of the triangles AOE and OMN. We have that

$$\text{Area } \triangle OMN = \tfrac{1}{3} \cdot \tfrac{1}{2} \text{ Area } \triangle AOE$$
$$= \tfrac{1}{6} \text{ Area } \triangle AOE.$$

There are eight such triangles in the case of the parallelogram. Thus the area of the inner portion of the parallelogram is equal to one-sixth of the area of the parallelogram.

The readers will note that the inner portion of the parallelogram is an octagon. Furthermore, it is a special kind of an octagon. Its opposite sides are parallel to one another. Dr. Edward Kasner in the book *Mathematics and the Imagination*, pp. 14–16 mentions a polygon on this type and he names it *parhexagon*. There is no reason why we should not call our octagon *paroctagon*. As a matter of fact, any polygon whose opposite sides are parallel to one another is a *parpolygon*. Note that a *parpolygon* must have an even number of sides. Would any one of the readers offer a proof of this statement? A short one, please!

The above problem should offer very few difficulties in a geometry class when the topics on parallelograms and areas are completed. Incidentally, can the readers offer some variations of the above problem in terms of recreational material?

The 30th Annual Meeting of the National Council of Teachers of Mathematics will be held at Des Moines, Iowa, April 16-19, 1952.

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, N. Y.

An Example of the Method of Duplation

NOTE: The following comment is contained in a letter sent by Mr. Hirsch to the editor.

We widh to mention that as far back as the tenth century Rabbi Saadia Gaon, famed philosopher and religious leader[1], made use of the method of duplations (or doubling) to carry out the process of multiplication. One finds in *Scripta Mathematica*, March, 1933,[2] the following description of the method used by the Rabbi to multiply $61+1/3+1/9$ by itself. He proceeds as follows:

$61\times10=610$ $61\times20=1220$
$61\times40=2440$ $61\times60=3360$
$61\times61=3721$

To this he adds the double of 1/3 of 61 and of 1/9 of 61, and approximates the total to 3775.

It will be noted that the multiplication of 61 by itself is performed by the method of duplations. What motivates Rabbi Gaon to add to this product the double of 1/3 of 61 and 1/9 of 61 is not mentioned. The reader will observe that these terms arise in the expansion of $(61+1/3+1/9)^2$ although Newton's statement of the binomial theorem was not yet to be formulated until about 700 years later (1676).[3] If we expand $(61+1/3+1/9)^2$ we have

$$\frac{(61)^2+2\cdot1/3\cdot61+2\cdot1/9\cdot61}{\text{terms in the approximation}}+$$

$$\downarrow$$

$$3775.222$$

$$\frac{(1/3)^2+(1/9)^2+2\cdot1/3\cdot1/9}{\text{terms omitted from the approximation}}$$

$$\downarrow$$

$$+.197=3775.419$$

The Rabbi's approximation of 3775 thus proves to be correct to the nearest integer. It is unlikely that the Rabbi himself was aware of this.

MARTIN HIRSCH
Junior High School 237
Brooklyn, New York

[1] Saadia Gaon (892–942) was head of the academy at Sura in Babylonia at a time when Babylonia in general and the academies at Sura and Pumbedita in particular were the center of Jewish thought. The title "Gaon" meaning excellence was given to the heads of these academies. Rabbi Saadia Gaon, was a scholar of wide achievement and great influence. After his death, the intellectual center of Judaism shifted from Babylonia to Spain. See the *Jewish Encyclopaedia* and also George Sarton's *Introduction to the History of Science*, vol. I,

[2] Harry Polachek, "A Method of Multiplication Used by Saadia Gaon in the 10th Century," *Scripta Mathematica*, Vol. I, (March, 1933) pp. 245–246. The author states that "The mathematical methods used by him (Rabbi Saadia Gaon) are undoubtedly representative of the practices of the leading Jewish and Arabic mathematicians of the time." The computations cited are in connection with the division of inheritances, and beside the problem quoted by Mr. Hirsch, the multiplication of 86 by itself using duplation is given also.

[3] It should be noted here that the binomial theorem was known implicitly for exponents that are positive integers before Isaac Newton made a formula which applied also to exponents that were negative or fractional. In particular, the square of a binomial is stated in geometric form by Euclid in Proposition 4, Book II in these terms:

"If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments."

This in itself does not establish that Rabbi Saadia was acquainted with Euclid's work, but the fact that the Elements was one of the first books translated from the Greek into Arabic and that the translation was made at the request of the Caliph of Bagdad, would make it seem more than possible that a Hebrew scholar living in Babylonia would have been aware of this classic. That, however, is pure conjecture.

BOOK SECTION

Edited by JOSEPH J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

ELEMENTARY SCHOOL

Numbers in Action, Teachers Edition, by Maurice L. Hartung, Henry Van Engen, and Catharine Mahoney. Cloth, 144 pages+Instructions to the Teacher, 1951. Scott, Foresman and Company, 433 E. Erie Street, Chicago 11, Illinois.

COLLEGE

1. 1st Year Mathematics

College Mathematics, by Charles E. Clark, Emory University. Cloth, v+331 pages, + tables, 1950. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N.Y. $3.85.

2. Trigonometry

Plane Trigonometry, with Tables, by Lyman M. Kells, Willis F. Kern, and James R. Bland, all at the United States Naval Academy. Cloth, xi+220 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N.Y. $3.50.

3. Statistics

Statistical Methodology Reviews, 1941–1950, Edited by Oscar Krisen Buros, Rutgers University. Cloth, x+457 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N.Y. $7.00.

4. Special Courses in Mathematics

Mathematics for Engineers, 3rd Edition, by Raymond W. Dull and Richard Dull. Cloth, xix+822 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N.Y. $7.50.

Mathematics of Finance, by Albert E. May, University of Wisconsin. Cloth, viii+264 pages, 1951. American Book Company, 55 Fifth Avenue, New York 3, N.Y. $3.00.

5. Advanced Mathematics

The Albegra of Vectors and Matrices, by Thomas L. Wade, Florida State University. Cloth, ix+189 pages, 1951. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $4.50.

TEACHING OF MATHEMATICS

Developing Meaningful Practices in Arithmetic, A Third Report by the Committee on Flexibility, Central New York School Study Council. Paper, xii+123 pages, 1951. Central New York School Study Council, 219 Slocum Hall, College Place, Syracuse 10, N.Y. $2.00.

Vitalizing Secondary Education, The First Commission on Life Adjustment Education for Youth. Paper, 106 pages, 1951. Federal Security Agency, Office of Education, Bulletin No. 3. Available through the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.30.

MISCELLANEOUS

Anthology in Educology, by Lowry W. Harding, Ohio State University. Paper, xv+78 pages, 1951. Wm. C. Brown Company, Dubuque, Iowa. $1.50.

How to Study—How to Solve, by H. M. Dadourian, Trinity College. Paper, vi+121 pages, 1951. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $.60.

REVIEWS

Arithmetic 8: The World of Numbers, Dale Carpenter and George F. Drake, Jr. New York, The Macmillan Company, 1950. 372 pp., $1.68.

The outstanding physical characteristic of this book is the excellent use of bold type—superior in size and spacing.

The simplicity of the wording of the table of contents and index make them satisfactory guides for both student and teacher.

The topics developed in this text are "Whole Numbers and Common Fractions," "Decimal Fractions," "Percentage," "Measurements," "Using Geometry," "Mensuration," "Using Your Income Wisely," "Arithmetic in the Home," and "Using Algebra."

Sections on "Factors" and "Rounding Off" are especially clearly treated by the author. The stress on vocabulary is a noteworthy feature of this book. In addition to this it contains other desirable aids to learning such as practice exercises, remedial tests, and tables of measures. Magic squares are included as enrichment material.

While this book gives an understanding of how number ideas work and stresses meaning, it does not, however, provide ample opportunity for developing speed and skill through many practice exercises.

The authors could have dispensed with Chapter IX, "Using Algebra," and could have

utilized this space more advantageously for more work in arithmetic.

I would recommend the use of this book in a slow eighth grade class.—GERTRUDE HOLZAPFEL, Hamilton Junior High School, Baltimore, Maryland.

First Algebra, Virgil S. Mallory. Chicago, Benj. H. Sanborn and Co., 1950. vii+444 pp., $2.24.

This textbook for ninth year algebra classes is not a revision of Mr. Mallory's earlier texts but, in his words, is "the culmination of many years of experience in teaching high school students, of consultations with teachers throughout the United States and in the observation of the teaching of algebra in many high school classes." It is written in clear concise language with new topics developed with the concepts which the pupil understands.

Each type of difficulty is given in boxed examples with full explanations followed by "Test Yourself" exercises with the answers given. In fact, the text seems to be as nearly self-teaching as it is possible to achieve.

A few pages of story problems are given in practically every chapter. These are about everyday life situations that are familiar to the ninth year pupil which leads to easy understanding of equations and formulas.

Most of the exercises are divided into three levels of difficulty: one covers the minimum requirements; the second demands somewhat greater skill; the starred problems challenge the student who is capable of doing superior work. Exercises of various types which are amply supplied make work books and supplementary problems unnecessary. The eight pages of remedial practice in arithmetic at the end of the book as well as diagnostic tests throughout the book encourages continued proficiency in arithmetic.

Cumulative and chapter reviews, keyed practice material and chapter tests furnish invaluable aids for the busy teacher. Numerous historical notes, accounts of modern mathematical applications and illustrations of topics of interest are included.

The combination of good format and large type make easy ninth grade reading.—ELIZABETH VAN LIEW, Mooresville High School, Mooresville, Indiana.

Plane Trigonometry, Alternate Edition. E. Richard Heineman. New York, McGraw-Hill, 1950. xiv+184 pp. and tables. $2.50.

Professor Heineman has written another fine textbook on trigonometry—especially for those students whose mathematical background is meager. The principal objective of the author has been teachability. The reviewer finds that the textbook bears testimony to the author's conscientious observance of this goal.

The contents are improved by the new rule for finding the characteristic of a logarithm, and a worthwhile note to the students. The subject matter of trigonometry is well covered and topics that are omitted in the textbook are those which are usually omitted in a high school course, anyway, due to lack of time.

Answers are given to all problems except those whose numbers are multiples of four. For the convenience of teachers, brief outlines for two-semester-hour and three-semester-hour courses are given.—ELBERT C. HUBBARD, Parks College, East St. Louis, Illinois.

College Algebra, Herman K. Fulmer and Walter Reynolds. Boston, Ginn and Company, 1951. vi+218 pp., $2.85.

The authors of this text have written a really brief college algebra. They have condensed into less than two hundred pages all the topics usually found in books twice the length except permutations, combinations and probability which have been omitted.

No attempt is made to prove or even to rationalize any of the operations discussed in the first six chapters. In the remaining chapters most proofs are indicated but the authors make liberal use of the rules illustrated by examples.

The chapter on Theory of Equations contains elementary topics through Horner's Method and Cramer's Rule is applied to systems of three equations in three unknowns only.

There are nearly fourteen hundred exercises seemingly well selected and well graded. The answers to all of the exercises are given in the back of the book. A four place logarithm table is included. The print is large and clear and the general appearance is attractive.—FRANK C. GENTRY, Arizona State College, Tempe, Arizona.

Lecture on Classical Differential Geometry, D. J. Struik. Cambridge, Addison-Wesley Press, Inc., 1950. viii+221 pp., $6.00.

This is an excellent account of the classical theory of differential geometry. The book is an outgrowth of a course which the author teaches at the Massachusetts Institute of Technology and is designed as a textbook, with an ample supply of problems. There is also a large collection of diagrams and photographs of models.

Anyone who is thoroughly grounded in advance calculus has adequate preparation to read the book; a knowledge of the rudiments of vector analysis would be highly desirable, but it can probably be picked up from the text.

The subject of the book is the application of calculus to the study of geometry of curves and surfaces. The presentation differs from that of many textbooks in English in that vector methods and vector notation are used throughout; the reader who has studied differential geometry without the use of vectors will be startled by the simplification that results.

The author has included numerous thumbnail biographies of the men who have contributed to the development of the subject.—M. P. GAFFNEY, Northwestern University, Evanston, Illinois.

The Arithmetic Theory of Quadratic Forms (Carus Monograph Number 10), Burton W. Jones. Buffalo, The Mathematical Association of America, 1950. x+212 pp., $3.00.

The author presupposes knowledge of the fundamentals of matrix theory and elementary number theory. Otherwise, the book gives a self-contained presentation of the central ideas of the arithmetic theory of quadratic forms. It begins with the classical theory in the field of reals from the point of view of representation theory, and successively narrows the fields and rings of coefficients until the classical problems in the ring of rational integers can be dealt with. Results for binary and ternary forms are then obtained as special cases of the preceding general theory. Use of the concepts of p-adic numbers and ideals in quadratic fields adds elegance to the presentation.

A comprehensive set of computational and theoretical problems is included after the bibliography. The general purpose of these problems seems to be clarification rather than extension of the theory.—T. C. Holyoke, Northwestern University, Evanston, Illinois.

Coefficient Regions for Schlicht Functions, A. C. Schaeffer and D. C. Spencer. New York, American Mathematical Society, 1950. xv+311 pp., $6.00.

This monograph is Volume 35 in the Colloquium Publications series and is a presentation of the results which the authors have obtained in the investigation of the family of extremal schlicht functions in the large. Most of the results are new. An additional chapter entitled "The region of values of the derivative of a schlicht function" by Arthur Grad has been included in the book.

A function $f(z)$ is said to be schlicht in a domain if, for any two points z_1 and z_2, $f(z_1) = f(z_2)$ only if $z_1 = z_2$. The investigation is concerned with the class of functions $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ which are regular and schlicht in the unit circle $|z| < 1$. More specifically, the problem is to determine the possible values of $a_2, a_3, \cdots$.

Necessary background for reading this monograph is a knowledge of the material contained in a standard course on functions of a complex variable.—D. H. Potts, Northwestern University, Evanston, Illinois.

Methods and Materials for Teaching General and Physical Science, John S. Richardson and G. P. Cahoon. New York, McGraw-Hill Book Company, 1951. viii+485 pp., $4.50.

This book is a companion volume to Miller and Blayd's in the biological sciences. The treatment is concerned almost entirely with student activities as related to apparatus and materials in physics, chemistry, and general science. For science teachers, the detailed description of projects, of classroom demonstrations, and of individual laboratory experiments, make the book a veritable mine of source materials careful explanations of classroom method enable the teacher to attain, to a much gr extent than otherwise, the author's objecti this type of teaching, namely an experie meaningful learning, motivated and adapt the various individual abilities, to create cal thinking, as well as initiative, resour ness, and cooperation. For the beginning te the book is an almost indispensable guid teachers of mathematics who are interest laboratory procedures, the book suggests opens many avenues of approach. The ch on teaching for thinking through laborator periences will be of special significance to m matics teachers. The lack of applied m matics (there is very little of it) in expla most of the physical properties, and the r of laboratory experiments, will appear as fect to most teachers of mathematics. This be explained in part by the author's major cern for general education rather than fo lege preparatory education.

Part I of the text treats: planning the u apparatus, using demonstrations, using stu projects and field trips, providing and visual materials, and providing and ca equipment. Part II gives specific demon tions, laboratory procedures, and project general science, physics and chemistry cla

The format of the book is excellent. print, the drawings, the paper, all mak book easy to use. There is a brief but pert bibliography, and an excellent index.—Ho F. Fehr, Columbia University, New York

Differential Algebra, Joseph Fels Ritt. York, American Mathematical So 1950. v+184 pp., $4.40.

Professor Ritt's book is volume 33 in Colloquium Publications series of the Ame Mathematical Society. The subject consid is the study of the algebraic structure o theory of certain types of differential equat namely those which are polynomials in th known function and its derivatives, the c cients being from a certain type of alge field. The present work goes back to the aut earlier *Differential Equations from the Alge Standpoint* (Colloquium Publications, Vol 1932). In the ensuing years the author an associates have advanced the subject to su degree that the present work must be rega as more than a revision and enlargement o earlier publication.

The problems considered are those of classical theory of differential equations, the abstract method employed serves to sepa the algebraic from the analytic and, as abst methods usually do, serves to give the subj coherence and unity which the classical met cannot produce.

As to necessary background required, exposition should be within the reach of a

(*Continued on page 598*)

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ture person skilled in the reading of mathematical notation provided that he has some knowledge of classical algebra, differential equations, and a smattering of abstract algebra.—D. H. Potts, Northwestern University, Evanston, Illinois.

The Theory of Algebraic Numbers, Harry Pollard. New York, John Wiley and Sons, Inc., 1950. xii+143 pp., $3.00.

This volume is number nine in the series of Carus Mathematical Monographs. These monographs are of expository nature and are designed to be comprehensible to "thoughtful people having a moderate acquaintance with elementary mathematics." This particular monograph fits the bill rather well, as much of it could be digested by a person who has had a second year of algebra, although there could be some notational difficulties.

The subject of the book is indicated by the title. More specifically, the material presented constitutes the elementary parts of classical algebraic number theory. Briefly an algebraic number is a root of a polynomial equation with rational coefficients. (The concept can be extended to equations with a more general type of coefficients.) It is proved that there exist non-algebraic numbers (i.e. transcendental numbers). The book not only introduces a subject which is fascinating in itself but also would serve as a fine background for courses in modern algebra.—D. H. Potts, Northwestern University, Evanston, Illinois.

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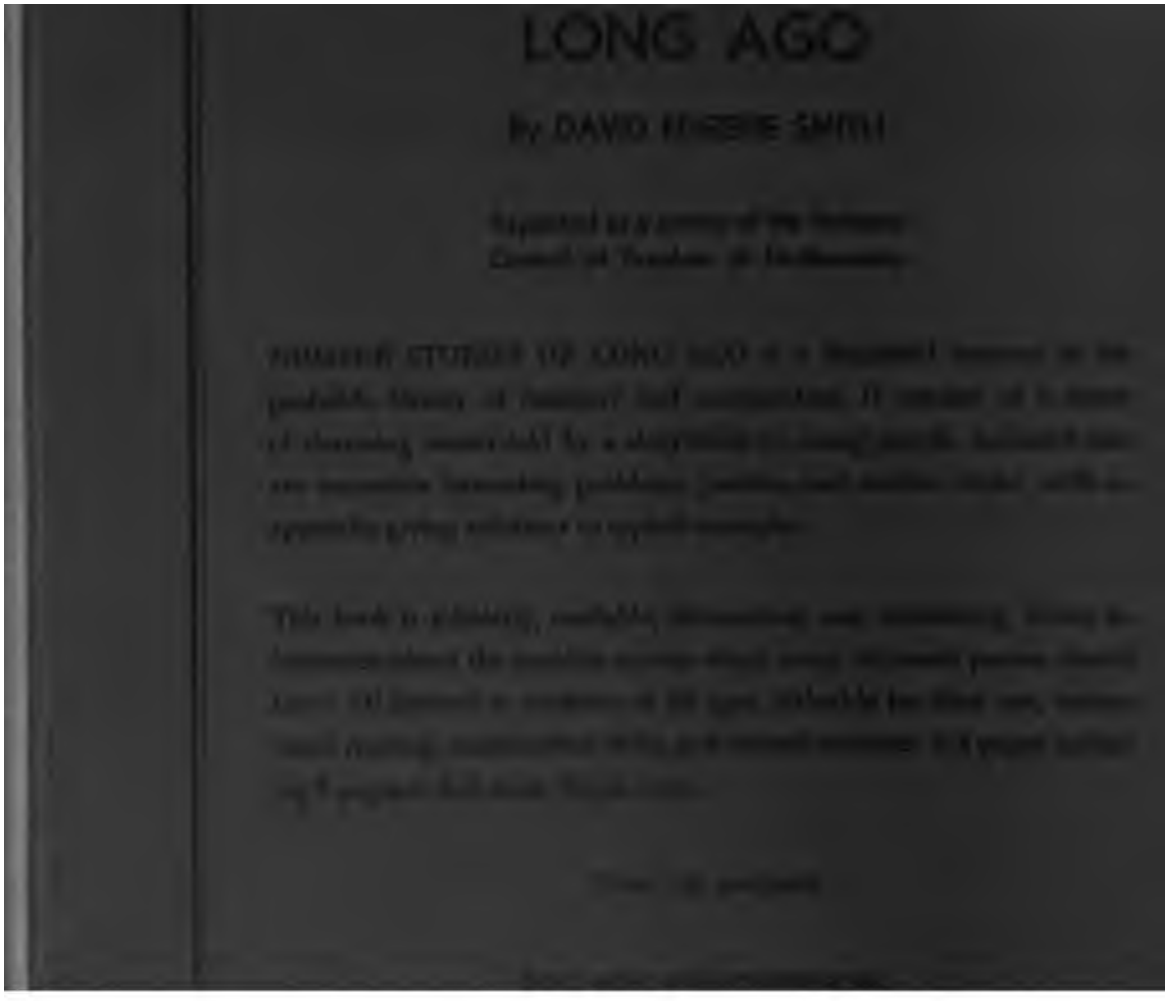
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